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Łukasz Brzezicki

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Łukasz Brzezicki
Ustka City Hall, Wyszyńskiego 3, 76-270 Ustka, Poland
corresponding author: brzezicki.lukasz@wp.pl

Origin and Evolution of Malmquist Productivity Index: Review of the Literature

Abstract
The aim of this review was to present research work on the development of the Malmquist index (MI) in various aspects based on the Data Envelopment Analysis (DEA) method. The MI is one of the three methods of dynamic analysis and also the main one, as it is the most commonly used to measure productivity changes. However, over the years, from being a simple productivity index like the models within the DEA method, it has become a more complex and useful index as it has seen many developments and extensions. Modifications and developments of the MI have taken place in many directions. A review of MI studies from 1990 to 2025 shows that the authors considered various aspects of the MI: Alternative ways of decomposing, different ways of measuring, different types of efficiency, different nature of data and variables and network structure or combined dynamic analysis methods into a single framework. The direction of changes and modifications of the MI can be divided into three groups: Adapting DEA solutions within the MI, presenting new MI solutions without being inspired by the DEA method and combining two methods of dynamic analysis into one framework.

Keywords
Malmquist index | productivity | productivity index | review | DEA

JEL Codes
C43, O47, C83

1. Introduction

Although efficiency and productivity are the major categories in economics and management assessing activities in different areas, their analytical analysis of quantitative methods in a dimensional situation was only presented in the 1980s–1990s. A review of the research conducted using frontier methods to estimate efficiency and productivity (Daraio et al., 2020) shows that the most commonly used method was the non-parametric Data Envelopment Analysis (DEA) (Charnes et al., 1978) and the Stochastic Frontier Approach (SFA)(Aigner et al., 1977; Meeusen & van Den Broeck, 1977), although to a lesser extent. As the original Malmquist index (MI) is based on the DEA methodology, as shown later, it is therefore the DEA that is focused on in this paper.

Charnes et al. (1978) proposed the DEA method for estimating the relative technical efficiency in a situation with multiple inputs and outputs. In the DEA method, the authors used the concepts of the distance function presented by Shephard (1953) and the efficiency measurement framework, including the division of overall efficiency into allocative and technical efficiency proposed by Farrell (1957). Technical efficiency is related to the technological production capacity achieved by an entrepreneur, excluding factors related to costs or prices (Brzezicki, 2017). Measuring technical efficiency can be done in two ways by adopting an appropriate orientation in the DEA model. Input orientation (minimising inputs) indicates how much current inputs can be reduced to make the unit under study efficient. Conversely, an orientation to outputs (maximising outputs) provides an indication of how much outputs can be maximised with given inputs to make the unit efficient. Technical efficiency is the inverse of Shephard's (1953) distance function.

The analysis of technical efficiency by means of the DEA method comes down to the solution of the objective function under given constraining conditions by means of linear programming, which makes it possible to determine the efficiency frontier (best production practice frontier). The identification of the efficiency of the unit under investigation takes place in relation to the efficiency frontier (Brzezicki, 2017). The efficiency ratios obtained by means of the DEA method are in the range 0–1, where one indicates a 100% efficient entity, which is on the frontier of efficiency. In other cases, we are dealing with the inefficiency of the economic entity (Brzezicki, 2017). Often, the technical efficiency calculated using DEA is referred to as relative efficiency, as its value is determined from other units in the sample and not, as in parametric methods such as SFA of an independent production benchmark. Charnes et al. (1978) estimated efficiency measures using linear programming and proposed a practical way to measure technical efficiency. Charnes et al. (1978) presented the first model, called CCR after the initials of their names, assuming constant returns to scale (CRS). Later, Banker et al. (1984) presented the second BCC model, which assumed variable returns to scale (VRS). However, the proposed DEA models (Charnes et al., 1978; Banker et al., 1984) only allowed evaluation over a single period. A solution was sought to analyse dynamically and measure productivity changes between two periods. It should be noted that when measuring efficiency over several years using the DEA method, efficiency is measured relatively on a frontier basis in each period separately. Even when efficiency increases from period to period, absolute productivity may not increase due to a regression of the frontier between the periods under study. The MI can be used to capture this change in the frontier.

Caves et al. (1982) presented the MI, building on the work of Malmquist (1953) who proposed constructing quantitative indices as ratios of distance functions. Their approach requires one to take the direction of measuring either output maximisation or input minimisation. Caves et al. (1982) defined the input-based MI as the ratio of two input distance functions. Subsequently, Färe et al. (1989a, 1990, 1992, 1994a) formulated a model of the MI corresponding to the geometric mean of two time-adjacent indices, thus extending the original Caves et al. (1982) index construction. The MI provides information on how much the ratio of aggregate output to aggregate input has changed between two periods. Färe et al. (1992) presented the input-based MI and output-based MI (1994a). The output-based MI is defined as

$$M_0(x^{t+1}, y^{t+1}, x^t, y^t) = \left[\frac{D_o^t(x^{t+1},\ y^{t+1})}{D_o^t(x^t,\ y^t)} \frac{D_o^{t+1}(x^{t+1},\ y^{t+1})}{D_o^{t+1}(x^t,\ y^t)}\right]^{1/2} \quad (1)$$

Where y^t, x^t are outputs and inputs in period t, while y^{t+1}, x^{t+1} are outputs and inputs in period $t+1$. The calculation of the MI thus defined consists of solving four linear programming problems. Two of linear programming problems relate to measurements over the same period ($D_o^t(x^t, y^t)$ and $D_o^{t+1}(x^{t+1}, y^{t+1})$, and the other two relate to intertemporal measurements ($D_o^t(x^{t+1}, y^{t+1})$ and $D_o^{t+1}(x^t, y^t)$. One of these is the output distance function $D_o^t(x^t, y^t)$ in period t which is calculated as the solution linear programming problem, but Färe et al. (1994a, p. 259) indicates that "this problem can be rewritten to reveal its relation to the traditional Farrell output-oriented measure of technical efficiency as well as the standard DEA models". Most significantly, Färe et al. (1989a, 1990, 1992, 1994a) presented a practical way of calculating the MI by means of linear programming problems also used within the DEA method. The model presented by Färe et al. (1994a) for calculating the $D_o^t(x^t, y^t)$ shows similarity to the radial output-oriented DEA (CCR) model, with two changes in terms of the objective function and the adopted returns to scale (Färe et al., 1994a). This was only due to some of the problems of measuring MI productivity mentioned by Grosskopf (2003), which are presented later in this article. However, this does not change the fact that the MI can be calculated using any DEA model and transformed into linear programming problems, provided that the same DEA model is used to calculate the four elements of the MI equation.

If the MI value is one (MI=1), productivity between the two periods has not changed, it has remained the same. However, when the MI is higher than one (MI>1), then there has been an increase in the productivity of the economic unit under study. Conversely, when the value is less than one (MI<1), it is interpreted that there has been a decrease in productivity.

A review of the literature on productivity studies shows that the work of the Färe team is the most frequently cited in the literature (Zelenyuk, 2023). It should be noted that the MI is one of three dynamic analysis tools in addition to Dynamic DEA and Window DEA (Peykani et al., 2021). While reviews can be found in the literature regarding Dynamic DEA (Mariz et al., 2018) and Window DEA (Peykani et al., 2021), no such review is available regarding the

MI. Although, attempts to do so can be found in the work of Färe et al. (1998, 2008). In the reviews by Färe et al. (1998, 2008), the focus was almost exclusively on the analysis of technical efficiency and the several ways in which the MI can be decomposed. It was only mentioned that the MI can be used to measure other types of efficiency. Previous reviews have not considered modifications of the MI in terms of (1) dealing with the measurement problem in a situation of VRS, (2) using a more complex production structure, both network and dynamic, (3) taking into account the different nature of data and variables, and (4) other solutions related to the MI. The narrow scope of analysis due to the small number of new proposals for modifications of the MI in various aspects have appeared since the publication of the Färe et al. (1998, 2008) paper, indicating a research gap to be addressed in this literature review.

The aim of this review was to present research work on the development of the MI in various aspects based on the DEA method. The research problem was to test whether the modifications and extensions of MI currently available in the literature indicate a more complex and comprehensive approach to productivity measurement than its prototype presented by Färe et al. (1989a, 1990, 1992, 1994a, 1994b). It is hypothesised that currently available MI solutions significantly outperform the simple productivity index presented by Färe et al. (1989a, 1990, 1992, 1994a, 1994b). The author hopes that this review will serve two functions. On the one hand, it will allow researchers involved in empirical research to more effectively select an appropriate modification or extension of the MI, better suited to their research. In addition, it should raise awareness among researchers wishing to apply the MI for the first time that there are various extensions and modifications of it, and not only the classical MI widely described in the literature. On the other hand, researchers focusing more on theoretical and methodological solutions within the MI should find research gaps and present other modifications of the MI in the future. Based on the above assumptions, the following research questions were formulated:

> Q1: What are the origins of the MI and why is it used in research?
>
> Q2: What are the types of MI decomposition?
>
> Q3: What types of efficiency can the MI be used for?
>
> Q4: What modifications of the MI are available in the literature?
>
> Q5: What types of variables and data can be used within the MI?

2. Research Methodology

It was decided that the literature review would be conducted in two consecutive stages. In the first stage, publications on the MI that the author of this article had collected during his research work of several years and during his work on other review articles related to the DEA method were analysed. These publications are the primary source of this review. The research collected over the years has brought together a large number of them, not only from international databases, but also publications from non-indexed journals with a local focus and often containing interesting scientific research on the MI. In the second stage, the collection of publications gathered in the first stage was expanded by performing searches in databases and websites. Before proceeding with the systematic literature review in the second stage, it is first necessary to define the research assumptions (Table 1) concerning the criteria for accepting or rejecting publications for review, the study period and how to search publications for review.

It was decided that the review would include studies written in English, as well as MI analyses based on the DEA method. However, publications belonging to the so-called 'grey literature' area, i.e., technical reports, doctoral theses, conference presentations, unofficial documents and working papers, will be excluded. Application (empirical) publications that do not present new (theoretical assumptions) modifications and developments of the MI were also excluded from this review.

This review will present studies published between 1990 and the beginning of 2025. The initial period adopted is due to the fact that a publication presenting the MI calculated using linear programming was presented for the first time. Grosskopf (2003, p. 460) indicates that the "first paper about Malmquist index was sent in 1989 (Färe, et al., 1989a) to Economic Journal, where it was promptly rejected. It ultimately appeared in 1994 in book (Färe, et al., 1994a). In that paper we imposed nonincreasing returns to scale (NIRS) on the reference technologies - we had originally used a variable return to scale technology but encountered repeated problems with infeasibility with the mixed period problems. The least restrictive technology that solved that problem was the NIRS".

Table 1. Assumptions of the Review

Periods	1990–2025
Database	Scopus, Web of Science Core Collection, Semantic Scholar, ResearchGate.com, Academia.com, Google Scholar, search engine Google
Inclusion criteria	- Papers (articles and chapters) which were written in English - Publication presenting new (theoretical assumptions) modifications and developments of the MI based on DEA methodology
Exclusion criteria	- Technical reports, doctoral theses, conference presentations, unofficial documents, working papers – “grey literature”, books - Application (empirical) articles that do not present new (theoretical assumptions) modifications and developments of the MI
Search term used (keywords)	(1): Malmquist index OR Malmquist Productivity Index OR Productivity Index AND (2): decomposition OR constant returns to scale OR variable returns to scale OR radial OR non-radial OR network OR two-stage OR dynamic OR window OR cost OR profit OR revenue OR allocation OR negative data OR undesirable outputs OR interval data OR fuzzy data OR non-discretionary variable OR boostrap OR stochastic OR double frontiers OR weights OR distance function

However, the Färe team, in order to present their research more quickly to the scientific world than only as a chapter in a book (Färe et al., 1994a), decided to publish information on the MI in a scientific article in 1990 (Färe et al., 1990)[1].

The searches were conducted in international databases of peer-reviewed publications: Scopus and the Web of Science Core Collection. However, not all journals are indexed in these databases, so it was decided to use online services for researchers: Semantic Scholar, ResearchGate.com, Academia.com and Google Scholar. In addition, the search engine Google was used. Searches for publications were performed using keywords consisting of two groups. The first group of keywords referred to the MI name and related words. The second group of keywords, in contrast, identified various MI assumptions and potential modifications found in DEA models. It was also decided to search for publications in databases and websites using citations to other publications and links between thematically similar articles (snowball effect). It was assumed that new developments and modifications of the MI should refer to previous publications. In addition, publications with the characteristics of a literature review on MI were discerned (e.g., Färe et al., 1998; Färe et al., 2008; Balk, 2001; Balk et al. 2020; Lovell, 2003) in order to gather a database of source publications (snowball effect), as well as to obtain information on the state of knowledge of MI modifications and extensions.

It was decided that the review would present various modifications and extensions of the MI in order to illustrate the full spectrum of capabilities of the index that were not available when it was first presented in 1990. It was decided to focus on modifications to the MI in terms of return to scale, decomposition of the MI, the way productivity was measured, consideration of the specifics of different data and variables, as well as the estimation of different efficiency and network linkages within the MI and other issues. The complete summary of the publications collected during the systematic literature review, together with their classification into specific thematic groups of MI developments or modifications were presented in Brzezicki (2025).

3. Results of Review Studies

The beginning of the practical calculation of the MI is attributed to Färe et al. (1989a, 1990, 1992, 1994a). However, as Althin (2001, p. 107) points out: “They

1 Førsund (2016, p. 141) indicates: “The history of the DEA-based Malmquist productivity index is presented in Färe et al. (1998), Grosskopf (2003) and Färe et al. (2008). The first working paper that established an estimation procedure based on DEA was published in 1989, was presented at a conference in Austin in the same year, and appeared as Färe et al. (1994a); a book chapter in a volume containing many of the conference presentations. The first journal publication appeared as Färe et al. (1990) with an application to electricity distribution. (However, this paper is not referred to in the 2003 and 2008 reviews and neither in Färe et al. (1992), although the methodological approach in the latter is the same)”.

calculated an adjacent Malmquist productivity index consisting of the geometric mean of two Malmquist indexes as defined by Caves, et al. (1982). Later, Berg, et al. (1992), introduced a base period Malmquist productivity index, which except for the fixed base period technology is the same as one of the indexes defined by Caves, et al. (1982)". In the MI presented by Färe et al. (1989a, 1990, 1992, 1994a), the productivity between two periods is determined sequentially for the following two survey periods (t →t+1, t+1→t+2, etc). In contrast, the MI proposed by Berg et al. (1992) measures productivity between two periods but always compares the subsequent survey period to a predefined one, usually the first period (t→t+1, t→t+2). Pastor and Lovell (2007, p. 591) presented the circularity of the MI, pointing out that it is a response to perceived problems: "The geometric mean version of the Malmquist productivity index does not satisfy the circular test, and its component adjacent period indexes can give different productivity change measures for the same data. A fixed-base version of the index solves both problems, but it is not independent of the base period".

The scientific debate on the MI heated up after the publication of the paper by Färe et al. (1994b). The authors decomposed the MI into three components in addition to the use of VRS. This was in contrast to an earlier paper by Färe et al. (1989a, 1990, 1992, 1994a), which used CRS and decomposed into two components, i.e., change in efficiency (catch-up effect) and technical change or equivalently change in the frontier technology (frontier shift effects). Färe et al. (1994b) decomposed the MI into technical change, pure efficiency change, and scale change. Thereafter, many authors (Ray & Desli, 1997, Färe et al., 1997, Balk, 2001, Lovell, 2003, Balk et al., 2020) presented other alternative decompositions of the MI. It is worth noting that they differ not only in the number of MI components decomposed, but also in the extent of the definition of the individual components, as well as in the way they are measured. An example of this is the decompositions of Färe et al. (1994b) and Ray and Desli (1997), which decompose into three similarly named components but differ in their definition and method of measurement (Zofio, 2007). The original decomposition of the MI into two components (Färe et al., 1994a), i.e., frontier shift and catch-up effect, is fully accepted in the literature (Lovell, 2003). Other decompositions of the MI have not gained as much popularity due to the imperfect factor of production scale in a model that uses VRS to measure the MI.

Grifell-Tatjé and Lovell (1995) indicated that the MI may incorrectly measure productivity changes if a model with VRS is used for calculations. The literature (Pastor, Lovell, 2005) indicates that when calculating the MI and its decomposition using linear programming, a situation of suboptimal solutions may occur, and this is particularly noticeable for a model with VRS (Cooper, et al., 2007; Tone & Tsutsui, 2017c). In response to the measurement problems noted, several modifications of MI have been proposed (Figure 1) by which productivity can be estimated, taking into account VRS.

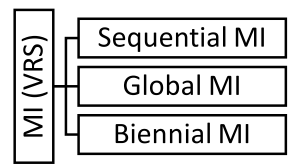

Figure 1. MI under VRS

Shestalova (2003) proposed the sequential MI, Pastor and Lovell (2005) proposed the global MI, and Pastor et al. (2011) proposed the biennial MI. Brzezicki and Łącka indicated (2022, p. 87) that "It should be noted, however, that sequential Malmquist (Shestalova, 2003) ignores the natural technological regression during the study and, therefore, does not indicate negative technological changes. In the case of global Malmquist (Pastor & Lovell, 2005), it is difficult to constantly add a new period to the survey, which entails the need to calculate the index multiple times. The MI biennial does not pose such problems (Pastor et al., 2011)".

Originally, radial DEA models were used to measure the MI (Färe et al., 1989a, 1990, 1992, 1994a), then Grifell-Tatjé et al. (1998) and Chen (2003) proposed the use of a non-radial DEA model to estimate MI productivity. Färe and Lovell (1978) presented non-radial efficiency (also called Russell efficiency), which has begun to be used to estimate efficiency in DEA models. In contrast to radial models (e.g. CCR and BCC), which proportionally reduce all inputs (input orientation) or proportionally increase all outputs (output orientation), non-radial models allow efficiency to be estimated at different levels of individual inputs or outputs. It is worth noting that Førsund (1998) provided a critique of Grifell-Tatjé et al. (1998) proposed quasi - MI, pointing out that "quasi-distance function lacks the fundamental requirement of output homogeneity and that slack in output or input constraints (for DEA models) is irrelevant for productivity measurement"

Table 2. Definitions of Different Types of Efficiency (Apart From Technical Efficiency)

Name of efficiency	Definition
Cost efficiency (CE)	"The cost efficiency of a producer using input vector x to produce output vector y when input prices are w is measured by the ratio of minimum cost to actual cost".
Allocative efficiency (AE)	"The input allocative efficiency of a producer using input vector x to produce output vector y when input prices are w is measured by the ratio of cost efficiency to input technical efficiency". AE=CE/TE "The output allocative efficiency of a producer producing output vector y with input vector x when output prices are p is measured by the ratio of revenue efficiency to output technical efficiency". AE= RE/TE
Revenue efficiency (RE)	"The revenue efficiency of a producer producing output vector y with input vector x when output prices are p is measured by the ratio of maximum revenue to actual revenue".
Profit efficiency (PE)	"The profit efficiency of a producer facing input prices w and output prices p is measured by the ratio of maximum profit to actual profit".

Source: Lovell et al. (1994, pp. 182–183)

(Fukuyama & Weber, 2001, p. 135). In contrast, Tone (2004) used the non-radial SBM model (Tone, 2001) to estimate the MI. Previously Chung et al. (1997) presented the Malmquist-Luenberger index, which integrated MI with directional distance function (DDF) (Chambers et al., 1996) and the concept of undesirable outputs (Färe et al., 1989b). In standard radial and non-radial DEA models, the efficiency measure is determined by the assumed orientation of the DEA model. In the DDF model, the measurement is made on the basis of the directional vector. It can be arbitrarily specified, thus determining the direction of measurement. In contrast, Zofío and Lovell (2001) proposed the use of the hyperbolic distance function (Färe et al., 1985) to calculate the MI. The hyperbolic approach involves reducing all inputs and increasing all outputs by the same proportion.

The MI was initially presented as an indicator for measuring productivity changes in technical efficiency, but it should be remembered that there are also other types of efficiency that can be calculated using the DEA method (see: Chapter 8 in Cooper, et al., 2007), and thus it is also possible to measure productivity changes in this efficiency using the MI (Table 2). The latest developments in the measurement of different types of efficiency and their decomposition can be found in the Pastor et al.'s work (2022).

Several papers can be found in the literature outlining the use of the MI to measure different types of efficiency (Figure 2). Originally, the MI was used to measure productivity changes in technical efficiency, but in the work of Färe & Grosskopf (1992), as well as the comment by Balk (1993), indicated that it could also be used to measure productivity changes concerning other types of efficiency. Proposals for measuring productivity in terms of cost efficiency were made by Maniadakis and Thanassoulis (2004), who developed the concepts and made a proposal for the decomposition of the cost MI. This was followed by Tohidi et al. (2012) using modifications on the use of VRS, presenting the global cost MI and biennial cost MI (Tohidi & Tohidnia, 2014). Measuring profit efficiency using the MI has been suggested by Tohidi et al. (2010) and Tohidi and Razavyan (2013). Navanbakhsh et al. (2006) proposed the revenue MI and Zhu et al. (2017) proposed the allocation MI.

Developments within DEA models in the use of the different nature of variables and data has led to the adaptation of these solutions within the MI (Figure 3). The first DEA models assumed that the researcher had full knowledge of the data needed for the study and that all variables positively influence the production process, but also that they were fully controlled by the units under study. However, business practice indicated that the simple assumption of the DEA models, that outputs are generated from the inputs, proved to be insufficient to estimate the efficiency of units with specific business profiles or in emergency situations.

The first of the problems confronting the researchers was incomplete or estimated data described as uncertain. There are situations where, for various reasons, the data are not defined in terms of absolute numbers, but only in terms of approximations. Therefore, DEA models have started to implement different ways of taking interval or fuzzy data (see Emrouznejad et al., 2014).

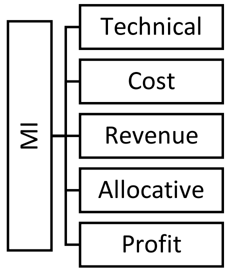

Figure 2. Measuring Various Efficiency With the MI

The second instance of problems was the inclusion of negative data. This was due to the fact that standard DEA models only assume consideration of positive (non-negative) data. The various ways of transforming negative data (e.g., by adding a small or large positive constant) into positive data adversely affected the estimated efficiency measures (Lu et al., 2024). In economic practice, there are situations where either inputs have been consumed but outputs have not been generated or there has been an emergency situation such as financial losses. Therefore, several DEA models have been proposed in the literature (the Range Directional Model - RDM, Variant of Radial Measure - VRM, Semi-Oriented Radial Measure – SORM and Base Point Slacks-Based Measure - BP-SBM) that allow negative data to be taken into account (Łącka & Brzezicki, 2023). Another research problem was the consideration of relational or ratio data, especially when estimating output-oriented efficiency. Consequently, DEA models have been developed to allow for the inclusion of ratio data (see Hatami-Marbini & Toloo, 2019).

However, it is not only data problems that researchers have encountered, but also the specificity of some of the variables influencing production. Standard DEA models assume that all variables are fully controlled by the units under study, but there are situations where variables, e.g., in terms of the number of people who can benefit from an entity's products or services, are independent of the entity, but this variable influences the efficiency measures obtained by the units. Another example is financial resources granted to an entity by external institutions, mostly public. Although the funds are used in the production process, the amount depends on another public institution. Therefore, the need to take into account uncontrolled (non-discretionary) variables when estimating efficiency using the DEA method has been noted (see: Cooper et al., 2007).

Measuring the efficiency of manufacturing companies through classical DEA models also deviates from economic practice, as companies in the production process, for various reasons, e.g., technological problems/failures, etc., may produce not only desirable outputs, but also incomplete (defective) outputs that are not desired by the companies. Another example related to the energy sector concerning undesirable outputs could be environmental pollution, e.g., in the form of air pollution. The assumptions of undesirable outputs have been implemented in the SBM model (Cooper et al., 2007) and DDF (Chung et al., 1997), among others.

Shahkooeei et al. (2022) point out that in traditional DEA models, the role of a variable is strictly defined and unambiguously assigned to either inputs or outputs. However, in some cases, the specific management rules of an organisation make the input or output variables play a dual role. Variables whose role is not entirely clear are called flexible variables. Unfortunately, the dual role of variables cannot be included in standard DEA models. Therefore, many developments of DEA models have been proposed that take these specific variables (see Amirteimoori et al., 2013).

Studies estimating productivity changes using the MI have considered the different nature of the data: negative (Portela & Thanassoulis, 2010; Mohammadi & Yousefpour, 2014), interval (Khalili-Damghani & Haji-Sami, 2018; Hosseinzadeh Lotfi et al., 2007), fuzzy (Peykani & Seyed Esmaeili, 2021; Emrouznejad et al., 2011), and ratio data (Dorri & Rostamy-Malkhalifeh, 2017).

In contrast, other studies estimating productivity changes using the MI have taken into account the different nature of the variable: undesirable outputs (Bansal & Mehra, 2022; Khoshroo et al., 2022), non-discretionary variables (Brennan et al., 2014, Essid et al., 2014), and dual-role factors also influence flexibility in the input or output role (Shahkooeei et al., 2022).

The MI modifications presented so far have only focused on the quantitative nature of the variables. However, it is possible to find work in the literature that considers qualitative variables when estimating the MI. Färe et al. (1995, 2001) used the concepts of Fixler and Zieschang (1992) to estimate quality in the MI. Subsequently, Färe et al. (2006) extended the original approach (Färe et al., 1995, 2001) to define quality augmented MI, the MI and quality index. Another concept of including quality variables in the MI was presented by Nankali et al. (2022).

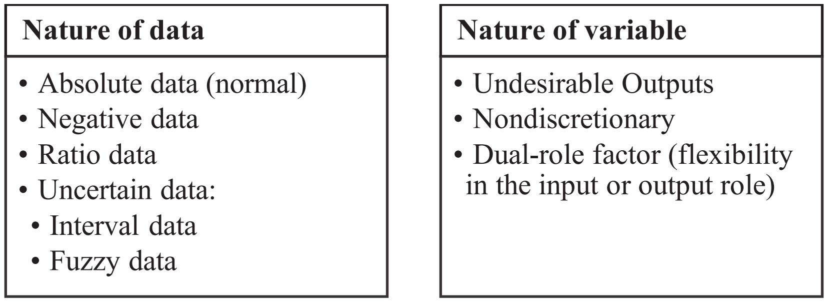

Figure 3. The Different Nature of Variables and Data Used by the MI

The literature also notes specific uses of the MI to measure intergroup productivity (e.g., Camanho & Dyson, 2006; Walheer, 2022). Other authors (e.g., Oh & Lee, 2010; Afsharian et al. 2018) have extended the MI with a meta-frontier (O'Donnell et al., 2008) to make comparisons between groups considering different technologies. There are also solutions to make a cumulative (Tone & Tsutsui, 2017c) or aggregation (Zelenyuk, 2006) of the MI. Färe and Zelenyuk (2003) proposed an aggregation of the Farrell (1957) approach. They introduced conditions to aggregate distance functions to analyse firms and the industry. Later, Zelenyuk (2006), applied the assumptions of Färe and Zelenyuk (2003) and proposed an aggregated MI to analyse a group of companies. In the literature, one can find attempts to use partnership evaluation by means of cross efficiency within the MI (Ding et al., 2019) instead of the self-assessment used in a classical MI.

The MI solutions presented so far, taking into account the different nature of the data and variables, were based on DEA models and referred to in the literature as 'black box', i.e., not taking into account the processes or production stages of the unit under study. However, the complexity of production processes indicated the need to develop DEA network models that would allow the determination of not only the overall efficiency of the entire production process as in classical DEA models, but above all the efficiency of individual production stages. The concept of DEA network models presented by Färe and Grosskopf (2000) quickly gained popularity among researchers, who incorporated its assumptions into existing DEA models (radial and non-radial), proposing different forms (Kao, 2014; Koronakos, 2019) from sequential models (two-stage, multi-stage) where each stage follows the previous one, to parallel models and other specific structures. The use of DEA network models to estimate productivity using the MI allows not only the determination of total productivity changes for the entire production process, but more importantly for individual production stages, which can be particularly useful information for management. The use of the above network assumption of various DEA models to estimate productivity changes using the MI can be found in works Kao and Hwang (2014), Kao (2017), Tavana et al. (2020), Seyed Esmaeili et al. (2022) and many others.

Another concept of measuring efficiency in dynamic terms (dynamic DEA models), i.e., over multiple periods at the same time instead of just one as in standard DEA models, was presented by Färe and Grosskopf (1996). This concept has also gained popularity because of its usefulness, especially when it is necessary to analyse the efficiency of projects implemented over a longer period than one. Examples of the use of dynamic DEA models to estimate productivity changes using the MI can be found in the work of Färe et al. (2018) and Tone and Tsutsui (2017a). The DEA network models and the DEA dynamic models share a common feature regarding variables, as pointed out by Färe and Grosskopf (1996, 2000). In DEA network models, individual stages are linked to each other by intermediate variables, whereas in dynamic DEA models, subsequent study periods are linked by inter-period variables. Since the two concepts of network models and dynamic DEA are related, Tone and Tsutsui (2014) presented a Dynamic Network SBM model to estimate efficiency from three perspectives, i.e., (1) overall efficiency, (2) period efficiency, and (3) stage efficiency. Tone and Tsutsui (2017b) proposed using Dynamic Network SBM to estimate the MI, calling it the Dynamic Divisional MI.

The concept of the double frontier can also be found in the literature on the DEA and the MI method. Although it is not very practical from a usability perspective, it was decided to present it in order to indicate and emphasise the positive research perspective prevailing in the measurement of efficiency and productivity. Research using the DEA method assumes that the efficiency of a unit is estimated on the basis of the best practice frontier (positive perspective). However, when the reverse is adopted, then efficiency is estimated in relation to the worst-practice frontier, the anti-frontier (negative perspective). The double frontier concept, most often involves estimating efficiency from a positive and a negative perspective. Overall efficiency is determined as the average of the two efficiency perspectives. Examples of the use of the double frontier concept within the MI, can be found in the work of Wang and Lan (2011).

Considering that the DEA method is deterministic, this makes it difficult to analyse the different types of uncertainty associated with the model adopted through the use of statistical inference. Therefore, the so-called statistical approach within DEA, which enables the analysis of the accuracy of the estimation of the efficiency measure, is of particular importance. The smoothed bootstrapping presented by Simar and Wilson (1998) is most commonly used for this purpose. This approach also has its analogue in the MI framework (Simar & Wilson, 1999). More information on bootstrapping in the DEA method and the MI, can be found in Simar and Wilson (2015) and Wang and Zelenyuk (2024).

The collected and classified research material (Brzezicki, 2025) made it possible to group the ongoing research on MI development and modification into several groups. The groups adopted are only conventional, as not all studies can be straightforwardly classified due to the fact that proposed MI developments or modifications consist of multiple elements simultaneously. Fallahnejad et al. (2024) proposed cross common weights global MI based on bargaining games (Nash bargaining). Bansal and Mehra (2022) presented Malmquist-Luenberger productivity indexes (also sequential) for dynamic network DEA with undesirable outputs and negative data. In contrast, Tohidnia and Tohidi (2019) presented a global cost MI with network structures. The MI containing combinations of multiple elements at the same time shows that the process of measuring productivity can be more complex and adapted to the defined research objectives.

Summarising the main areas of MI modification as groups is shown in Figure 4. The summary has two main objectives. On the one hand, they are meant to present the different possibilities that are available within the MI. Young researchers may not know that there are more complex solutions that can be used within the MI. Given that the MI is used in the vast majority of studies to measure changes in technical efficiency, they may not see that it can also be used to measure other types of efficiency or to consider different types of nature of data or variables. Advanced researchers may not follow the new solutions available within the DEA or MI method presented in the literature that can be used to analyse a more complex production process. Particularly in terms of network and dynamic, DEA models for estimating productivity changes using the MI.

In carrying out the searches for this review, it was noted that while in terms of measuring efficiency using the DEA method researchers use various modifications of DEA models, for estimating productivity changes over time using MI, they are far more careful, as in most cases they choose the standard MI calculated using two radial DEA models (CCR or BCC).

On the other hand, the groups presented in Figure 4 are meant to indicate ways to solve research problems that researchers may confront in terms of taking into account uncertain data, specific variables, choosing how to decompose MI.

The literature review indicated that the development and modification of MI have proceeded in three main directions (Figure 5). The first group is concerned adaptations of solutions found in DEA models, in terms of taking into account the different nature of variables and data and network relationships for the MI, among other things. Portela et al. (2004) first presented a RDM allowing the inclusion of negative data within the DEA method, then Portela and Thanassoulis (2010) used its assumptions in the MI. Recent literature reviews of the DEA method (Mergoni et al., 2025; Emrouznejad et al., 2025) indicate that many more solutions can be implemented in future developments and modifications of the MI (e.g., shared inputs, desirable inputs, bounded variable, big date, the nearest point on the efficient frontiers also knows maximum frontier, leader–follower model, forecast and stochastic approach, etc.).

The second group includes publications that either present completely new MI solutions or combine several MI solutions together into a single MI framework. Tohidi and Razavyan (2013) presented a circular global profit MI by building on previous publications that included information on its individual elements. The last group included publications in which the authors attempted to combine two methods of dynamic

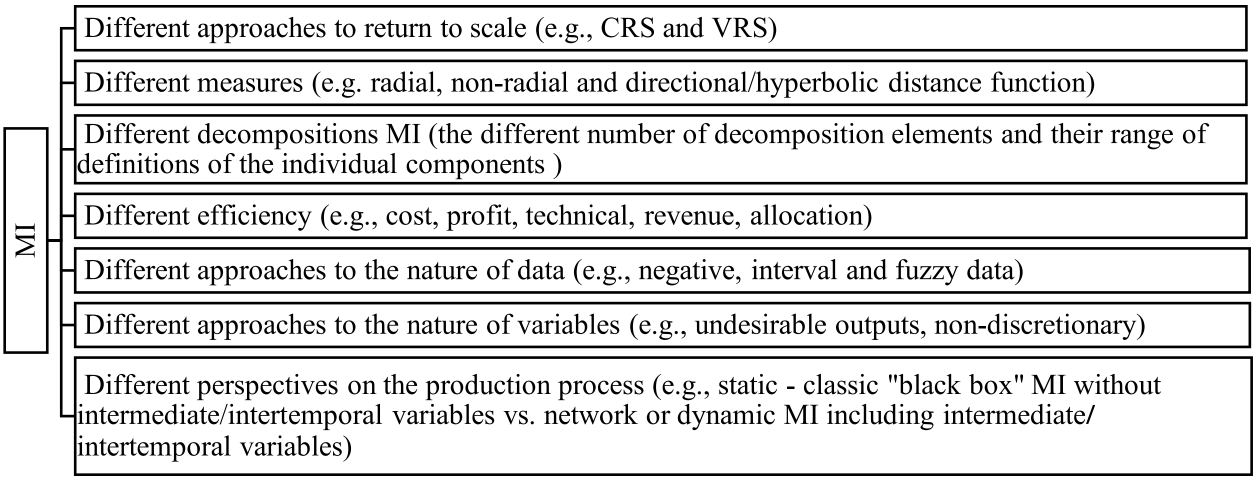

Figure 4. Various Developments and Modifications of MI

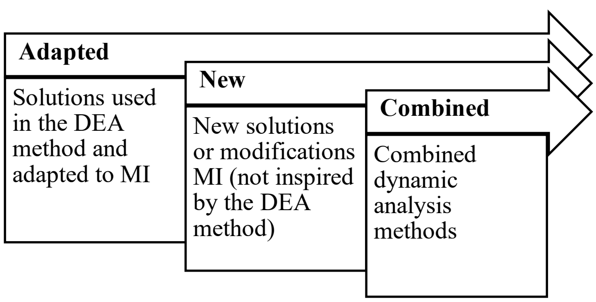

Figure 5. MI Development and Modification Pathways

analysis into a single framework. The combination of Window DEA and the MI can be found in the works of Sueyoshi and Aoki (2001) and Emrouznejad and Thanassoulis (2010). However, these combinations are not always appropriate. Asmild et al. (2004, p. 67) "show that for both the adjacent and the base period Malmquist index and for all suggested definitions of same period frontier, the standard decomposition into frontier shift and catching up effects gives inappropriate results when Malmquist indices are based on DEA window analysis scores".

To conclude the discussion, it is worth mentioning another productivity index which, due to its similar name, may be confused with the MI (Lovell, 2003). It is known as the "Malmquist Total Factor Productivity Index" which was proposed by Bjurek (1996). It is also referred to as the (Färe & Zelenyuk, 2021) "Hicks–Moorsteen Productivity Index", "Moorsteen-Bjurek Productivity Index", "Bjurek productivity index", or "Diewert–Bjurek productivity index". Lovell (2003) points out that the MI (Färe et al., 1989a, 1990, 1992, 1994a, 1994b) is either input-oriented or output-oriented, while the Hicks-Moorsteen index proposed by Bjurek (1996) is simultaneously input- and output-oriented. The Hicks-Moorsten productivity is defined as the ratio of Malmquist output quantity index and Malmquist input quantity index.

However, it is worth noting that other indices for measuring productivity are also available in the literature (Färe et al., 2008), e.g., the Paasche, Laspeyres, Fisher, Törnqvist, Luenberger, Färe-Primont index and others. However, they are not the subject of this review, which focuses exclusively on the MI. More on other productivity indices and how they are calculated can be found in the following papers: Färe et al. (2008), Russell (2018) and Sickles and Zelenyuk (2019).

Choosing a DEA model other than the radial CCR or BCC to estimate the MI or a different MI decomposition than the standard one (Färe et al., 1992, 1994b) always implies the problem of making appropriate calculations. Although a large number of programmes are available for calculating efficiency and productivity (Daraio et al., 2019) and have the computational algorithms of the DEA method and the MI implemented, only a few of them offer a choice of more complex solutions (different MI decompositions, choice of DEA model to calculate MI, etc.). Software and other IT solutions for calculating the MI include: the Total Factor Productivity Toolbox for MATLAB (Balk et al., 2020), the DEA Toolbox for MATLAB (Álvarez et al., 2020), the DEA Solver Pro, deaR-Shiny (Benitez et al., 2021), MaxDEA (Pro, Ultra or X), PIM DEA and packages for the R programme (FEAR, deaR, productivity).

4. Conclusions, Limitations and Direction for Future Research

The following conclusions can be drawn from this review of the origins, development and modification of the MI. The MI was undoubtedly only popularised by Färe's team, even though its theoretical assumptions had been presented several years earlier. The lively academic discussion followed the publication of the paper by Färe et al. (1994b), in which the authors decomposed the MI into three components, including a variable return to scale. Initially, the authors mainly focused on the MI decomposition, with few studies extending the standard MI. However, after the publication of the paper by Maniadakis and Thanassoulis (2004), there has been a significant development of the MI in various aspects. To date, studies have emerged that take into account different returns to scale, alternative ways of decomposing MI, different ways of measuring, consideration of different types of efficiency, consideration of different specificities of data and variables, consideration of network structure, development of statistical approaches within the MI, combining dynamic analysis methods or adopting two approaches to efficiency (e.g., an optimistic and a pessimistic efficient frontier) and many other aspects. The main direction of changes and modifications of the MI can be divided into three groups: Adaptation of DEA solutions in the MI, presentation of new MI solutions without inspiration from the DEA method and combining two methods of dynamic analysis into one design.

Although this review has filled the research gap found, it also has limitations that should be addressed in the future. Firstly, only studies that presented the development of the MI based on the DEA method were included in this review. Therefore, it makes sense for future literature reviews to use a systematic literature review to consider different approaches to productivity analysis over time, whether they are based on a non-parametric or parametric method, as well as a partial frontier. Secondly, this review only focuses on presenting the different approaches within the MI, without considering the thematic areas in which the MI has been used. Therefore, it would also be worth considering in a future review a thematic analysis of the areas in which the MI has been used. Another interesting area could be a bibliometric analysis of the MI.

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