Computing dynamical curlicues

A curlicue $\Gamma = \Gamma(u)$, where $u = (u_n)_{n=0}^{\infty} \subset \mathbb{R}$, is a piece-wise linear curve in $\mathbb{C}$ passing consecutively through the points $z_0 = 0 \in \mathbb{C}$, and z_1, z_2, ..., where

$$z_n = \sum_{k=0}^{n-1} \exp(2\pi\imath u_k), \quad n = 1, 2, ...$$

A curlicue can be obtained from an arbitrary sequence $(u_n)_{n=0}^{\infty}$ of real numbers. However, when this sequence is given by iterates of some dynamical system $F : X \to X$ at a given point $x_0 \in X$, where $X \subset \mathbb{R}$, we can speak about dynamically generated curlicues.

This dataset contains source codes of the Matlab functions Rotation.m, Arnold.m and Sequence.m which can be used to plot the first N points of a curlicue generated, respectively, by rotation on the circle by $2\pi\varrho$ angle, the Arnold circle map (with different parameters) and the sequence $u_n = n\log(n)$. Additionally, these functions allow us to calculate other properties of a curlicue, such as corresponding Birkhoff average and diameter of a curlicue. Description of the functions and variables involved is provided as comments in m-files. It's worth pointing out that the function Sequence.m can be easily modified to compute and draw a curlicue generated by an arbitrary sequence u_n given by explicit formula. We also include txt-files with exemplary data obtained by these functions and four figures (eps-files) generated by a function Sequence.m for, respectively, $u_n = n\log(n)$ (Figure1.eps, $N = 4000$), $u_n = \pi n^2$ (Figure2.eps, $N = 4000$), $u_n = n^{2/5}$ (Figure3.eps, $N = 3000$) and $u_n = \sqrt{2}n^2$ (Figure4.eps, $N = 2000$).

More details on dynamically generated curlicues, especially obtained via circle homeomorphisms, can be found in pre-print paper J. Signerska-Rynkowska, *Curlicues generated by circle homeomorphisms*, arXiv:1909.09892 [math.DS] which makes use of the abovementioned programs and data.