ACCURACY AND PARAMETER ESTIMATION OF ELASTIC AND VISCOELASTIC MODELS OF THE WATER HAMMER

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Abstract: The water hammer problem is considered, one of the most important questions of unsteady flows in pipelines. Although first mentioned in the scientific literature more than a hundred years ago and widely analyzed since in many research centers, the problem is not fully recognized yet. It may be considered on two levels: practical and theoretical. In both cases, several difficulties arise rendering the results less than fully satisfactory. The most important difficulties are the proper mathematical description of the phenomenon, the choice of the solution method, estimation of the model parameters and numerical aspects of solving the equations governing the phenomenon's run. They are presented in the paper and typical approaches to their solution are discussed. Numerical solutions are compared with experimental results.

Keywords: water hammer, unsteady flow in pipelines, parameter estimation, numerical methods

1. Introduction

Water hammer is a popular name of a rapidly varied unsteady flow in a pipeline, with violent changes of pressure values due to sudden changes of flow velocity. These changes are usually caused by rapid valve operation (opening or closure), turbomachinery regulation, abrupt changes in the operation of pumps, pump failures and other accidents in pipelines, mechanical vibrations of elements, *etc.* The pressure changes propagate through the pipeline in the form of rapid pressure waves, being sequences of sudden, consecutive pressure increases and decreases. Due to liquids' compressibility and the elasticity of pipe walls, the celerity of such disturbances may exceed 1000 m/s, and the pressure values oscillate from very high (often exceeding the pipelines' permissible values) to very low (sometimes leading to underpressure). Waves of this kind may be reflected at pipe-end boundaries or internal features such as changes in cross-section, changes in pipe material or branches, further modifying the frequencies or periods of oscillations. From the practical point of view, the phenomenon is dangerous for pipelines and fittings, as the propagation and reflection of such waves often causes noises, leaks, serious damage to pipelines, various forms of cavitation, gas release, column separation and other negative consequences.

As the importance of the water hammer, mainly associated with water distribution systems, extends well beyond sanitary engineering, into acoustics, solid mechanics, civil engineering, widely understood water management and even hemodynamics, the phenomenon has been the subject of numerous analyses since the late 19^{th} century.

Ménabréa [1], Michaud [2], Korteweg [3], Joukowsky [4], Frizell [5] and Allevi [6, 7] are usually mentioned as authors of the first works focused on the water hammer problem (*e.g.* [8, 9]). However, there were other important but less known contributions to the history of water hammer research, such as [10] or [11] (see also [12]). Although those early analyses were preliminary in character, they did discuss the basic theoretical problems: the wave celerity formula was derived, the description of wave transformation and the influence of pipeline junctions and fixtures were taken into consideration.

Many other works have been presented since then (the most popular being [13–16]) and significant progress has been made in the phenomenon description. However, despite the tremendous progress in mathematical modeling and measurements, the water hammer is still one of the most interesting problems of pipeline hydraulics and the subject of numerous publications (*e.g.* [17–23] or [9, 24–28] by Polish authors).

2. Practical and theoretical aspects of water hammer analysis

The phenomenon of water hammer may be considered in its two aspects [26]. From the practical point of view, the most important question is to determine the extreme values of pressure, usually considered equal to the peak of the pressure wave's first amplitude. Later on the pressure wave is attenuated due to various reasons including flow resistance. The intensity of this damping influences the phenomenon's duration, depending on the material of the pipes, the initial value of flow discharge, the kind of fluid and other factors (usually a matter of a few seconds). Thus, for some researchers the most important task is to determine the first amplitude and the increase in pressure. In this approach, the oscillation period, attenuation intensity and other characteristics are of lesser importance. The maximal value of pressure during a simple water hammer may be calculated from a theoretical formula developed in the 19^{th} century as one of the first basic equations describing the water hammer (often referred to as the Joukovsky or Ellievi equation). As it often produces values close to the observed ones, the water hammer problem is sometimes considered to be solved. However, the situation is relatively easy only as long as a simple case of a single pipe of constant diameter is considered. In most practical cases, the situation is far more complicated and additional factors influence the water hammer, such as obstacles in the pipeline, diameter changes, complex pipeline systems, *etc.* For example, when a pipeline network is considered, not only the maximal value of pressure cannot be easily calculated from the simple formulas, but the location of its occurrence is not easily recognizable. The pressure wave may propagate through complicated piping networks, waves may be superimposed or reflected from obstacles and predicting the extreme pressure values is not easy. Experiments have proven that even for relatively simple pipeline systems – such us a single pipeline with one diameter change – the maximal value of pressure may occur not in the first amplitude and be inconsistent with the value obtained from the theoretical formula [29, 30]. Thus, both from the practical and the theoretical point of view, there remain some aspects that have not been recognized to a satisfactory degree.

From the theoretical point of view, proper description of the phenomenon is the most important. Thus, not only should coincidence of the first pressure wave amplitude be achieved, but also consistency of calculated and measured values of pressure for the whole duration of the water hammer phenomenon. It is not only interesting from the cognitive point of view, but also important for more detailed analyses, in which the observations and predictions of pressure characteristics may be applied to solve practical problems of a different nature, *e.g.* localization of leaks in a pipeline [21]. However, in most cases it is very difficult or impossible to obtain good coincidence between numerical solutions and measurements and calculation results are often significantly different from observations. The problems arising when solving water hammer equations are of various kinds, the most important being proper mathematical description of the phenomenon, parameter estimation and numerical aspects of the solution [31].

3. The traditional mathematical description of the water hammer

The fundamental equation of water hammer theory and one of the first approaches to the phenomenon's analysis is a relation governing pressure changes due to velocity changes in a pipeline:

$$\Delta p = \varrho c \Delta v, \tag{1}$$

where p is pressure, ϱ – fluid mass density, v – velocity and c – the speed of sound in the pipeline's material.

Equation (1) is commonly known as the Joukovsky equation (or the Joukovsky-Frizell equation) or the Allievi formula, and its first explicit statement in the water hammer context is usually attributed to Joukovsky [4], although some authors claim that it had been derived earlier, in a more general context, by Rankine [10]. This simple relation enables calculating theoretical values of maximal and minimal pressure in a pipeline. However, as has been mentioned above, it is a theoretical formula, valid for simple cases of water hammer in single pipelines of constant diameter, constant fluid density and constant c. Nevertheless, the importance of this relation for water hammer analysis is significant.

The speed of sound, c, in Equation (1), which also defines the speed of pressure wave propagation in a pipeline, was originally proposed by Korteweg [3] for cylindrical pipes of constant diameter:

$$c = \sqrt{\frac{K/\rho}{1 + KD/Ee}}, \tag{2}$$

where D is the pipe's diameter, e – the thickness of its walls, E – the pipe walls' modulus of elasticity and K – the bulk modulus of the fluid.

The two formulas presented above were the first, fundamental relations of water hammer theory, thus often treated as the statements most important for the phenomenon's practical analysis. However, they did not describe the phenomenon, merely specifying the range of pressure changes and the speed of wave propagation.

The traditional description of a water hammer in a pipeline, presented by Chaudhry [32] and Streeter and Wylie [16], can be expressed as a set of two equations:

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+R_0Q|Q|=0, \tag{3a}$$

where $R_0=\frac{8}{g\pi^2}\frac{\lambda}{D^5}$, and

$$\frac{\partial H}{\partial t}+\frac{c^2}{gA}\frac{\partial Q}{\partial x}=0, \tag{3b}$$

where Q is the rate of discharge, H – water head, g – acceleration due to gravity, c – wave celerity, A – cross-section area, D – internal pipe diameter and λ – the linear friction factor.

For many years, this set of equations had been treated as a complete description of the phenomenon. The next great step in water hammer analysis was possible thanks to improvements in measurement techniques and mathematical modeling. It brought unexpected conclusions and significant altered the problem's description and solution.

4. Problems in solving water hammer equations

4.1. Accuracy of mathematical description

As has been mentioned above, thanks to the appreciable improvements in measuring techniques and fast progress in mathematical modeling, comparison of observed water hammer characteristics (mainly pressure changes during the phenomenon) with calculation results based on Equation (3) became possible. It demonstrated significant differences between the two.

Examples of water hammer observations and calculations for selected cases of single pipelines are shown in Figure 1. In each case the main part of the system was a single pipeline of constant diameter, wall thickness and material (steel or polymer).

The pipe was fed from a large pressure reservoir in which a constant value of pressure was enforced during the experiment. Rapidly varied unsteady water flow was caused by a sudden closure of a ball valve mounted at the pipe's end. The water hammer pressure characteristics were measured with a tensiometer indicator and the time of valve closing was measured with a precise electronic stop-watch connected to the valve. The characteristic parameters of the pipeline and initial conditions for each case are given in the Figure's caption. Results of measurements and calculations for steel and polymer pipelines are presented.

The differences between observations and calculations are significant. They concern not only the amplitude of pressure changes but also the shape of the oscillations' characteristic, the phenomenon's duration and the frequency of oscillations. Notably, the examples show quite simple cases of single pipelines of constant diameter. For more complicated situations, *e.g.* changes in diameter or piping material, and for pipeline systems, the discrepancies are even greater. This led to the conclusion that the traditional description of the water hammer is incomplete and inadequate. Attempts to

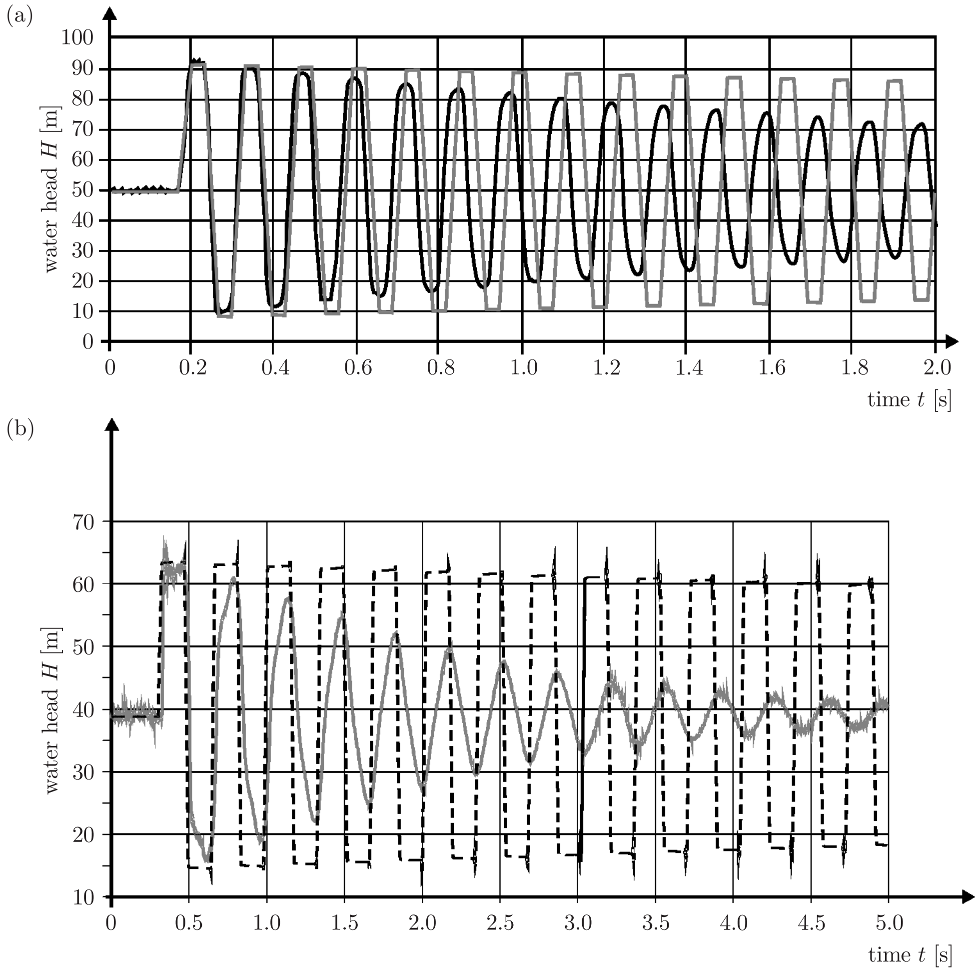

Figure 1. Water hammer in a single pipeline. Comparison of measurements and calculations with the traditional model (black line – measured, gray line – calculated pressure characteristics):

(a) steel pipeline, data and measurements after [9]:
method of characteristic (Cr = 1, $dx = 1.367$m, $dt = 0.00108$s), pipe length $L = 41$m, intrinsic diameter $D = 42$mm, wall thickness $e = 3.0$mm, wave speed $a = 1260$m/s, friction coefficent $\lambda = 0.055$, initial discharge $Q = 0.453\,\mathrm{dm}^3$/s, initial water head 50m, valve closure time 0.034s;

(b) polymer pipeline, data and measurements after [26]:
four-point scheme ($\theta = 0.5$, $\psi = 0.5$, Cr = 1, $dx = 1$m, $dt = 0.00236$s), pipe length $L = 36$m, intrinsic diameter $D = 40.8$mm, wall thickness $e = 4.6$mm, wave speed $a = 423$m/s, pipe roughness 0.004m, initial discharge $Q = 0.744\,\mathrm{dm}^3$/s, initial water head 38.8m, valve closure time 0.024s

improve the classical description and recognize the main factors influencing the phenomenon have been carried out since. As a result, the original set of equations (3) was often replaced with a more complicated description, taking into account different potential reasons of the appreciable lack of consistence between measurements and calculations.

In one approach, the main reason of the above-mentioned discrepancy is considered to be improperly estimated friction. As it was proved that the equation governing friction losses in steady flow did not reproduce the unsteady conditions properly, particular emphasis was put on modification of the friction term in the momentum equation. Many different modifications can be found, from the simplest idea of multiplying the friction coefficient, λ, by a constant (as high as 10 or more) [9] to introducing additional terms to the friction formula, dependent on space and/or time velocity derivatives [23, 33–38]. The results obtained from calculations approached the observations, what gave an impression that such modification is the answer to the problems of water hammer solution. However, when the question was thoroughly analyzed, it turned out that the problem was still not solved to a satisfactory degree. Szymkiewicz [28] proved that modification of the friction term, even in the more sophisticated approach of the Bergant *et al.* [38] or Brunone *et al.* [34] models, could not solve the problem, even if apparently satisfactory coincidence between calculations and observations was obtained. This is due to the features of Equations (3), which are of the hyperbolic type. An analysis of its solution's accuracy has proven that the coincidence between observations and calculations is a result of numerical dissipation rather than the modification of the friction term. Furthermore, as long as the set of equations is hyperbolic, proper reproduction of the "natural" dissipation observed in measurements is impossible (the shape of the pressure characteristics is not "smooth") unless "artificial" factors are introduced (*e.g.* numerical dissipation). A thorough consideration of this problem, with detailed analysis of the models recommended by various authors, can be found in [28]. Some of the effects discussed above that influence the calculation results for the pipeline example shown in Figure 1a may be illustrated as in Figure 2.

Another aspect of the accuracy of the phenomenon's mathematical description is connected with polymer pipelines. The differences between calculations and observations are particularly clear for such pipes and the effects mentioned above do not eliminate them. This is due to the material's viscoelastic behavior in reaction to stress. Equation (3), describing an elastic model, may be applied for steel pipes and in preliminary calculations for plastic pipes. If more accurate calculations are required, it is necessary to develop a form of mathematical description taking into account the viscoelastic character of pipe walls' deformations [20, 22]. A detailed analysis of the viscoelastic behavior of polymers and models capable of its reproduction can be found in [39] or [40]. The viscoelastic behavior is characterized by instantaneous elastic strain followed by gradual retarded strain for the applied load [41]. This time-dependent strain behavior resulting from constant loading is defined as creep and depends on the body's molecular structure, the stress-time history and temperature.

There are two main approaches to describing the specific behavior of polymers during a water hammer. In the more popular one, the phenomenon is described with the Kelvin-Voigt model of a number of elements (N elements in the general approach) [31, 41–43]. In this model, the behavior of the pipe's material is referred to the behavior of the set of N parallel systems of springs and dashpots connected as shown in Figure 3.

As a consequence, the set of equations governing the phenomenon assumes the following form:

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+R_0Q|Q|=0, \tag{4a}$$

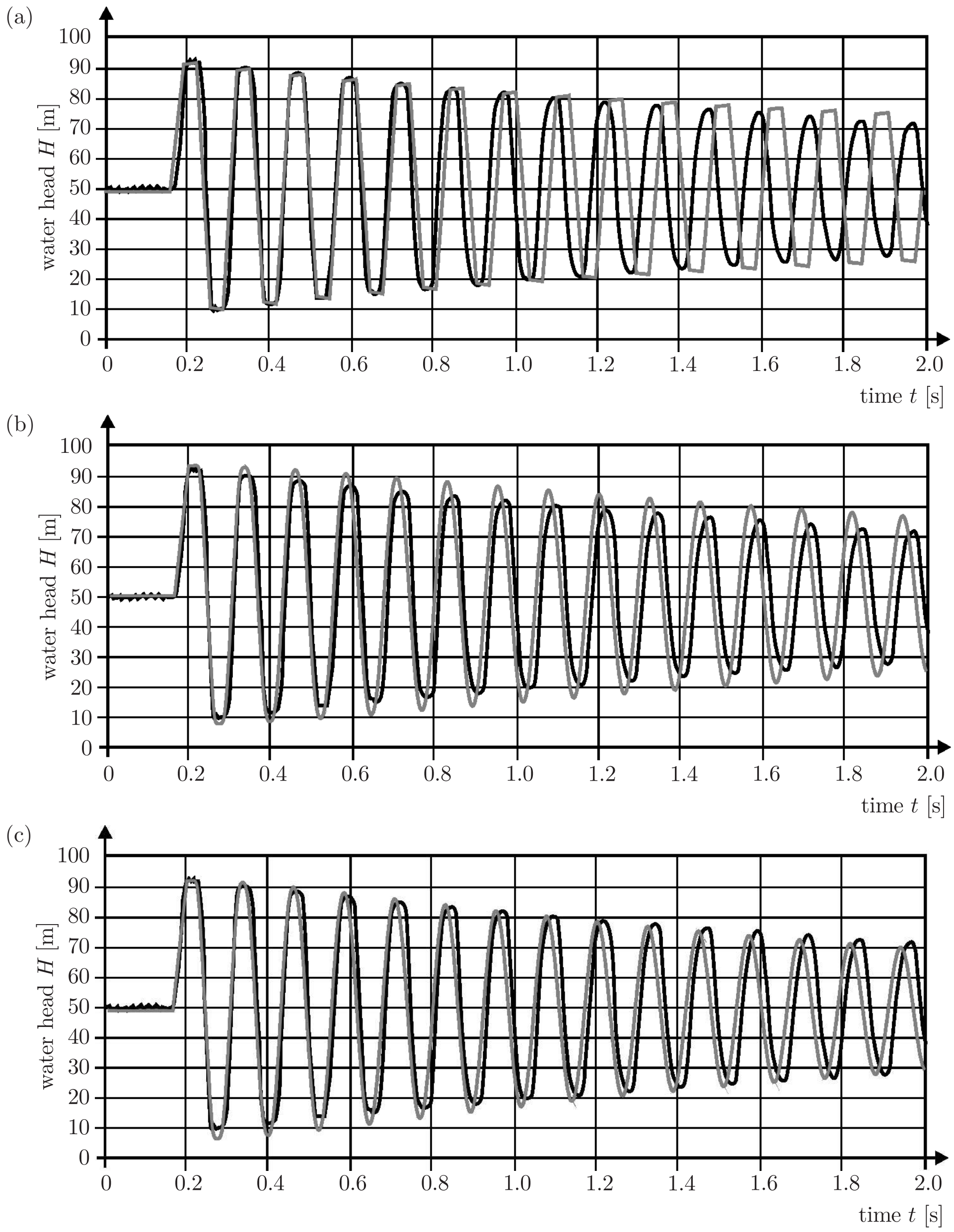

Figure 2. Influence of various factors on the water hammer solution (black line – measured, gray line – calculated pressure characteristics, pipeline data as in Figure 1a):

(a) increased friction coefficient (method of characteristic, $\mathrm{Cr}=1$, $dx=1.367\mathrm{m}$, $dt=0.00108\mathrm{s}$, $\lambda=0.250$);

(b) change of wave speed and slight numerical dissipation (four-point scheme, $\theta=0.6$, $\psi=0.5$, $dx=1.367\mathrm{m}$, $dt=0.00108\mathrm{s}$, $\lambda=0.055$, $a=1324\mathrm{m/s}$);

(c) change of wave speed, calculation of the friction coefficient and stronger numerical dissipation (four-point scheme, $\theta=0.65$, $\psi=0.5$, $dx=1.367\mathrm{m}$, $dt=0.00108\mathrm{s}$, λ calculated from Colebrook-White formula, $a=1324\mathrm{m/s}$)

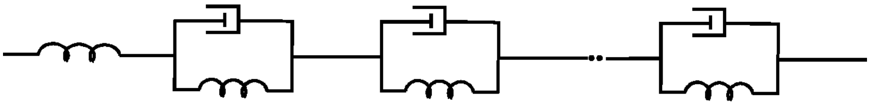

Figure 3. The Kelvin-Voigt model for the viscoelastic behavior of a polymer pipeline

where $R_0 = \dfrac{8}{g\pi^2}\dfrac{\lambda}{D^5}$,

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} + \frac{2a^2}{g}\sum_{i=1}^{N}\frac{\partial \varepsilon_i}{\partial t} = 0, \tag{4b}$$

where

$$\frac{\partial \varepsilon_i}{\partial t} = \frac{1}{\tau_i}\left(\frac{pDc_1}{2eE_i} - \varepsilon_i\right). \tag{4c}$$

In the above equations, ε_i ($i = 1,2\ldots,N$, N being the total number of elements) is the retarded strain of each i^{th} element of the Kelvin-Voigt model, while E_i and τ_i are the tensile modulus and retardation time for the i^{th} Kelvin-Voigt element. The speed of wave propagation is marked as a in this case, in order to distinguish it from the speed of sound from Equation (3). Coefficient c_1 is a constant dependent on the manner of pipeline fastening. A more detailed derivation of Equation (4) on the basis of the traditional water hammer model can be found in [42] and [31].

An exemplary application of the model (4), compared with (3), is shown in Figure 4. Introduction of the additional term representing the viscoelasticity of pipe walls to the mass equation enables obtaining improved calculation results for water hammers in polymer pipes. However, this approach poses its own problems: what is the optimal number of Kelvin-Voigt elements, what are the proper values of E_i and τ_i, and how to determine the proper value of wave speed a. These questions will be discussed below.

4.2. Choice of the solution method. Numerical aspects of the solution.

As has been mentioned above, the numerical approach to the problem is an important question connected with solving the water hammer equations. It is connected not only with the choice of solution method, but also with estimation of numerical parameters (such as distance and time steps influencing the Courant number and the specific parameters connected with the particular method) and proper interpretation of their influence on the solution. The role of numerical aspects is particularly important in the water hammer case. Numerical effects are known to significantly alter the solution, not only with respect to the numerical quantities but – even more importantly – the phenomenon's character. The water hammer is a distinct example of a problem wherein it is particularly easy to lose touch with the physical reality of the process and interpret numerical effects as physical ones. An example of such misinterpretation has already mentioned above after [28]. The amplitude and frequency of pressure oscillations are essential elements of the phenomenon and the values obtained in the solution may be the resultant effect of both the water hammer's physical character and the numerical dispersion and dissipation. It is thus important to realize the influence of numerical parameter values and state them clearly when the solution is presented. If this is neglected, the problem may be easily considered to have been successfully solved, even if the assumed success is due to numerical parameters individually chosen for the analyzed case rather than a physical recognition of the process and its proper mathematical description.

Up till now, the most popular method of solving water hammer equations has been the characteristics method (*e.g.* [9, 16, 18, 34, 41, 42]), which used to be considered the most efficient.

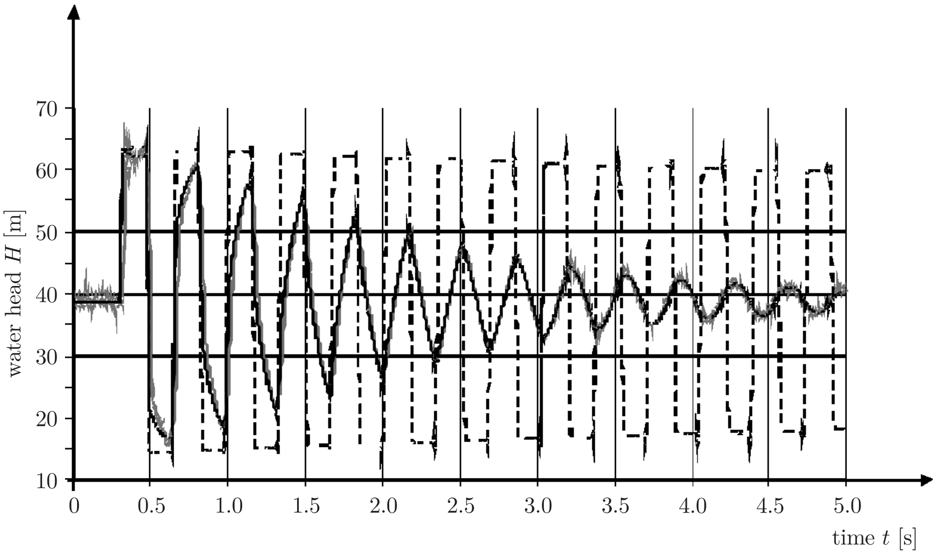

Figure 4. Comparison of elastic and viscoelastic models applied to a water hammer in a polymer pipeline [31]: gray line – observed, black line – calculated with use of viscoelastic model and dashed line – calculated with use of elastic model pressure characteristics; MDPE pipe (SDR 10.7 PN = 10), length $L = 36$m, extrinsic diameter $D = 50$mm, wall thickness $e = 4.6$mm, pipe roughness 0.004mm, initial discharge 0.744dm^3/s, water head in valve cross-section/steady state 38.8m, closure time0.024s, $a = 423$ m/s; numerical parameters of the Preissman scheme: $\theta = \psi = 0.5$, $dx = 1$m, $dt = 0.00236$s, Cr = 1; one-element Kelvin-Voigt model parameters: $\tau = 0.0541$s, $J = 1/E = 0.9 \cdot 10^{-10}$ Pa^{-1}

However, finite difference methods, *e.g.* the four-point scheme (one of the most popular) or its version (the Preissman scheme or the predictor-corrector method), have proved to be equally effective and often more convenient (*e.g.* [26, 27, 43]). A minority of authors propose the finite element method, its modified versions [28] or the finite volume method [44]. There is also a less known but very interesting space-time conservation method, originally proposed by Chang [45] and applied to the problem of unsteady flow in an open channel by Molls and Molls [46] and to the problem of reverse flow routing in an open channel by Weinerowska [47]. The attempts of the present author to apply the method to the simple case of a water hammer in a single elastic pipe of constant diameter have been successful and have demonstrated that it has significant advantages over the popular four-point scheme: better accuracy and explicit formulas producing relatively easy solutions. The method's disadvantage is its conditional stability, but the condition of a Courant number equal or less than unity is relatively easy to satisfy. However, the method's application to more complex problems (changes in diameter, pipeline networks, *etc.*) is more problematic, as the scheme loses its simple explicit formulas and the stability condition is usually more difficult to satisfy as the speed of the pressure wave is different for different diameters and the Courant number assumes more diversified values. Nevertheless, the method is interesting and worth further consideration.

To summarize, water hammer equations can be solved with any numerical method convenient for hyperbolic equations. Most importantly, one should be ever aware of the chosen method's properties (accuracy, stability) and the influence of the chosen values of numerical parameters on the solution, its character, the values of the calculated variables and their sensitivity to parameter changes. Only then the origin of effects influencing the solution can be properly recognized to be of physical or numerical character.

4.3. Identification of model parameters

Parameter estimation is a complex problem in its own right. It is an essential aspect of mathematical modeling of any physical phenomenon, but it becomes even more important in the water hammer context. There are several reasons for this, the most important of these being the relatively high sensitivity of the solution to parameter changes combined with the difficulty of physical interpretation of the parameters or their changes during the phenomenon. In the case of the water hammer, there are three groups of parameters appearing in the mathematical model (numerical parameters are a separate issue, analyzed above): parameters connected with friction, the speed of the pressure wave and the parameters of the Kelvin-Voigt model (number of elements, retardation time and tensile modulus of each element) when a polymer pipe is considered. Thus, even if the simplest approach to friction is applied and only one parameter connected with friction is considered (the friction coefficient), at least four parameters must be estimated for a polymer pipe. If one assumes (as in [22]) that a five-element Kelvin-Voigt model is applied, the number of parameters is increased to twelve. However, the problem with parameter estimation is not limited to the number of parameters. Nowadays, when computational techniques have been developed so well, estimation of even more parameters with various optimization methods is no longer a mathematical challenge. The problem is connected with the uniqueness and physical interpretation of the solution.

In the case of the water hammer, the parameters which can be related to physical characteristics (the friction coefficient, the pressure wave speed) become more difficult to estimate than in steady flows in a pipeline. Friction during rapid changes of flow is unlike in thr traditional steady state approach and the correct formula is still being sought. The pressure wave speed is not easily estimated even when Equation (2) is applied, due to a problem with proper estimation of the value of E, which changes for each pipe within a range typical for its material. Besides, experiments have shown values of wave speed measured for polymer pipes to be considerably greater than those obtained from Equation (2) (cf. [22]). The value of wave speed changes in time and space during the phenomenon and the variation may be about 10% of its value. All this renders the relatively easy estimation of wave speed on the basis of (2) improper, while proper estimation is extremely difficult.

Another group of parameters are retardation time and tensile modulus for each Kelvin-Voigt element. As the "elements" are purely conceptual, the parameters have only a mathematical meaning and no physical interpretation. As such, they must be estimated on the basis of the trial-and-error method or determined with an optimization procedure. Therefore, a set of parameters is only valid for its particular case and the values for other similar cases may differ significantly. Moreover, measurement results must be available in order to determine the parameters, which restricts us to the cases for which experimental research is possible.

The above remarks are merely a brief survey of the difficulties encountered during model calibration; the problem of parameter estimation was already analyzed in detail in [31]. In many water hammer examples presented by various authors, there is no clearly stated general approach to parameters determination. Parameters are usually estimated on the basis of experiments using various methods (optimization, trial-and-error, *etc.*), but the results are valid for the analyzed case. Thus, there is no easy way to predict parameters in cases for which experiments cannot be performed. Considering that the above remarks concern simple single-pipeline cases, we can realize that more complicated cases (varying diameters, pipeline networks, *etc.*) will pose even more problems. Thus, in spite of many examples of successfully calibrated water hammer models, estimation of their parameters remains an open question.

5. Conclusions

The most important aspects of water hammer modeling have been presented. Despite the progress in measurement and computational techniques made during the last century, the problem of proper mathematical reproduction of the water hammer is still an open question. The most important reasons are the phenomenon's incompletely recognized mathematical description, influence of relatively many physical effects it (changes in friction and wave speed, viscoelastic properties of pipe walls, *etc.*), difficulties in proper parameter estimation and, last but not least, numerical aspects of modeling. As there is still no clear procedure of model parameters' calibration, it is important to find an effective approach of relatively global character, any distinct regularity in parameter estimation, to enable better modeling of more complicated cases or those for which experiment is impossible.

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