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An advanced Thermal-FSI approach to flow heating/cooling

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Abstract. Actually, two-way thermal-energy exchange between working fluid and solid material of a casing is a leading problem for modern – semi automatic – design techniques. Many questions should be solved, especially, the turbulent mode of thermal energy transport both in fluid and solid, should be re-examined and reformulated from the primary principles. In the present paper, a group of researchers from Energy Conversion Department of IMP PAN at Gdańsk, tries to summarise a last three-years efforts towards to mathematical modelling of advanced models of thermal energy transport. This extremely difficult problem in "thermal-FSI" ("Fluid Solid Interaction") means that the both for solid and fluid mathematical model of a surface layer should be self-equilibrated and self-concise. Taking these requirements into account, an advanced Reynolds-Stanton analogy has been discussed and implemented. Some numerical examples concerning of the benchmarks experiments and industrial applications have also been developed and presented.

1. Governing equations for the temperature field

Heating is a well known basing process responsible for energy transport from a hot continuous body to a cold one. Mechanism of heating is based on appearing of driving force in a form of temperature difference between cold and hot places. Flux of heating energy generously vanish when the temperature of contacting media becomes equal.

Due to the complex nature of heating, in the literature there is no one common mechanism responsible for transport of mass, energy, momentum during heating. Therefore, according to many different theoretical view point we have a lot of governing equations related to the temperature field being a fundamental unknown. Among them the most popular are (see; Gyftopoulos, Beretta [1], Cano-Andrade et al [2], Bejan [3,17], Feidt [4]):

- Intrinsic Quantum Thermodynamic (IQT) [2]:

$$\frac{d\rho}{dt} = -\frac{i}{h}[H,\rho] - \left(\frac{1}{\tau_1} D_a \oplus \rho_F + \frac{1}{\tau_F} \rho_a \oplus D_F\right) \tag{1}$$

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- Locally maximum entropy production principle (LMEPP) [3]:

$$\frac{\partial \sigma}{\partial T} + \mathrm{div}\left(\frac{\partial \sigma}{\partial \vec{g}} - \mathrm{div} \frac{\partial \sigma}{\partial \vec{\vec{G}}} \right) = 0 \tag{2}$$

- Balance of entropy:

$$\frac{\partial}{\partial t}(\rho \eta) + \mathrm{div}(\rho \eta \vec{v} + \vec{h}_\eta) + \rho \sigma = 0 \tag{3}$$

- Balance of energy:

$$\frac{\partial}{\partial t}(\rho e) + \mathrm{div}(\rho e \vec{v}) = \mathrm{div}\left(\vec{\mathsf{F}}_{mech} + \vec{\mathsf{F}}_{heat} + \vec{\mathsf{F}}_{chem} + \vec{\mathsf{F}}_{e-m} + \vec{\mathsf{F}}_{mass} + ... \right) \tag{4}$$

The above four governing equations for the temperature field are only examples of efforts of scientific community to understand the fundamental behaviour of nature. For instance, balance of entropy and energy are in the core of explaining the transition between macroscopic and microscopic worlds [4]. Furthermore, there is no one common picture for describing complexity of types of communication phenomena. Thus, thinking on transport of heating across the solid-fluid contact layer one should consider an exact nature of a non-linear equation of thermal motion. The examples given above by equations (1)(2)(3) and (4) are the most known – anthers, like "steepest entropy ascent" (SEA), "dissipative quantum dynamic (DCD), etc., are under permanent efforts of scientific community [1].
Let recall some basic elements of above governing equations of thermal motion. In IQT equation (1) the evolution of density state operator of thermal motion is given by two terms. The first term on the right-hand side describes the unitary Hamiltonian dynamics of the system and the second the non-Hamiltonian dissipation dynamics. The operator $[H,\rho] = H\rho - \rho H$ is the commutator between Hamiltonian and density state operator, and $\rho_A = Tr_F \rho$; $\rho_F = Tr_A \rho$ are reduced state operator. Additionally, τ_A, τ_F are characteristic time for subsystems A (internal) and F (external). Important ingredients of the IQT model are facts that it can exactly describe the *locally perceived energy* and *locally preserved entropy* [2] – it means that having these local observables one can defines – using the Gyftopoulos-Beretta and von Neumann definitions – both energy and entropy for the overall system.

The equation (2) is known to be governing equation of LMEPP (Bejan [3]) and is generously based on the notion of *entropy production* : σ . Assuming that this entropy production depends on the temperature gradient $\vec{g} = \mathrm{grad} T$ [thermal diffusion] and the second gradient $\vec{\vec{G}} = \mathrm{grad} \vec{g}$ [Maxwell thermal transpiration] we can reinterpret equation (2) to be a Euler-Lagrange variational condition for maximal entropy production [6].

The equation (3) presents the most problematic equation of the whole classical field theory – the balance of specific entropy η. Here the continuum density is denoted by ρ, the specific momentum vector by $\vec{v}$ and the Rankine entropy flux by $\vec{h}_\eta$. Like above the specific entropy production is denoted by σ.

In practices, the balance of energy (4) is a governing equation for the field of temperature $T(\vec{x},t)$. According to the principle of producing of energy "ex nihilo" in equation (4) there is no internal sources of energy and the changing of energy e is governed only by energy fluxes $\vec{\mathsf{F}}_a$, $a = mech, heat, chem, e-m, mass, ...$. The most important for describing of thermal motion dynamics is the Stokes-Rankine heating energy flux $\vec{\mathsf{F}}_{heat}$, frequently denoted in the literature by letter

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$\vec{q}$. By the Rankine formulae : flux $\vec{F}_{heat} = T\vec{h}_{\eta}$ the equation of energy is indirectly connected with the thermal equation of motion or the balance of entropy, therefore, in numerical practices, the balance of energy leads to determine the temperature field.

Efforts of the Department of Energy Conversion IFFM PAS-ci are concentrated on finding a proper boundary conditions for thermal equations of motions [5]. Our aims are focused around the novel scientific tool called "thermal-FSI" and our basic assumption is that both fluid flow and solid deformation, governed by the same kind of momentum conservation, should be adequately formulated for proper advanced fluid-solid thermal energy exchange. In practice, it means that the advanced thermal-FSI should leads to using the same discretization moving lattice for momentum and energy equations both in fluid and solid. Let us mention, that from the point of view of energy and entropy transport the acronym FSI should be understood to be "Fluid Solid Interaction" not as "Fluid Structure Interaction". It is because thermal phenomena cannot be treated as a "thermal-plate" or a "thermal-shell" and always must be taken to be three-dimensional. Here, the main aim of our study is only restricted to introduce a correct form of boundary condition for the energy equation, especially the boundary condition within the contact layer between solid and fluid.

2. Balance of energy in the solid-fluid contact layer

2.1. Thermal-FSI

Thermal-FSI is a part of science dedicated to describing of energy transport and conversion within a thin layer occurring in a contact of solid and fluid material. In difference to the *Momentum-FSI* where the exchange of momentum between solid and fluid is the main phenomena, in the Thermal-FSI the equation of energy plays the main role in two-way (from solid to fluid and vice versa) energy transport [15].

Speaking about the two-way interaction between solid and fluid authors have in mind a problem of interphase surface mechanical motion. The interphase layer or the interphase surface of contact in a mechanical motion is governed by extended boundary condition between CFD and CSD approaches [15]. Usually, these mechanical conditions leads to motion of discretization lattices (finite volumes or finite elements), since both CFD and CSD obeys discretization of whole domain, divided into solid and fluid parts. It is known that motion of fluid domain discretization lattices follows from a large displacement of solid boundaries, and, vice versa, motion of solid discretization lattices follows from an action like mass sedimenting, swelling, etc.

From the point of thermal motion view also the problem of a thermal interphase motion is important and should be taken into account in formulation a general mathematical model for the Thermal-FSI. Therefore, we shall consider a thermal contact between hot fluid and cold solid continua to be described by a thin but finite contact thermal layer described as a sum of the von Smoluchowski thermal length jump [5]:

$$l_T = l_T^f + l_T^s \tag{5}$$

where l_T^s , l_T^f are the thermal length jump adequate to solid and fluid side (figure 1).

2.2. Boundary condition for energy in the solid-fluid contact layer

Going into a generalized boundary condition for energy equation let us start from the traditional forms used in CSD:

$$-\alpha^s (T^s - T_s^s) - \vec{q}^s \cdot \vec{n} = 0 \tag{6}$$

and CFD:

$$\alpha^f (T^f - T_s^f) - \vec{q}^f \cdot \vec{n} = 0 \tag{7}$$

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Here - $\vec{q}^{\,s}$ and $\vec{q}^{\,f}$ means the flux of thermal energy on solid and fluid side which for isotropic materials, according to the Fourier law, depend on the coefficients of thermal conductivity ei., $\vec{q}^{\,s} = \lambda^s \mathrm{grad}T$, $\vec{q}^{\,f} = \lambda^f \mathrm{grad}T$. Additionally, the transfer coefficients α^f, α^s are responsible for overall transport of thermal energy in macroscopic vicinity of the contact layer. In the classical approach there is no the von Smoluchowski thermal jump, therefore fluid and solid temperature coincidences: $T_s^f = T_s^s$.

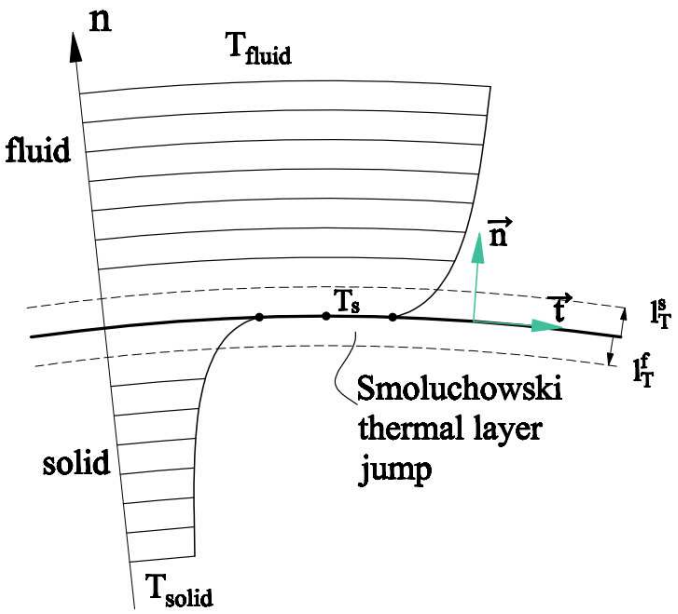

Figure 1. Temperature profile in a solid-fluid contact layer

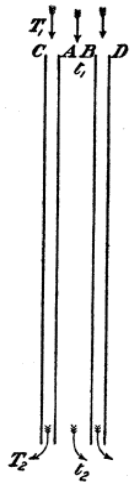

Figure 2. Exchange of thermal energy In the Stanton experiment.

Turning into a generalized boundary condition that reflects a CFD-CSD type of coupling one shall take into account as a surface energy contribution as well as the thermal jump phenomena. Without going into details, let $e_s = c_{\rho s} T_s$ be a simplest form of surface internal energy governed by surface temperature T_s Defining the surface metric tensor to be [6]: $\vec{I}_s = \vec{I} - \vec{n} \otimes \vec{n} = a_{\alpha\beta} \vec{a}^{\alpha} \otimes \vec{a}^{\beta}$ and the surface gradient and divergence operator [5] grad_s , div_s we are able to express the boundary condition for energy equation within a thermal contact layer. Removing from balance other energy fluxes and keeping only mechanical $(\vec{\mathsf{F}}_{mech})$ and thermal $(\vec{\mathsf{F}}_{heat} \equiv \vec{q})$ energy vectors on the boundary the energy balance within the thermal contact layer can be written to be:

$$\partial_t(\rho_s c_{\rho s} T_s) + \mathrm{div}_s(\rho_s c_{\rho s} T_s \vec{I}_s \vec{v}_s) + \mathrm{div}_s(\lambda_s \mathrm{grad} T_s) + \mathsf{l}_T^f (T_s^f - T_s) - \mathsf{l}_T^s (T_s^s - T_s) +$$

$$\alpha^f (T^f - T_s^f) - \alpha^s (T^s - T_s^s) + (\vec{q}^{\,f} - \vec{q}^{\,s}) \cdot \vec{n} + (\vec{\mathsf{F}}_{mech}^{\,f} \vec{v}_{slip}^{\,f} - \vec{\mathsf{F}}_{mech}^{\,s} \vec{v}_{slip}^{\,s}) \cdot \vec{n} = 0 \qquad (8)$$

According to equations (6) (7) in the thermal contact layer an additional surface density ρ_s and the surface temperature T_s appears. The surface velocity vector $\vec{v}_s$ and the slip velocity vectors $\vec{v}_{slip}^{\,s}$,

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$\vec{v}_{slip}^{\,f}$ should be independently calculated from the boundary conditions for the momentum equations [5]. The von Smoluchowski jump coefficients are related with the thermal jump length as: $l_T^f = l_T^f / \lambda_f$, $l_T^s = l_T^s / \lambda_s$. This last contribution to equation (8) is especially important in micro- and nano-scales where the mechanism of the von Smoluchowski thermal jump becomes most valid. Also in nano-scale the energetic contribution from the mechanical slip – for instance the Navier slip – gives the considerable values

3. Numerical Thermal-FSI treatment

On the contemporary level of development Thermal-FSI has no possibilities for numerical solution of equation (8) even in a simplified form. The first reason is an insufficient developed of Finite Element Method as well as Finite Volume Method. Actually both methods are prepared only to simultaneous solving only a part of equation (8) in a form:

$$(\vec{q}^{\,f} - \vec{q}^{\,s}) \cdot \vec{n} = 0 \tag{9}.$$

It is obvious that keeping further $\vec{q}^{\,s} = \lambda^s \mathrm{grad}T$, $\vec{q}^{\,f} = \lambda^f \mathrm{grad}T$ with constant λ^s, λ^f one can obtain false value of heating vectors and, as a result, false value of temperature profile. Therefore, Thermal-FSI is based on a concept of functional changing of diffusional coefficients in the nearest neighborhood of surface of contact, for instance $\lambda^f = \lambda^f(y^+, \mathrm{Re}_y, \delta)$. Unfortunately, this approach needs an independent from the momentum equation the near-surface discretization. Using of two different discretizations - one for momentum and second for energy equation – is time consuming and numerically uncomfortable. Therefore, the better solution is to applied some analogies with momentum equations that, even if physically unfortunately, leads to unified CSD/CFD system of equations solved only in one discretization lattice.

4. The Reynolds-Stanton analogy

Let, according to literature tradition, the letter y means the coordinate normal to solid-fluid contact surface. Then the condition $(\vec{q}^{\,f} - \vec{q}^{\,s}) \cdot \vec{n} = 0$ can approximate the condition (8) if we take an idea that the passage of thermal energy from solids to fluids moving past them is governed by the same principles as apply in virtue of viscosity to the passage of momentum originated with Reynolds [8] and it has been further developed by Stanton [7] (see Lord Rayleigh[9], L. Prandtl [10], A. Eagle, M. Ferguson [11], L.H. Loitsiantski [12]). The analogy, originally written for viscous stress tensor $\vec{q}(y) \sim \vec{\tau}(y)\vec{n}(y)$ can be also pressure dependent, both fluid and solid:

$$\vec{q} = a(-p\vec{\vec{I}} + \vec{\tau})\vec{n}\,, \tag{10}$$

where a coefficient of Reynolds-Stanton analogy discussed extensively by Prandtl [10] and Schlünder [14]. The above analogy responses on the following question – It is possible to have simultaneously high rates of thermal energy transfer and low rates of momentum transfer ? To answer this very practical question leads immediately to the scientific problem whether – and if so – how are thermal energy and momentum transfer independent ?

Now the question arises – how equation (10) can be adopted to CFD/CSD modelling. Practically it means that for small y^+ the length and direction of $\vec{q}$ vector becomes independent of direction of $\mathrm{grad}T$ and length coming from laminar and turbulent Prandtl number [15],[16]. In our work we have simply implemented of condition (10) to a commercial code.

5. An example

Stanton's experiment [7] has been prepared to proof Reynolds' mechanism of thermal energy transport [1]. It was thought to be simplest measurements of transport of energy from flowing hot

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water (39.6 ^{o}C) to flowing cold water (18 ^{o}C) across a metal wall. The pipes in the Stanton experiment is presented in figure 2. Total length of pipes was 67 cm and internal diameters are: 13.9 (cold) and 15,7 mm (hot)(figure. 2). The mass flux of hot water was constant – 148 g/s, mass flux of cold water was changed from 148 to 27 g/s.

For the same mass flux 148 g/s of hot and cold water the results is symmetrical – the increase of temperature of cold water on 4.4 ^{o}C and decrease temperature of hot water on 4.4 ^{o}C. For decreasing of mass flux of cold water from 148 to 27 g/s it has been observed the systematic increase of the inlet temperature t_2 from 22.4 to 31.8 ^{o}C and simultaneously, increasing of T_2 from 35.2^{o}C to 27.4 ^{o}C.

The quite similar results has also been numerically obtained by using of implemented the Reynolds -Stanton analogy. It was calculated that during the change of mass flow rate of cold water (average velocity change from c =98 to 18 m/s) the total change of energy by the heating increases from 336 [W] to 390 [W] and harmonise with the formulae proposed by Reynolds and empirically proved by Stanton :

$$H = (A + B\rho c)(T_1 - t_1) \tag{11}$$

with $A =$ 343 and $B = 0{,}334$ calculated by Stanton and repeated in Thermal-FSI from overall integration of normal component of heating vector ; $H = \iint_{\partial V} \vec{q} \cdot \vec{n} dA$.

6. Conclusions

It has been shown that the simplest Thermal-FSI condition has its adequate formulation of boundary condition for energy equation, and it does not allow correct simulation of evolution of the temperature scalar field in the homogeneous flow of hot fluid over a cold solid. But the great advantages of Thermal-FSI is that whole conservation equation can be solved on one common discretization grid. Our study throws doubt on the wisdom of obtaining experimental and numerical values of the Nusselt and Stanton numbers.

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