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A model of damaged media used for describing the process of non-stationary creep and long-term strength of polycrystalline structural alloys

Abstract The main laws of the processes of creep and long-term strength of polycrystalline structural alloys are considered. From the viewpoint of continuum damaged media (CDM), a mathematical model is developed that describes the processes of viscoplastic deformation and damage accumulation under creep. The problem of determining material parameters and scalar functions of the developed constitutive relations based on the results of specially set basic experiments is discussed. An experimental–theoretical methodology for determining material parameters of the derived constitutive relations of CDM is developed based on analyzing the viscoplastic deformation and failure processes of laboratory specimens in the conditions of soft loading (stress controlled). Experimental results of short-term creep of the VZh-159 heat-resistant alloy are presented. The obtained numerical results are compared with the test data using the numerical modeling method of experimental processes. Qualitative and quantitative agreement between numerical results and experimental data is shown. It is concluded that the developed constitutive relations are reliable, and that the proposed methodology accurately determines the material parameters of the model under degradation of initial strength properties of structural materials according to the long-term strength mechanism.

Keywords Nonstationary creep · Long-term strength · Numerical modeling · Constitutive relations · Mechanics of damaged medial · Temperature · Damage · Viscoplastic deformation · Material parameters

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1 Introduction

Service life of structural elements working in the conditions of elevated temperatures T ($T > 0.5\ T_{metl}$, where T_{metl} is melting temperature) and cyclic mechanical effects are mainly determined by physical processes of degradation of initial strength properties of structural materials (processes of low-cycle fatigue and creep damage accumulation, that lead to one of the most serious types of failure – brittle fracture of structures made of plastic materials.

The currently available standard methods of evaluation of life of structural elements do not account for actual processes taking place in materials. Elastic analysis used in the standard approach does not make it possible to account for actual characteristics of viscoplastic deformation of materials, which, to a considerable degree, determine service life of structural elements. In a general case, strength of a structure must take into account loading time and history. As a result, the failure criterion will be closely connected with defining relations describing the failure process [1].

In its turn, this calls for developing new methods of evaluating life of structural elements, based on related equations of thermoviscoplasticity, equations of damage accumulation and failure criteria, with comprehensively substantiating them with the help of related full-scale and numerical experiments on laboratory specimens and numerical analyses of processes of deformation and failure of structural elements in the exploitation conditions [1,2].

The dependence of durability of materials on the duration (frequency) of a cycle and on the presence and duration of holdings in a cycle taking place against the background of elevated constant or varying temperatures is determined by developing creep strains and defects resulting from these strains, which, in contrast to fatigue defects developing over the grain body, accumulate along grain boundaries and result in inter-crystalline fracture.

One of the most popular methods of studying long-term strength is constructing an experimental 'tensile stress $\sim$ time to failure' curve in uniaxial tension experiments on specimens at a constant temperature [3]. Creep curves have three characteristic parts. From the viewpoint of studying the characteristics of the damage accumulation process, the most interesting points are time of the beginning of the third part, creep rate at the steady-state part, the law of change of the creep rate over the third part, time to failure and creep strain by the time of failure. The presence of the third transient part on the curve is caused by the effect of accumulated damage on the deformational characteristics of the material (creep rate), whereas the law of change of creep rate over this part is determined by the damage accumulation rate in the material over this part. The beginning of the third part is the starting point where the accumulated damage begins to affect the deformational characteristics of the material.

The three stages of creep deformation are, in fact, a manifestation of succeeding changes in the material on the macro-scale. After a continuous initial period where viscous deformation prevails, abrupt increase in strain rate, characteristic of the third stage, is a direct result of the accelerated growth of microdefects, followed by their merging leading to creep failure. In connection with structural materials, the important issue is the interconnection of deformation and failure processes, as the existence of such an interconnection makes it possible to seek ways of evaluating creep damage of the material, using deformational characteristics. Experimental studies revealed a distinct relation between kinetic processes of creep deformation and damage accumulation: there exists a linear relation between creep deformation at all stages and volume ratio of microdefects; at the third stage, a directly proportional relation between the increase in the volume ratio of micropores and the increase in creep rate is observed [4,5]. It appears impossible to divide the process of nucleation and growth of discontinuities and that of deformation into the driving and driven ones. These processes are interconnected and interdependent.

Hence, experimental study of the creep process in the third part of the curve is of considerable importance for studying laws of the damage accumulation process, though, in actual cyclic deformation processes with different holding times of the material, failure (the formation of a macrocrack) due to damage accumulation occurs for small creep deformations (but large trajectories thereof). As in actual exploitation conditions of structural materials the beginning of the steady-state part is attained after several tens of hours, and total time to failure amounts to several hundreds or even thousands of hours, the problem of conducting experiments of sufficiently large duration arises. To solve this problem, it is necessary to establish a correspondence between short-term creep experiments at elevated temperatures and characteristics of creep and long-term strength in large-duration experiments at more moderate temperatures. There exist several different relations defining this correlation [6–9].

Most studies of creep and long-term strength of structural materials are based on tensile tests of cylindrical specimens at constant temperatures and stresses. However, actual problems of creep and long-term strength of structural elements involve much more complex conditions, such as variable stresses, with a varying type of stress state and cyclically varying temperatures. As both creep and long-term strength of metals are rather sensitive to changing stress states and temperatures that affect all their parameters, it becomes important to study laws of non-isothermal creep and long-term strength for different stress state types and develop methods for evaluating creep and long-term strength of materials of structural elements in the conditions of a multiaxial stress state, based on the experimental data obtained in tests with a uniaxial stress state.

Durability curves are normally constructed for constant stresses (loads), whereas in actual conditions stresses and temperatures vary according to rather complex laws. As a result, the problem of damage summation for rather irregular loading histories arises. When evaluating time to failure in the conditions of variable stresses and temperatures, the main issue is to be able to use in the analyses the results of constant stress and temperature tests. In engineering applications, the linear damage summation law is normally used to this end, where each part of spent life, or accumulated damage, is assumed independent of the other. Failure occurs when the sum of such parts of spent life is equal to unity. This law is widely used in engineering analyses for structures working in the conditions of varying temperatures and loads. Very few attempts have been made to assess the accuracy of separate analyses for varying temperatures and stresses, although even a superfluous examination of kinetics of damage processes reveals a noticeable difference. The available experimental programs have two main drawbacks. The first one is that creep damage is treated as a theoretical notion, rather than an actual physical phenomenon that can be measured. The second one is connected with the fact that experiments are mainly planned to verify only a particular damage summation law without aiming at revealing general laws of the process. As a result, for each experimental series (varying the temperatures or the stresses), it is only possible to find whether the summation law holds, without being able to explain why it does not. Analysis of the available experimental results makes it possible to reveal a considerable effect of stress and temperature histories on long-term strength and to conclude that the linear damage summation law is inadequate for the conditions of varying stresses and temperatures. Good correlation has been found between room temperature hardness of a creeping specimen and its long-term strength. Comparison with the linear law of summation of relative lives leads to the following inconsistencies [10]:

- creep damage accumulation in a tested alloy is not, in a general case, a linear function of preloading time;
- it is rather difficult to compare damage states for parts of life of equal time, as separate effects of each of the factors (temperature or stress) on the process of nucleation and growth of pores has not been studied. It was found, however, that, for each given part of life, the total pore volume increases with decreasing stress (at a constant temperature) and decreasing temperature (at a constant stress). For the same effect during a part of life, the degree of softening (pore volume) increases with decreasing stress, but it is almost insensitive to temperature;
- there is no common value of creep damage always causing failure. Each combination of tests or exploitation conditions yields its own characteristic value of critical damage.

The evaluation accuracy of service life of structural elements under given operating conditions will depend on how reliably the applied equations of state describe the kinetics of the stress-strain state under these conditions. To date, a large number of equations have been developed that describe the processes of material damage. However, most of these equations are focused only on certain classes of loading that are not related to specific equations of deformation processes and, therefore, cannot reflect the dependence of damage accumulation processes on history of the stress-strain state, strain rate and temperature. In fact, the history of viscoplastic deformation, the type of deformation trajectory, the nature of temperature change, the type of stress state, the history of its change, etc., significantly affect the rate of damage accumulation processes. The purpose of research in this area is not so much to clarify the various formulations necessary for determining macroscopic deformations for a given loading history but to understand the main patterns of processes that prepare and determine the failure [1,2,5,11–15].

The present paper develops, in the framework of mechanics of damaged media, a mathematical model of CDM describing damage accumulation processes in structural materials (metals and their alloys) with a degradation mechanism caused by developing creep deformations. Processes of long-term strength of the VZh-159 heat-resistant alloy have been numerically analyzed, and the obtained results compared with the data of full-scale experiments.

2 Defining relations of mechanics of damaged media

The main assumptions of the version of CDM under consideration are as follows:

- the material of the medium is initially isotropic and free of defects (only the anisotropy caused by deformation processes is taken into account; the anisotropy of elastic properties caused by damage processes in the material is not accounted for);
- components of strain tensors e_{ij} and strain rates $\dot{e}_{ij}$ are a sum of the 'instant' and the 'time' components. The 'instant' component consists of elastic components e^{e}_{ij},$\dot{e}'^{e}_{ij}$ independent of the deformation history and determining the final state of the process, and plastic components e^{p}_{ij},$\dot{e}'^{p}_{ij}$ depending on the deformation history. The time component (creep strains e^{c}_{ij},$\dot{e}'^{c}_{ij}$) describes deformation processes at low loading rates as a function of time;
- evolution of equipotential creep surfaces is described by the change of its radius C_c and the displacement of its center ρ^{c}_{ij};
- the volume of the element changes elastically, i.e., $e^{p}_{ii} = e^{c}_{ii} = 0$;
- the considered processes are characterized by small deformations;
- the only structural parameter characterizing the damage degree of the material on the macroscale is scalar parameter ω – ≪damage value≫ ($\omega_0 \le \omega \le \omega_f$);
- the effect of the accumulated damage value on the deformation process in the material is accounted for by introducing effective stresses $\tilde{\sigma}_{ij}$.

The damaged medium model consists of three interrelated parts:

- relations defining viscoplastic behavior of the material, accounting for its dependence on the failure;
- equations describing damage accumulation kinetics;
- a strength criterion of the damaged material.

a) Relations of thermal creep

The constitutive relation between the stress and elastic strain tensor is established using equations of thermoelasticity:

$$\sigma = 3K\left[e - \alpha T\right)]; \quad \dot{\sigma} = 3K\left[\dot{e} - \dot{\alpha}T - \alpha\dot{T})\right] + \frac{\dot{K}}{K}\sigma, \sigma'_{ij} = 2Ge'^{e}_{ij},$$
$$\dot{\sigma}_{ij} = 2G\dot{e}'^{e}_{ij} + \frac{\dot{G}}{G}\sigma'{}_{ij}, e'^{e}_{ij} = e'_{ij} - e^{c}_{ij} \quad (1)$$

where σ, e are spherical, and σ'_{ij}, e'_{ij} are deviatoric tensor components of stresses σ_{ij} and strains e_{ij}, respectively; $G(T)$ is shear modulus, $K(T)$ is volumetric elasticity modulus, $\alpha(T)$ is thermal expansion coefficient, all are functions of temperature T.

In order to describe creep processes, a family of equipotential creep surfaces F_c (on which creep strain rate has a constant value) is introduced in the stress space. The surfaces have a common center ρ^{c}_{ij} and different radii C_c determined by the current stress state, see [16–21]:

$$F^{(i)}_c = S^{c}_{ij}S^{c}_{ij} - C^2_c = 0, S^{c}_{ij} = \sigma'_{ij} - \rho^{c}_{ij}, i = 0, 1, 2, ... \quad (2)$$

According to the associated law we get

$$\dot{e}^{c}_{ij} = \lambda_c \frac{\partial F^{(i)}_c}{\partial S^{c}_{ij}} = \lambda_c S^{c}_{ij}, \quad (3)$$

where λ_c corresponds to current surface $F^{(i)}_c$ defining current stress state S^{c}_{ij}.

Among these equipotential surfaces, a surface of radius $\bar{C}_c$ can be chosen, that corresponds to zero creep rate:

$$F^{(0)}_c = \bar{S}^{c}_{ij}\bar{S}^{c}_{ij} - \bar{C}^2_c = 0, \bar{S}^{c}_{ij} = \bar{\sigma}'_{ij} - \rho^{c}_{ij}, \quad (4)$$

where $\bar{S}^{c}_{ij}$ and $\bar{\sigma}'_{ij}$ is a set of stress states corresponding (with a certain allowance) to zero creep rate.

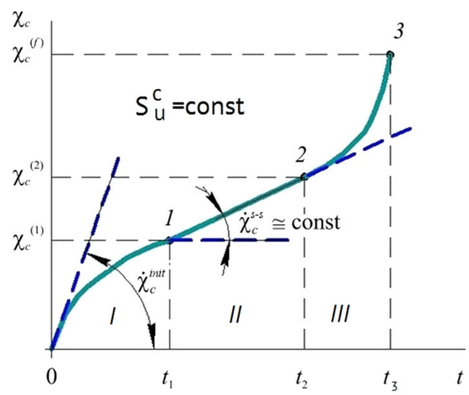


Fig. 1 The curve for χ_c as a function of time t of the process for $S_u^c = const$

It is assumed that

$$\bar{C}_c = \bar{C}_c(\chi_c, T), \dot{\chi}_c = \left(\frac{2}{3}\dot{e}_{ij}^c, \dot{e}_{ij}^c\right)^{1/2}, \chi^c = \int_o^t \dot{\chi}^c dt, \lambda_c = \lambda_c(\psi_c, T), \psi_c = \left[\frac{\left(S_{ij}^c S_{ij}^c\right)^{1/2} - \bar{C}_c}{C_c}\right],$$

$$\lambda_c = \begin{cases} 0, & \psi_c \leq 0 \\ \lambda_c, & \psi_c > 0 \end{cases}, \qquad (5)$$

where $\bar{C}_c$ and λ_c are experimentally determined functions of temperature T.

The evolution equation of the change of the coordinates of the creep surface center has the form [2,18–21]:

$$\dot{\rho}_{ij}^c = g_1^c \dot{e}_{ij}^c - g_2^c \rho_{ij}^c \dot{\chi}_c, \qquad (6)$$

where g_1^c and $g_2^c > 0$ are experimentally determined material parameters.

Concretizing relation (3), the orthogonality law can be represented as in [22]:

$$\dot{e}_{ij}^c = \lambda_c(\psi_c, T) S_{ij}^c = \lambda_c \psi_c S_{ij}^c = \lambda_c \left(\frac{\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c}{C_c}\right) S_{ij}^c, \qquad (7)$$

whence the expression for $\dot{\chi}_c$ will take the form of [23]:

$$\dot{\chi}_c = \sqrt{2/3}\dot{e}_u^c = \sqrt{2/3}\lambda_c\left(\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c\right). \qquad (8)$$

The curve for χ_c as a function of time t of the process for $S_u^c = const$ in the case of multiaxial deformation according to a ray trajectory has the form depicted in Fig. 1

Curve $\chi_c \sim t$ (Fig. 1) can be conventionally subdivided into three parts, describing three stages of creep process (I, II and III, called primary, secondary and tertiary creep stages , respectively):

I. the part of transient creep $(0 - \chi_c^{(1)})$, where creep strain rate $\dot{\chi}_c$ decreases;
II. the part of steady-state creep $(\chi_c^{(1)} - \chi_c^{(2)})$, where creep strain rate $\dot{\chi}_c$ is approximately constant, $\dot{\chi}_c \cong const$;
III. the part of transient creep $(\chi_c > \chi_c^{(2)})$, where creep strains increase rapidly (preceding failure) and $\dot{\chi}_c$ abruptly rises.

The lengths of the parts depend to a large degree on value $S_u^c = const$.

From (8) for the three parts of the creep curve, the expression for λ_c will take the form [17]:

$$\lambda_c = \begin{cases} 0, & \psi_c \leq C_c \ \vee \ \chi_c = 0 \\ \lambda_c^I, & 0 \leq \chi_c \leq \chi_c^{(1)} \\ \lambda_c^{II}, & \chi_c^{(1)} \leq \chi_c \leq \chi_c^{(2)} \\ \lambda_c^{III}, & \chi_c^{(2)} \leq \chi_c \leq \chi_c^{(3)} \end{cases}, \qquad (9)$$

where $\lambda_c^I = \lambda_c^{(0)}\left(1 - \frac{e_{11}^c}{e_{11}^{c(1)}}\right) + \lambda_c^{(1)}\frac{e_{11}^c}{e_{11}^{c(1)}}$, $\lambda_c^{II} = \frac{3}{2}\frac{\dot{e}_{11}^{s-s}}{(\sigma'_{11} - 3\rho_{11}^c/2 - \bar{\sigma}_c)}$, $\lambda_c^{III} = \lambda_c^{II}/(1-\omega)^{r_c}$ is obtained from the experiments with uniaxially stress laboratory specimens [17].

In formulas (9): $\lambda_c^{(0)}$ and $\lambda_c^{(1)}$are values of λ_c at the initial and final points of the first part of the creep curve of the material; $e_{11}^{c(1)}$, $e_{11}^{c(2)}$ and $e_{11}^{c(3)}$ are boundaries of the parts of the creep curve for a uniaxial stress state; $\dot{e}_{11}^{c(init)}$ is creep strain rate at an initial time, $\dot{e}_{11}^{c(s-s)}$ is creep strain rate over the part of steady-state creep (the second part of the creep curve); ω is damage value of the material; $\bar{\sigma}_c = \sqrt{2/3}\bar{C}_c$ is creep strength for a uniaxial stress state; r_c is material parameter [16,17].

Equations (1)–(9) describe the transient and steady-state parts of the creep curve for different stress levels and the main effects of the creep process for alternating stresses.

At the stage of the growth of defects scattered over the bulk of the material, the effect of damage degree on the physical–mechanical characteristics of the material is observed. In the first approximation, this effect can be taken into account using the concept of a degrading continuum (by introducing effective stresses) [2]:

$$\tilde{\sigma}'_{ij} = F_1(\omega)\,\sigma'_{ij} = \frac{G}{\tilde{G}}\sigma'_{ij}, \tilde{\sigma} = F_2(\omega)\,\sigma = \frac{K}{\tilde{K}}\sigma, \tag{10}$$

where $\tilde{G}$, $\tilde{K}$ are effective elasticity moduli determined using McKenzie's formulae [24]:

$$\tilde{G} = G(1-\omega)\left[1 - \frac{(6K + 12G)}{(9K + 8G)}\omega\right], \tag{11}$$

$$\tilde{K} = 4GK(1-\omega)/(4G + 3K\omega). \tag{12}$$

Effective internal variable $\tilde{\rho}_{ij}^c$ is determined in a similar way:

$$\tilde{\rho}_{ij}^c = F_1(\omega)\,\rho_{ij}^c = \frac{G}{\tilde{G}}\rho_{ij}^c. \tag{13}$$

b) Evolutionary equations of damage accumulation

Experimental and theoretical analyses of damage of materials make it possible to represent an evolutionary equation of damage accumulation in an elementary volume a material in the following general form [2,4,5,25–27]:

$$\dot{\omega} = f_1(\beta)\,f_2(\omega)\,f_3(Z_c)\left\langle \dot{Z}_c \right\rangle, \left\langle \dot{Z}_c \right\rangle = \begin{cases} \dot{Z}_c, & \dot{Z}_c > 0, \\ 0, & \dot{Z}_c \le 0, \end{cases} \tag{14}$$

where $\beta = \sigma/\sigma_i$ (σ is spherical (the first invariant of the stress tensor) component of the stress tensor, σ_i is the intensity of the stress tensor (the second invariant of the stress tensor)), ω is the damage parameter of the material, Z_c is the energy parameter characterizing the accumulated relative damage energy spent on the formation of microdefects during creep, $\dot{Z}_c$ is the rate of creep change. In (14), function $f_1(\beta)$ is type (voluminosity) of the stress state, $f_2(\omega)$ is level of accumulated damage, $f_3(Z_c)$ is accumulated relative energy spent on the nucleation of defects.

Considering the fact that currently there is no systematized and reliable enough experimental data characterizing creep of materials up to the moment of failure in a required range of working stresses and temperatures, as well as considerable scatter of the available experimental data, an evolutionary equation of creep must be formulated in the 'simplest' form [5,10,22,23,28]:

$$\dot{\omega} = \frac{\alpha_c + 1}{r_c + 1} f_c(\beta)\,Z_c^{\alpha_c}(1-\omega)^{-r_c}\left\langle \dot{Z}_c \right\rangle, \tag{15}$$

where

$$Z_c = \frac{W_c - W_c^a}{W_c^f}, \tag{16}$$

$$\left\langle \dot{Z}_c \right\rangle = \frac{\left\langle \dot{W}_c \right\rangle}{W_c^f}, \dot{W}_c = \rho_{ij}^c \dot{e}_{ij}^c, \tag{17}$$

$$f_c\,(\beta) = \exp(k_c \beta), \tag{18}$$

where ω is creep damage value; W_c^a is value of the failure energy at the end of the stage of nucleation of scattered defects due to creep, and W_c^f is value of the energy corresponding to the formation of a macroscopic crack [1,2]; $f_c(\beta)$ is function of the voluminosity parameter of the stress state β, $W_c = \int_0^t \dot{W}_c\, dt$ are energies spent on nucleation of scattered defects due to creep; k_c, α_c, r_c are material parameters depending on temperature T.

c) Strength criterion of the damaged material

The condition when the damage value reaches its critical point

$$\omega = \omega_f \leq 1 \tag{19}$$

is taken as a criterion of the termination of the phase of growth of scattered microdefects (the stage of formation of a macrocrack).

By integrating evolutionary equation of damage accumulation (15)–(18) together with defining relations of thermoviscoplasticity (1)–(9) and damage criterion (19) according to a known history of thermal-mechanical loading in this particular elementary volume of the material, it is possible to determine time of formation of a macroscopic creep crack.

The main characteristics of the process of viscoplastic deformation of damaged materials (the state parameters), which, in the general case, are described by tensors σ_{ij}, e_{ij}, e_{ij}^p, ρ_{ij}^p, e_{ij}^c and scalars χ, C_p, C_c, T and ω can be determined by particularly formulating the defining relations of CDM, and the linearization of the algorithm of determining λ reduces to writing defining relations of CDM in increments which depend on the chosen step Δt. Time step Δt can be corrected for complex parts of the deformation trajectory during the entire computation time, if the computations are stable. Such an approach [29] is especially useful when analyzing boundary-value problems of mechanics of deformable bodies and, thus, is used in the present paper.

In a general case, stresses, plastic strains and creep strains are determined by integrating equations of thermal creep (1)–(13), using the four-point Runge–Kutta method with correcting the stress deviator and then determining the stresses according to equations of thermoplasticity [5], taking into account the average creep strain rate at a time: $t^{n+1} = t^n + \Delta t$.

3 Methodology of determining the parameters of defining relations of CDM

For practical application of equations of thermal creep (1)–(18) the following information is required:

- relations $G(T)$, $K(T)$, $\alpha(T)$, where T is temperature;
- relation for a current creep surface radius corresponding to zero creep rate $\tilde{C}_c = \tilde{C}_c\,(\chi_c, T)$;
- relation $\lambda_c = \lambda_c(T)$; for the different parts of the creep curve;
- relations for kinematic hardening moduli $g_1^c(T)$, $g_2^c(T)$;

Material parameters of evolutionary equations of thermal creep are determined from basic experiments [2].

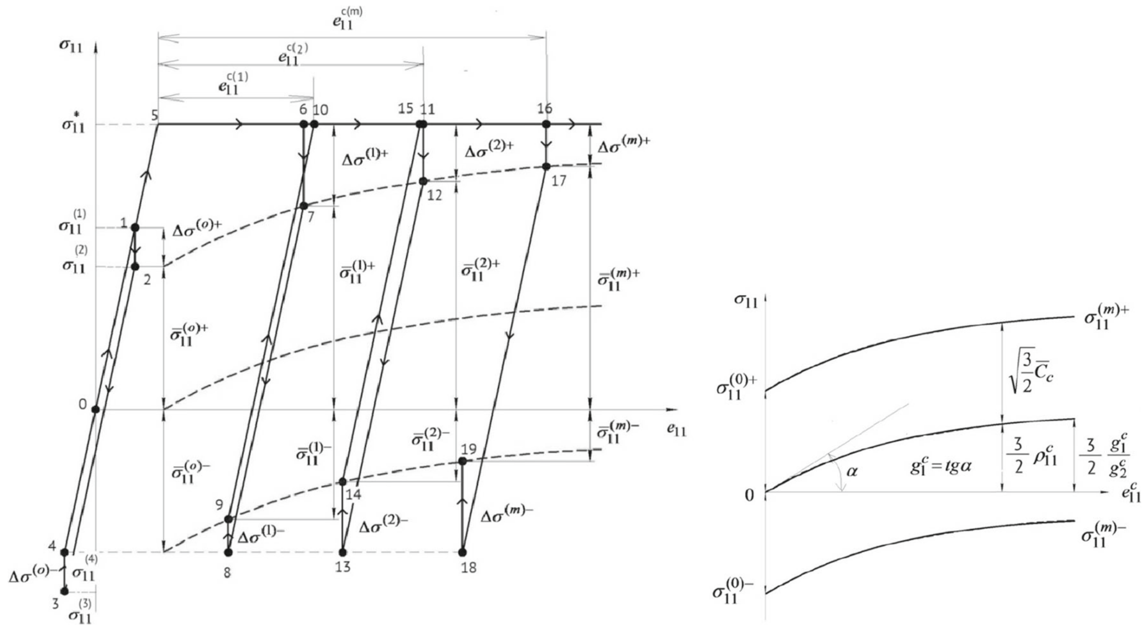
Such main basic experiments are uniaxial tension-compression tests of cylindrical laboratory specimens.

The main types of basic experiments are isothermal experiments at constant basic temperatures T_j (j =1, 2,...).

The main types of the specimens are cylindrical solid and cylindrical tubular ones. The chosen specimen types provide uniform distribution of stress, strain and temperature fields within the working part, preventing loss of stability and change of form of specimens under alternating loads and minimizing the effect of concentrators on the stress-strain state over the part of specimens between the working part and the thickening [2,16,17].

To determine kinematic (anisotropic) hardening moduli $g_1^c(T)$, $g_2^c(T)$ and a relation for the creep surface radius corresponding to zero creep rate $\tilde{C}_c = \tilde{C}_c\,(\chi_c, T)$, a specimen is heated up to the temperature value of the 'basic' experiment $T = T_j = const$ and tested on creep in a uniaxial stress state, using the 'soft testing' scheme.

First, the specimen is loaded up to stress value $\sigma_{11}^{(1)}$ at point 1 (Fig. 2a). This stress level is chosen from the analysis of the available fan of creep curves obtained at 'basic' temperature $T = T_j$ (the creep curve



(a) (b)

Fig. 2 Basic experiments for determining the parameters of the thermal creep model

corresponding to zero creep rate). As a result of relaxation, the process stops at point 2 (stress $\sigma_{11}^{(2)}$), where the creep strain rate tends to zero.

Then the specimen is loaded up to the stress of the opposite sign $\sigma_{11}^{(3)}$ (point 3 in Fig. 2a) and, as a result of relaxation, finds itself at point 4. Thus, stress $\tilde{\sigma}_{11}^{(0)+}$ (point 2) and stress $\tilde{\sigma}_{11}^{(0)-}$ (point 4) characterize (with a certain allowance for residual deformation) the initial upper and lower limits of the creep surface corresponding to zero creep rate.

To determine transformations of the creep surface on the same specimen at a prescribed stress $\sigma_{11}^{*} = const$, a series of analogous operations is done after the creep strain attains the prescribed creep strain levels $e_{11}^{c(1)}$, $e_{11}^{c(2)}$,..., $e_{11}^{c(m)}$.

A set of points 2, 7, 12 17, etc., obtained in this way, characterizes the change of the upper (for tension) limit of the creep surface as a function of the accumulated creep strain.

Points 4, 8, 13, 19, etc., characterize, respectively, the change of the lower (for compression) limit of the creep surface.

Thus, from the results of the experiment at basic constant temperatures $T = T_j$ one obtains:

- a locus of tensile creep strengths with an assigned allowance for residual deformation;
- a locus of inverse compression creep strengths.

The relation for the creep surface radius corresponding zero creep strain rate is determined using the formula:

$$\tilde{C}_c = \sqrt{2/3}\frac{\sigma_{11}^{(m)+} + \sigma_{11}^{(m)-}}{2}, m = 0, 1, 2, 3... \tag{20}$$

To determine kinematic (anisotropic) hardening moduli $g_1^c(T)$ and $g_2^c(T)$, it is necessary to integrate relation (6) for $T = T_j = const$.

As a result:

$$\rho_{11}^c = \frac{g_1^c}{g_2^c}\left(1 - e^{-g_2^c e_{11}^c}\right), \tag{21}$$

where e is base of natural logarithms; g_1^c is tangent of inclination of the tangent line to curve $\rho_{11}^c \sim e_{11}^c$ at the coordinate origin (Fig. 2b); $\rho_{\max}^c = g_1^c/g_2^c$ is limiting asymptotic value of ρ_{11}^c at a given temperature $T = T_j$.

Table 1 Physical–mechanical characteristics and material parameters of the CDM model of the VZh-159 heat-resistant alloy

$T, ^oC$	K, MPa	G, MPa	$\bar{C}_c, MPa$	$\lambda_c^{(0)}, 1/MPa \cdot hr$	$\lambda_c^{(1)}, 1/MPa \cdot hr$	g_1^c, MPa		
750	137000	64000	265	0.0005	0.0005	1100		
850	113000	52000	85	0.00068	0.00068	1000		
g_2^c	$W_c^f, MJ/m^3$	$W_c^a, MJ/m^3$	α_c	r_c	k_c	$\chi_c^{(1)}$	$\chi_c^{(2)}$	ω_f
150	57.5	14.3	1	0.1	1	0	0.025	0.83
150	14	5	1	0.1	1	0	0.06	0.83

The modules of anisotropic (kinematic) hardening g_1^c and g_2^c are derived from (20).

Material parameters of the evolutionary equations of damage accumulation are experimentally determined at the second stage of the damage accumulation process, from which accumulated damage begins to affect the physical–mechanical characteristics of the material. At the same stage, the experimental deformation processes are numerically analyzed, using relations of thermoviscoplasticity. The method consists in that all the deviations of the results of numerically modeling deformation processes without accounting for the effect of accumulated damage from the experimental results at the second stage of damage accumulation process are attributed to the effect of accumulated damage ω.

Boundaries W_c^a, W_c^f can be approximately determined from fatigue tests with an assigned stress (or strain) amplitude, based on the time of the onset of softening of the material.

To determine the parameters of evolutionary equation of creep (15)–(18), the third part of creep curves $e_{11}^c(\sigma_{11}, T_j)$ at different constant temperatures and stresses is used. This can be considerably simplified in the case of similarity of the creep curves [2]. In this case, the relative curve for $T = T_j$ is chosen as the basic curve. The known relation between the creep rate in the third part $e_{11}^{c\,(3)}$ and steady-state creep rate $e_{11}^{c\,(2)}$ makes it possible to find parameter $r_c\,(\omega, T)$ as a function of ω and T.

4 Results of the study

Paper [30] presents the results of experimentally studying the processes of short-term high-temperature creep of the VZh-159 heat-resistant alloy, which were obtained in the Laboratory for testing physical–mechanical properties of structural materials of Research Institute for Mechanics, National Research Lobachevsky State University of Nizhny Novgorod. The experiments were done on the ZWICKZ030 general-purpose test machine, with a MAYTEC-HT080/1 heating device. The measuring tools included a force transducer, class 1 according to ISO 7500-1 (deviation from the assigned force during one test is within 15H) and a high-temperature sensor of longitudinal strain (type PMA–12/V7-1). To control the temperature inside the high-temperature oven in its three zones and on the specimen, three thermocouples (type K) are used. The assigned temperature was maintained (to the accuracy of 2° C) using a HTO-08/1 electronic control unit.

Tests were done on cylindrical specimens of solid cross section with the length of $l = 50mm$ and diameter $d = 8mm$ of the working part, loaded in uniaxial tension for different levels of normal stresses σ_{11} and temperatures T.

The short-term creep process up to the formation of a macrocrack according to the long-term strength mechanism was numerically simulated, using program ≪EXPMODEL≫ designed for numerically modeling non-isothermal viscoplastic deformation and damage accumulation in structural materials (metals and their alloys) subjected to irregular nonstationary thermal-mechanical loading [31]. The physical–mechanical characteristics of the VZh-159 alloy and the material parameters of the CDM model are listed in the Table 1.

Figures 3, 4, 5 and 6 depict the creep curves for:

- temperature $T = 750^oC$ and stresses $\sigma_{11} = 350$ and 450 MPa, respectively (Fig. 3);
- temperature $T = 850^oC$ and stresses $\sigma_{11} = 120$ and 150 MPa, respectively (Fig. 4);
- temperature $T = 750^oC$ and a transfer from stress level $\sigma_{11} = 350$ to stress level $\sigma_{11} = 450MPa$ (Fig. 5);
- temperature $T = 850^oC$ and a transfer from stress level $\sigma_{11} = 120MPa$ to stress level $\sigma_{11} = 150MPa$ (Fig. 6).

The solid lines in the figures show the numerical modeling results, obtained using defining relations of CDM (1)–(19), whereas the dashed lines correspond to the related experimental results. Both qualitative and quantitative agreement of the experimental and numerical results can be seen.

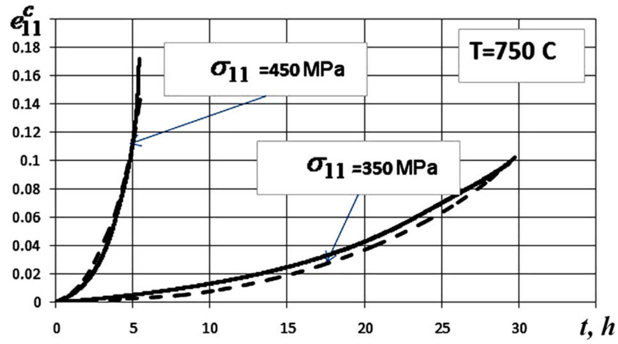


Fig. 3 The creep curves for temperature $T = 750^o C$ and stresses $\sigma_{11} = 350$ and 450 MPa

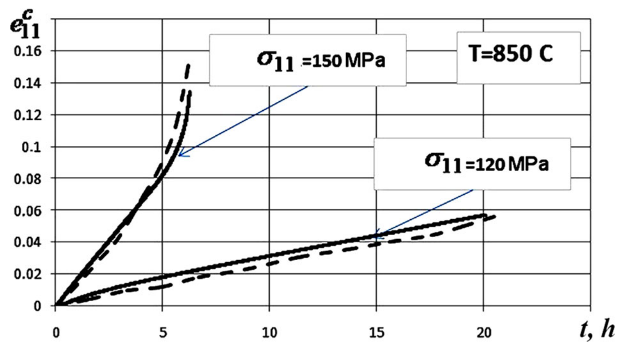


Fig. 4 The creep curves for temperature $T = 850^o C$ and stresses $\sigma_{11} = 120$ and 150 MPa

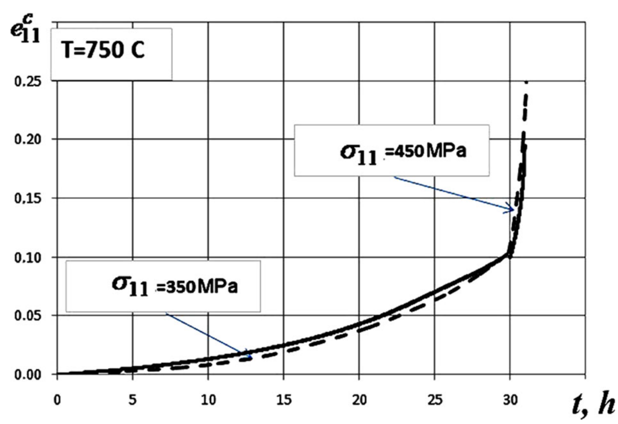


Fig. 5 The creep curves for temperature $T = 750^o C$ and a transfer from stress level $\sigma_{11} = 350$ to stress level $\sigma_{11} = 450 MPa$

Figures 7, 8 and 9 show the following relations for the experiment presented in Fig. 6:

- effective stresses $\tilde{\sigma}_{11}$ as a function of time t of the process (Fig. 7);
- effective radius of the zero level creep surface $\tilde{\bar{C}}_c$ as a function of time t of process (Fig. 8);
- damage value ω as a function of time t of the process (Fig. 9).

In general, analyzing the obtained numerical results, it can be noted that the introduced model of damaged media qualitatively and quantitatively describes the main effects observed in the process of nonstationary creep of structural materials (metals and their alloys) and degradation of the initial strength properties of materials when the degradation follows the long-term strength mechanism.

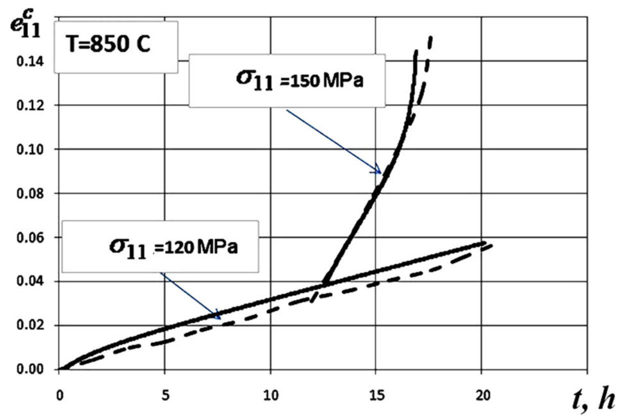


Fig. 6 The creep curves for temperature $T = 850^o C$ and a transfer from stress level $\sigma_{11} = 120 MPa$ to stress level $\sigma_{11} = 150 MPa$

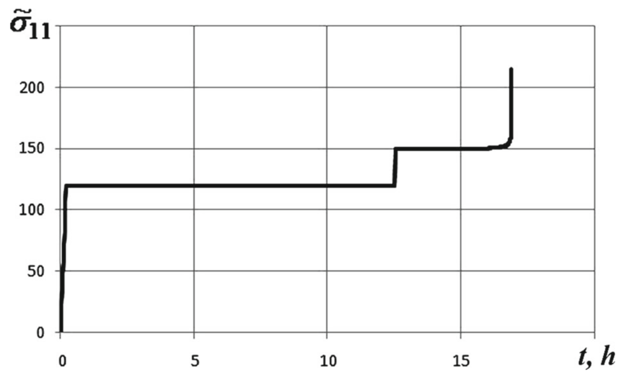


Fig. 7 Effective stresses $\tilde{\sigma}_{11}$ as a function of time t

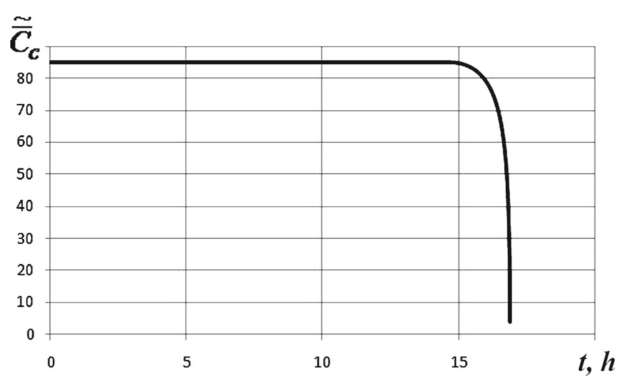


Fig. 8 Effective radius of the zero level creep surface $\tilde{\tilde{C}}_c$ as a function of time t

5 Conclusions

A mathematical model of damaged media has been developed. It describes coupled inelastic deformation and damage accumulation processes in structural materials (metals and their alloys), accompanied by degradation of the initial strength properties of materials according to the long-term strength mechanism. Let us note that inelastic behavior of materials is rather complex, so its proper description requires a lot of efforts form both experimental and theoretical points of view. As a result, in the literature are known a lot of sophisticated models of continuum. Here we proposed a new modification of the models considering some inelastic phenomena (creep, damage, temperature dependence, strength degradation among others). Finally, as an example, a particular material was studied which could be used as heat resistant material.

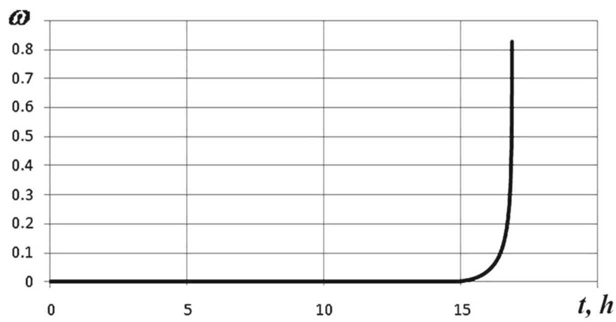


Fig. 9 Damage value ω as a function of time t

A relevance of the discussed constitutive relations of CDM for creep has been assessed, using both the numerical modeling methods and comparison with experimental data. As a result, one can conclude that the proposed methodology accurately determines the material parameters entering the above relations even in the case of limited experimental data. The provided analysis includes using some experimental observations, formulation of mathematical models and systems of basic experiments in order to find the material parameters, study of the applicability limits and validation of the proposed model by conducting numerical and full-scale experiments.

Further experimental and theoretical studies are required for more complex cases such as, for example, non-stationary multiaxial stress-strain states and non-isothermal processes for various arbitrary trajectories of complex loading. In general, the further analysis is also required for experimental data on the coupled mechanisms of material degradation (non-stationary creep, fatigue, their interaction, etc.), and further adaptation of the developed model of coupled processes of destruction of structural materials.

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