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A NEWLY-DEVELOPED MODEL FOR PREDICTING CUTTING POWER DURING WOOD SAWING WITH CIRCULAR SAW BLADES

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ABSTRACT

In the classical approach, cutting forces and cutting power in sawing processes of orthotropic materials such as wood are generally calculated on the basis of the specific cutting resistance k_c (cutting force per unit area of cut). For every type of sawing kinematics (frame saws, band saws and circular sawing machines) different empirical values of specific cutting resistance k_c have to be applied. It should be emphasised that sources in the scientific literature and handbooks do not provide any information about wood provenance, nor about cutting conditions in which cutting resistance had been determined. In analyses of sawing processes in which the offcut is formed by shear, Atkins's ideas that all cutting forms a branch of elastoplastic fracture mechanics can be applied. Thanks to this modern approach it was possible to reveal, using experimental results data of fracture toughness and shear yield stresses of Polish pine (*Pinus sylvestris*), the significant effect of the raw material provenance (source of wood) on cutting power. In the common model for circular sawing machine kinematics, which is similar to metal milling, the sum of all uncut chip thicknesses of the all the teeth simultaneously engaged represented the mean uncut chip thickness. In this work predictions of the newly-developed model for the circular sawing machine are presented. In the model, beside uncut chip thicknesses changes, appropriate changes in shear yield stress and toughness with tooth/grain orientation have been taken into account. The conducted analyses have demonstrated that values of RMS of cutting power obtained with the new developed model are slightly larger than experimental values. On the other hand computed values of cutting power with the use of the mean uncut chip thicknesses in the model are a bit lower from the empirical one.

Keywords: Circular sawing machine, cutting, fracture mechanics, fracture toughness, modelling, orthotropic material, sawing process, wood.

INTRODUCTION

Theoretical and experimental determination of values of forces acting in the cutting process belongs to the fundamental and simultaneously the most explored field of mechanics of the machining process. The process of cutting is analyzed in fracture mechanics terms with a view to quantifying the various parameters involved. The governing parameters in the models (predominantly of orthogonal cutting) are tool geometry, which are principally the rake and wedge angles, geometry of the uncut chip (mainly its thickness), the machined material properties yield stress and fracture toughness, and

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friction conditions between chip and rake surface (Atkins 2003, 2005, 2009, 2016, Williams *et al.* 2010, Williams and Patel 2016, Wyeth and Atkins 2009, Wyeth *et al.* 2009).

Wood may be described as an orthotropic material; having unique and independent mechanical properties in the directions of three perpendicular axes: longitudinal, radial, and tangential. The longitudinal axis direction is parallel to the fibers course. The radial axis is normal to the growth rings, while the tangential axis is perpendicular to the grain but tangent to the growth rings (Kretschmann *et al.* 2010, Laternser *et al.* 2003, Porankiewicz and Goli 2014). These specific features might be taken into account in analyses of energetic effects thanks to building surface energy for separation (i.e. material's fracture toughness) into the models for forecasting of energetic effects (cutting force or cutting power). In wood cutting processes it could be emphasised the use of some elements of fracture mechanics: in the calculation models of cutting forces developed by Laternser *et al.* (2003), analyses made by Merhar and Bučar (2012), theoretical and experimental studies of sawing of pine wood on sash gang saws by Orlowski and Palubicki (2009), in theoretical and empirical studies of circular sawing by Hellström *et al.* (2013) and Kopecký *et al.* (2014), in experimental studies on formation of wood surfaces and chips by Marchal *et al.* (2009).

Otto and Parmigiani (2015) in discussion of their empirical results of the sawing process noticed that shear yield stresses and fracture toughness of the cut raw material could be useful for better explanation of some saw-chain cutting mechanics phenomena, in numerical simulation of orthogonal cutting problems Nairn (2015) described a method called the material point method. The latter has been applied for numerical modelling of orthogonal cutting of wood with a bench plane, in which both full chip formation and the crack growth were able to be simulated (Nairn 2016). Such simulations can be used to optimise woodworking with a bench plane.

The sawing process with circular saw blades experimentally has been investigated in the laboratory conditions Beljo-Lučić *et al.* (2004), in case of industrial conditions by Cristóvão *et al.* (2013), and for wood crosscutting process (Krilek *et al.* 2014).

The presented by Orlowski *et al.* (2013) analytical analyses of energetic effects with the use of cutting models that include work of separation in addition to plasticity and friction corroborated their versatility and revealed the usefulness for every known type of sawing kinematics. The same methodology was applied for forecasting of the capacity of every type of sawing machines (frame sawing machine, circular sawing machine and bandsawing machine), for the real sawing patterns used in the sawmill, in terms of available power, feed rates and so on, leading to comments on the design of machines with different kinematic features (Orlowski *et al.* 2014). In the common model for circular sawing machine kinematics, which is similar to metal milling, the sum of all uncut chip thicknesses of the all the teeth simultaneously engaged represented the mean uncut chip thickness (Orlowski *et al.* 2013, 2014). This work's goal was to demonstrate predictions of the newly-developed model for the circular sawing machine tool. In the proposed model, beside uncut chip thicknesses changes, appropriate changes in shear yield stress and toughness with tooth/grain orientation have been taken into account.

THEORETICAL BACKGROUND

Making an assumption that cutting force F_c acting in the middle of the cutting edge is an equilibrium of forces related to the direction of primary motion for a single saw tooth the mechanical process of material separation from the sawn workpiece, i.e. chip formation, can be approximately described by the example of an orthogonal process (two dimensional deformation) (Orlowski *et al.* 2013).

In the case of circular sawing, identically as in analytical models for milling (Altintas 2000, Ammar *et al.* 2009, Budak 2006), the instantaneous uncut chip thickness $h_j(\varphi)$ at a certain location of the cutting edge (Figure 1) can be approximated as follows:

$$h_j(\varphi) = f_z \sin\varphi_j \tag{1}$$

where: f_z is feed per tooth, φ_j is the angular position of the *j*-th tooth (immersion angle), and its value changes as follows:

$$\varphi_j = \varphi + (j-1)\varphi_P \qquad j = 1,\ldots,z \tag{2}$$

φ_P is the pitch angle defined as $\varphi_P = \dfrac{2\pi}{z}$, and *z* is number of teeth.

If $\phi_{en} \le \phi_j \le \phi_{ex}$, then it has a value, otherwise it is null. ϕ_{en} is an angle of teeth entrance which is given by $\phi_{en} = \arccos\dfrac{2(H_p + a)}{D}$ (when the tool tooth gets into the workpiece for machining), and ϕ_{ex} is an exit angle (the tooth of the saw blade gets out of the workpiece) which can be determined as $\phi_{ex} = \arccos\dfrac{2a}{D}$, *D* is a diameter of the circular saw blade. In the case of cutting with circular saw blades the cutting power a new developed macro-mechanic model (Orlowski and Ochrymiuk 2013), which is based on the model proposed initially in work by Orlowski *et al.* (2013), can be expressed as:

$$P_{cj}(\phi) = v_c S_t \left[\frac{\tau_{\gamma\|\perp_j}(\phi)\cdot\gamma_j(\phi)}{Q_{shear_j}(\phi)} h_j(\phi) + \frac{R_{\|\perp_j}(\phi)}{Q_{shear_j}(\phi)} \right] \tag{3}$$

In the above model (Eq. 3) it was assumed that the cutting edges of teeth are sharp. Moreover, if we assume the cut is straight, and the cut deviation is low; then we can ignore the effect of lateral forces on the power consumption (Mohammadpanah and Hutton 2016a).

In Eq. (3) $\tau_{\gamma\|\perp_j}(\varphi)$ is the shear yield stress, $\gamma_j(\varphi)$ is the shear strain along the shear plane, which is given by:

$$\gamma_j(\varphi) = \frac{\cos\gamma_f}{\cos(\Phi_c(\varphi_j) - \gamma_f)\sin\Phi_c(\varphi_j)} \tag{4}$$

β_μ is friction angle which is given by $\tan^{-1}\mu = \beta_\mu$, with μ the coefficient of friction, γ_f is the rake angle, $\Phi_c(\varphi_j)$ is the shear angle which defines the orientation of the shear plane with respect to cut surface, $R_{\|\perp_j}(\varphi)$ is specific work of surface separation/formation (fracture toughness), and $Q_{shear_j}(\varphi)$ is the friction correction:

$$Q_{shear_j}(\varphi) = [1 - (\sin\beta_\mu \sin\Phi_c(\varphi_j) / \cos(\beta_\mu - \gamma_f)\cos(\Phi_c(\varphi_j) - \gamma_f))] \tag{5}$$

For least cutting force F_c, the shear angle $\Phi_c(\varphi_j)$ (for indirect tooth position) satisfies (Atkins 2003):

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$$\left[1-\frac{\sin\beta_\mu \sin\Phi_c(\varphi_j)}{\cos(\beta_\mu-\gamma_f)\cdot\cos[\Phi_c(\varphi_j)-\gamma_f]}\right]\cdot\left[\frac{1}{\cos^2[\Phi_c(\varphi_j)-\gamma_f]}-\frac{1}{\sin^2\Phi_c(\varphi_j)}\right]=$$

$$=-\left\{\cot\Phi_c(\varphi_j)+\tan[\Phi_c(\varphi_j)-\gamma_f]+Z\right\}\cdot\left[\frac{\sin\beta_\mu}{\cos(\beta_\mu-\gamma_f)}\left\{\frac{\cos\Phi_c(\varphi_j)}{\cos[\Phi_c(\varphi_j)-\gamma_f]}+\frac{\sin\Phi_c(\varphi_j)\sin[\Phi_c(\varphi_j)-\gamma_f]}{\cos^2[\Phi_c(\varphi_j)-\gamma_f]}\right\}\right] \quad (6)$$

in which $Z=\dfrac{R_{\parallel\perp_j}(\varphi)}{\tau_{\gamma|\perp_j}(\varphi)\cdot h_j}$ is the parameter which makes $\Phi_c(\varphi_j)$ material dependent, on the contrary to the classical approach by Merchant (Böllinghaus *et al.* 2009, Markopoulos 2013). Derivation of the formula (6) can be found in Appendix A.

Taking into account the position of the cutting edge in relation to the grains, for indirect positions of the cutting edge fracture toughness $R_{\parallel\perp_j}(\varphi)$ and the shear yield stress $\tau_{\gamma\parallel\perp_j}(\varphi)$ may be calculated from formulae:

$$R_{\parallel\perp_j}(\varphi)=R_{|}\cos^2\varphi_j+R_{\perp}\sin^2\varphi_j \quad \tau_{\gamma\parallel\perp_j}(\varphi)=\tau_{\gamma|}\cos^2\varphi_j+\tau_{\gamma\perp}\sin^2\varphi_j \quad (7)$$

Accordingly to Orlicz (1988), who applied the plane stress transformation equation for determination of specific cutting resistance in indirect positions of the cutting speed direction, the same approach has been implemented for computation of the shear yield stress and fracture toughness as tensor values (Orlowski *et al.* 2013, Hlaskova *et al.* 2015). It ought to be emphasized that the same method is commonly used in general mechanics of materials to transform the stress components from one set of axes to another (Gere 2004).

Maximum, average or RMS (root-mean square) values of power can be determined after one full revolution of the tool, i.e. φ: 0-360° is simulated (Budak 2006). Thus, the total cutting power can then be computed as:

$$P_c(\phi)=\sum_{j=1}^{j=z}P_{cj}(\phi) \quad (8)$$

The obtained values from the Eq. 8 could be augmented by the chip acceleration power P_{ac} variation as a function of mass flow and tool velocity (Orlowski *et al.* 2013, Pantea 1999, Atkins 2009).

MATERIALS AND METHODS

Materials

Scots pine (*Pinus sylvestris* L.) samples (15 items) originating from the Forest Inspectorate Lipusz in the Baltic Natural Forest Region (PL) were used as experimental samples in the sawmill conditions. Samples were in the shape of rectangular blocks with dimensions of 100 mm (*H*) ´ 50 mm (*W*) ´ 2200 mm (*L*) with moisture content MC 35%, density ρ = 509,2 kgm^{-3}. The value of friction coefficient *m* = 0,6 for pine wood was taken according to Glass and Zelinka (2010). The shear yield stresses τ_γ, for perpendicular cutting and for longitudinal cutting, were as follows: $\tau_{\gamma\perp}$ = 24,82 MPa and $\tau_{\gamma\parallel}$ = 18,40 MPa, and fracture toughness for the same directions were $R_\perp$ = 1212,69 Jm^{-2} and $R_\parallel$ = 34,99 Jm^{-2}. It should be emphasized that values of shear yield stresses and fracture toughness were obtained empirically in the cutting tests in the industrial conditions according to the methodology presented in the paper by Orlowski *et al.* (2015). Furthermore, cutting tests have been recommended for determination of the cut material properties as an alternative and effective way (Atkins 2005, Wang *et al.* 2013).

Machine and tool data

The cutting experiment was carried out on the single shaft multi rip sawing machine PWR301 (TOS Svitavy, CZ) at the Complex sawmill in Dziemiany (the Baltic Natural Forest Region, PL). In this type of the circular sawing machine, circular saw blades are clamped on the shaft with collars, on the contrary to the machines with guided spline circular saws which are often used in the North America (Mohammadpanah and Hutton 2016b). The machine settings were as follows: number of saw blades n_b = 1, spindle rotational speed 3800 rpm, cutting speed v_c = 69,64 ms^{-1}, feed speed v_f = 40 m×min^{-1}, (feed per tooth 0,58 mm, average of uncut chip thickness h = 0,373 mm), clearance of a circular saw blade over the workpiece 5 mm, electric engine power P_{EM} = 45 kW, cutting kinematics - up-sawing (conventional sawing). In those cutting conditions the uncut chip thickness starts at minimum and increases toward the end of the cut (Figure 1), it means that the feed direction of the workpiece is opposite to the rotation of circular saw blade (Figure 2).

Changes in current as a function of time during cutting and idling operations were measured with sampling frequency equal to 1 kHz. The details of the measurement system can be found in Appendix B. The cutting power was calculated as a difference of a total electric power P_{ET} and an electric idling power P_{Eid}. Since the measurements were carried out in one phase it was assumed that in other phases values of current have been the same. The real electric power can be calculated as (Engineering Tool Box 2014):

$$P_E = \sqrt{3} \cdot U \cdot I \cdot PF \tag{9}$$

where: U is voltage (V, U = 400 V), I is measured current (A), PF is power factor (PF = cos$F$). It was assumed that for the cutting process the value of PF = 0,8 and for idling PF was equal 0,3.

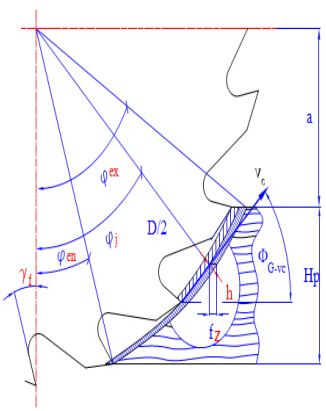

Figure 1. Sawing kinematics on circular sawing machine: H_p workpiece height (depth of cut), a position of the workpiece, φ_j angular tooth position, Φ_{G-vc} an angle between grains and the cutting speed direction, φ_{en} an angle of teeth entrance, φ_{ex} an exit angle.

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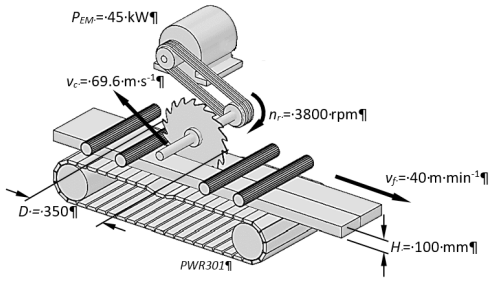

Figure 2. Scheme of the kinematic system of the single shaft multi rip sawing machine PWR301.

One circular saw blade was applied with data as follows: 350 mm (D) × 80 mm (d) × 2,5 mm (s), overall set S_t = 3,9 mm number of carbide tipped teeth z = 18. The tooth geometry in the tool-in-hand system (Astakhov 2010) is shown in Figure 3, and the values of angles applied in the circular saw blade were as follows: the tool side flank α_f = 12 º, the tool side rake γ_f = 25º, the tool back flank $\alpha'_{p1} = \alpha'_{p2}$ = 2º, tool cutting edge angle κ_r = 90°, minor tool cutting edge angle $\kappa'_{r1} = \kappa'_{r2}$ = 2°, the cutting edge inclination angle λ_s = 0°.

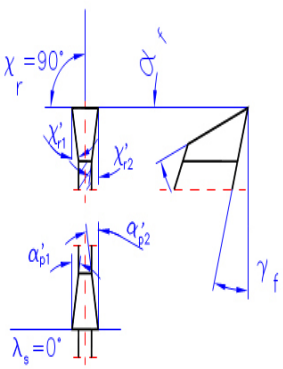

Figure 3. The circular saw blade tooth geometry in the tool-in-hand system: α_f the tool side flank, α'_p the tool back flank, κ_r tool cutting edge angle, κ'_r minor tool cutting edge angle, γ_f the tool side rake, λ_s the cutting edge inclination angle.

Analysis of cutting power consumption

Predictions of cutting model that includes work of separation in addition to plasticity and friction in the case of sawing pine wood on examined circular sawing machine have been carried out with the model presented in this paper. Besides uncut chip thicknesses changes, appropriate changes in shear yield stress and toughness with tooth/grain orientation had been taken into account.

Moreover, additional computations have been done for the model proposed in the work by Orlowski *et al.* (2013), in which the sum of all uncut chip thicknesses of the all the teeth simultaneously engaged represented the mean uncut chip thickness. In predictions the raw material data, computed with Equations (7), was as follows: shear yield stresses $\tau_{\gamma\|\perp}(\overline{\varphi} = 40{,}07°) = 23064099$ Pa (for the tooth position at the average an angle of tooth contact with a workpiece $\overline{\varphi}$, which was calculated from $\overline{\phi} = \frac{\phi_{en} + \phi_{ex}}{2}$), and fracture toughness $R_{\|\perp}(\overline{\varphi} = 40{,}07°) = 523$ Jm^{-2}. In this model the shear strain along the shear plane γ, the shear angle Φ_c, and the friction correction Q_{shear} were also calculated for the average angle of tooth contact with a workpiece $\overline{\varphi}$.

The both ploughing effect (Blackmann *et al.* 2013, Wang *et al.* 2013), and chip momentum were disregarded (Orlowski *et al.* 2013).

RESULTS AND DISCUSSION

Results of predictions of cutting power obtained with the use of a new developed cutting model that include work of separation in addition to plasticity and friction in the case of sawing of pine from the Forest Inspectorate Lipusz in the Baltic Natural Forest Region (PL) provenance with one circular saw blade, at the feed speed $v_f = 40$ m·min^{-1}, for one full revolution of the tool (the first one), are shown in Figure 4. In this computation beside uncut chip thicknesses changes, appropriate changes in shear yield stress and toughness with tooth/grain orientation had been taken into account. For a stable condition of cutting power changes in the new developed model (N_M) the RMS value was computed, which was equal to P_c(RMS_N_M) = 13774±147 W. Cutting power estimation can be found in Appendix C.

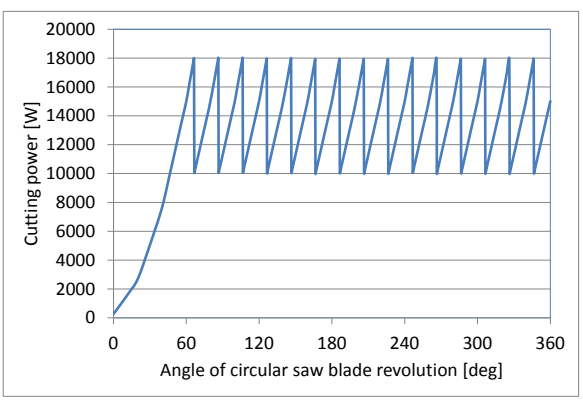

Figure 4. Predictions of cutting power obtained with the use of a new developed cutting model that include work of separation in addition to plasticity and friction for the circular sawing machine in the case of dry pine sawing with one circular saw blade.

In the model for circular sawing machine kinematics, presented in the paper by Orlowski *et al.* (2013), in which the sum of all uncut chip thicknesses of the all the teeth simultaneously engaged represented the mean uncut chip thickness, the predicted cutting power was equal to P_c(Av Thick) = 13100±147 W.

The cutting power measured in sawmill conditions was equal to P_c(Exper) = 13563±45 W. Figure 5 shows the comparison of cutting powers obtained: with the use of a new developed cutting model (N_M), in the experiment in sawmill conditions (Exper, an average value), and with a cutting model in which a mean value of uncut chip thicknesses is applied (AV Thick).

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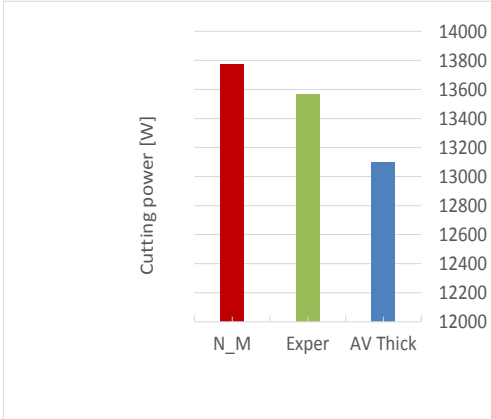

Figure 5. Comparison of cutting power obtained: with the use of a new developed cutting model (N_M), in the experiment in sawmill conditions (Exper), and with a cutting model in which a mean value of uncut chip thicknesses is applied (AV Thick).

This comparison revealed that the value of cutting power for a new model (N_M) is larger than the experimental one about 1,56% and the cutting power for the model (AV Thick) is lower about 3,41%. For the both models, differences in relation to the experimental values are rather small. Nevertheless, the presented model is time consuming: the computation time for one revolution of the circular saw blade takes about 1800 s, on the contrary for the cutting power model (AV Thick), which gives an insignificantly larger error of predictions, takes only 10 s. Hence, this new-developed cutting model could be recommended for forecasting changes of cutting forces in case of dynamical analyses and unsteady cases. Moreover, for predictions of cutting power for circular sawing process, the model that applied in a sum of mean value of uncut chip thicknesses is highly recommended.

CONCLUSIONS

This paper shows that estimation of energetic effects using the newly-developed cutting model that include work of separation in addition to plasticity and friction is capable of predictions not only average values of cutting power but also its dynamical changes. Furthermore, this approach extends opportunities to modeling of the cutting process of with circular saw blades. The conducted analyses have demonstrated that values of RMS of cutting power obtained with the new developed model are slightly larger than experimental values. On the other hand computed values of cutting power with the use of the mean uncut chip thicknesses in the model are a bit lower from the empirical one. Nevertheless, since the time consuming process of computation for industrial application this model should not be recommended. Moreover, it could be changed by application of the more power computer, or the use of highly optimized simulation code dedicated to the process control units.

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Appendix A

According to (Atkins 2003) internal work is described by three parts: plasticity along shear plane (a), friction along the underside of the chip (b) and formation of new cut surface (c)

$$F_c \cdot v_c = S_t \cdot \tau_y \cdot h \cdot v_c \cdot \left(\cot(\phi_c) - \tan(-\phi_c + y_f)\right) + \frac{F_c \cdot v_c \cdot \sin(\beta_\mu)\sin(\phi_c)}{\cos\left(\beta_\mu - y_f\right)\cos\left(-\phi_c + y_f\right)} + R_c \cdot S_t \cdot v_c \quad \text{(A1)}$$

a) *b)* *c)*

From (A1) it can be obtained

$$\frac{F_c}{S_t \cdot \tau_y \cdot h} = \frac{-\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f)(\cot(\phi_c) - \tan(-\phi_c + y_f) + z)}{\sin(\beta_\mu)\sin(\phi_c) - \cos(\beta_\mu - y_f)\cos(-\phi_c + y_f)} \quad \text{(A2)}$$

where

$$Z = \frac{R}{\tau_y \cdot h} \quad \text{(A3)}$$

Hence, the parameter Z makes Φ_c material dependent. According to the principle of minimum work, the work done in cutting should be at minimum, and thus its derivative should be zero, i.e.

$$\frac{\partial F_c \cdot v_c}{\partial \phi_c} = v_c \frac{\partial F_c}{\partial \phi_c} = 0 \quad \text{(A4)}$$

and the cutting velocity v_c is not equal to zero, it follows from Eq. (A4) that

$$\frac{\partial F_c}{\partial \phi_c} = 0 \quad \text{(A5)}$$

Substituting Eq. (A2) to Eq. (A5) we can obtain

$$\begin{aligned}\frac{\partial F_c}{\partial \phi_c} &= \frac{\cos(\beta_\mu - y_f)\sin(-\phi_c + y_f)(\cot(\phi_c) - \tan(-\phi_c + y_f) + z)}{\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f) - \sin(\beta_\mu)\sin(\phi_c)} \\ &+ \frac{\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f)\left(-(\cot(\phi_c))^2 + \left(\tan\left(-\phi_c + y_f\right)\right)^2\right)}{\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f) - \sin(\beta_\mu)\sin(\phi_c)} \\ &- \frac{\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f)\left(\cot(\phi_c) - \tan\left(-\phi_c + y_f\right) + z\right)\left(\cos(\beta_\mu - y_f)\sin\left(-\phi_c + y_f\right) - \sin(\beta_\mu)\cos(\phi_c)\right)}{\left(\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f) - \sin(\beta_\mu)\sin(\phi_c)\right)^2} = 0\end{aligned}$$

Or

$$\left(\left(\sin(\phi_c)\right)^2\left(\begin{array}{l}\left(\cos(\beta_\mu - y_f)\right)^2\left(\cos(-\phi_c + y_f)\right)^2 - 2\cos(\beta_\mu - y_f)\cos(-\phi_c + y_f)\sin(\beta_\mu)\sin(\phi_c)\\ +1-\cos(\phi_c)^2 - \cos(\beta_\mu)^2 + \cos(\beta_\mu)^2\cos(\phi_c)^2\end{array}\right)\right)^{-1}$$

$$\left(\cos(\beta_\mu - y_f)\left(\begin{array}{l}2\sin\left(-\phi_c + y_f\right)\sin(\beta_\mu)\cos(\phi_c) - 2\sin\left(-\phi_c + y_f\right)\sin(\beta_\mu)(\cos(\phi_c))^2\\ +\sin(-\phi_c + y_f)Z\sin(\beta_\mu)\sin(\phi_c) - \sin(-\phi_c + y_f)Z\sin(\beta_\mu)\sin(\phi_c)(\cos(\phi_c))^2\\ --2\cos(-\phi_c + y_f)(\cos(\phi_c))^2\sin(\beta_\mu)\sin(\phi_c) - \cos(\beta_\mu - y_f) + \cos(\beta_\mu - y_f)(\cos(\phi_c))^2\\ +\cos(\beta_\mu - y_f)\left(\cos(-\phi_c + y_f)\right)^2 - \cos(-\phi_c + y_f)Z\sin(\beta_\mu)\cos(\phi_c)\\ +\cos(-\phi_c + y_f)Z\sin(\beta_\mu)\left(\cos(\phi_c)\right)^2\end{array}\right)\right) = 0$$

Manipulating the trig function we obtain equation proposed in (Atkins 2003)

$$\left(1 - \frac{\sin(\beta_\mu)\sin(\phi_c)}{\cos(-\beta_\mu + y_f)\cos(-\phi + y_f)}\right)\left(\frac{1}{\cos(-\phi_c + y_f)^2} - \frac{1}{\sin(\phi_c)^2}\right) + \frac{(\cot(\phi_c) - \tan(-\phi_c + y_f) + Z)\sin(\beta_\mu)\left(\frac{\cos(\phi_c)}{\cos(-\phi_c + y_f)} + \frac{\sin(\phi_c)\sin(-\phi + y_f)}{\cos(-\phi_c + y_f)^2}\right)}{\cos(-\beta_\mu + y_f)} = 0 \quad \text{(A8)}$$

Appendix B

Cutting power measurement system. The cutting power measurements were performed using a system consisting of a probe AC/DC current transducer DHR 100C10 (LEM USA Inc.), high-accuracy isothermal terminal block NI SCXI 1328 (National Instruments, USA), an 8-channel isolation amplifier NI SCXI 1125 (National Instruments, USA), 4-Slot Chassis NISCXI 1000DC (National Instruments, USA) and computer with the NI PCI 6281 card (National Instruments, USA).

Appendix C

Based on equation (3) the error of the cutting power could be expresses in the form

$$\delta P_c = \frac{v_c S_t}{Qshear}\left(y \cdot h \cdot \delta\tau_{y\|\perp} + \delta R_{\|\perp}\right) \quad \text{(C1)}$$

Where: $\delta\tau_{y\|\perp}$ and $\delta R_{\|\perp}$ are the shear yield stress and fracture toughness errors of the following values (Chuchala and Orlowski 2016): $\delta\tau_{y\|\perp} = 0,27$ MPa and $\delta R_{\|\perp} = 384 J/m^2$.

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Assuming average values depending on angular position variables in Eq. (C1), it can be obtained a value of the estimated cutting power error $\delta P_c = 147W$.