

A NOTE ON POLYNOMIAL ALGORITHM FOR COST COLORING OF BIPARTITE GRAPHS WITH $\Delta \leq 4$

KRZYSZTOF GIARO AND MAREK KUBALE

Gdańsk University of Technology, ETI Faculty
Gabriela Narutowicza 11/12
80-233 Gdańsk, Poland

e-mail: giaro@pg.edu.pl
kubale@eti.pg.gda.pl

Abstract

In the note we consider vertex coloring of a graph in which each color has an associated cost which is incurred each time the color is assigned to a vertex. The cost of coloring is the sum of costs incurred at each vertex. We show that the minimum cost coloring problem for n -vertex bipartite graph of degree $\Delta \leq 4$ can be solved in $O(n^2)$ time. This extends Jansen's result [K. Jansen, *The optimum cost chromatic partition problem*, in: Proc. CIAC'97, Lecture Notes in Comput. Sci. 1203 (1997) 25–36] for paths and cycles to subgraphs of biquartic graphs.

Keywords: bipartite graph, chromatic sum, cost coloring, NP-completeness, polynomial algorithm.

2010 Mathematics Subject Classification: 05C15.

REFERENCES

- [1] J. Cardinal, V. Ravelomanana and M. Valencia-Pabon, *Minimum sum edge colorings of multicycles*, Discrete Appl. Math. **158** (2010) 1216–1223.
doi:10.1016/j.dam.2009.04.020
- [2] K. Giaro and M. Kubale, *Efficient list cost coloring of vertices and/or edges of bounded cyclicity graphs*, Discuss. Math. Graph Theory **29** (2009) 361–376.
doi:10.7151/dmgt.1452
- [3] K. Jansen, *Complexity results for the optimum cost chromatic partition problem*, Forschungsbericht, Trier University (1996) 96–41.
- [4] K. Jansen, *The optimum cost chromatic partition problem*, in: Proc. CIAC'97, Lecture Notes in Comput. Sci. **1203** (1997) 25–36.
doi:10.1007/3-540-62592-5_58

- [5] K. Jansen, *Approximation results for the optimum cost chromatic partition problem*, J. Algorithms **34** (2000) 54–89.
doi:10.1006/jagm.1999.1022
- [6] V. King, S. Rao and R. Tarjan, *A faster deterministic maximum flow algorithm*, J. Algorithms **17** (1994) 447–474.
doi:10.1006/jagm.1994.1044
- [7] L.G. Kroon, A. Sen, H. Deng and A. Roy, *The optimal cost chromatic partition problem for trees and interval graphs*, in: Proc. WG'96 IWGTCCS, Lecture Notes in Comput. Sci. **1197** (1997) 279–292.
doi:10.1007/3-540-62559-3_23
- [8] A. Kosowski, *A note on the strength and minimum color sum of bipartite graphs*, Discrete Appl. Math. **157** (2009) 2552–2554.
doi:10.1016/j.dam.2009.03.008
- [9] E. Kubicka, *The Chromatic Sum of a Graph* (Ph.D. Thesis, Western Michigan University, Kalamazoo, 1989).
- [10] M. Małafiejski, K. Giaro, R. Janczewski and M. Kubale, *A 27/26-approximation algorithm for the chromatic sum coloring of bipartite graphs*, in: Proc. APPROX'02, Lecture Notes in Comput. Sci. **2462** (2002) 136–146.
doi:10.1007/3-540-45753-4_13
- [11] J.B. Orlin, *Max flows in $O(nm)$ time, or better*, in: Proc. STOC'13 (ACM, New York, 2013) 765–774.
doi:10.1145/2488608.2488705
- [12] K. Supowit, *Finding a maximum planar subset of a set of nets in a channel*, IEEE Trans. Comput.-Aided Design **6** (1987) 93–94.
doi:10.1109/TCAD.1987.1270250
- [13] X. Zhou and T. Nishizeki, *Algorithm for the cost edge-coloring of trees*, in: Proc. COCOON'01, Lecture Notes in Comput. Sci. **2108** (2001) 288–297.
doi:10.1007/3-540-44679-6_32

Received 18 October 2018

Accepted 19 March 2019