

# An Analysis of Scattering from Ferrite Post of Arbitrary Convex Cross Section with the Use of Field Matching Method

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**Abstract**—A problem of electromagnetic wave scattering from ferrite post is presented. The post is assumed to be located in closed areas as waveguide junction, or in open area illuminated by a plane wave. The object is of arbitrary convex cross section and the method of analysis is semi-analytical, based on the direct field matching technique.

## I. INTRODUCTION

The scattering problems in both closed and open areas have been extensively investigated for applications in communication systems for decades [1]–[10]. Different methods of analysis can be used for the scattering problems depending on the shape of scatterers and type of excitation. For the case of open problems, i.e. plane wave scattering, where the posts are located in free space to shape the radiation pattern, there are several techniques such as partial differential equation with the eigenfunction expansion method [1], integral equation formulation [2] or the iterative algorithm [3]. For closed problems, where the objects are located e.g. in waveguide junctions, the most basic techniques are the modal expansion method [4] and the orthogonal expansion method [5]. With the rapid development of computer technology the utilization of commercial full-wave simulators employing the space-discretization techniques is very popular, which allows to investigate the structures with complex geometries. These methods are more general, however they are significantly more time and memory consuming, especially when dense meshes must be employed to model the complex shapes of the structures.

In this paper we utilize a simple semi-analytic method based on field matching technique to investigate electromagnetic wave scattering from cylindrical posts of arbitrary convex cross section made of ferrite material located in three-port waveguide junction and in free space illuminated by a plane wave.

## II. FORMULATION OF THE PROBLEM

The investigated structures are presented in Fig. 1. The scatterer, which is located inside the waveguide or placed in free space can be of arbitrary convex shape as it is schematically illustrated in Fig. 2. The analysis procedure, which is applied for both open and closed structures, is divided into two stages.

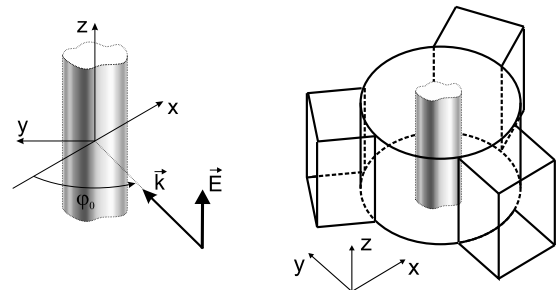


Fig. 1. Open and closed structures. (a) Perpendicular plane wave illumination. (b) Waveguide junction.

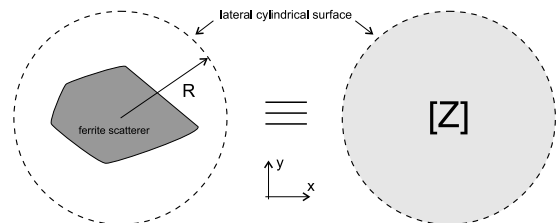


Fig. 2. The geometry of a scatterer and its impedance matrix representation.

In the first stage the sole object of arbitrary convex cross section and invariant along its axis (see Fig. 2) is investigated. The aim of this stage analysis is to determine the relation between the electric and magnetic fields on the surface of a hypothetical cylinder of radius  $R$ , which surrounds the post and express it in a form of multimode impedance matrix  $\mathbf{Z}$  [7]. This matrix describes the investigated structure and it is used in the second stage of the procedure, where it is matched with any known external incident fields as described in [8].

In order to calculate the impedance matrix of a single scatterer it is assumed that the object is illuminated by an unknown incident field which can be expanded in a series of cylindrical functions with unknown coefficients. Formulating and satisfying the continuity conditions for electric and magnetic tangential field components on the post surface and

using the projection of the fields at the boundary on a fixed set of orthogonal basis functions [7] allow to calculate the scattered field from the object, which is also formulated as a series of cylindrical functions with unknown coefficients. The relation between incident and scattered field coefficients can be described by a transmission matrix (T-matrix). For the case of dielectric or metallic posts this matrix has a form as presented in [7]. For the case of ferrite post the modification concerning the fields inside the ferrite material needs to be made. For ferrite magnetized along  $z$  axis the following material equations are assumed:

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}, \quad \vec{B} = \mu_0 \bar{\mu} \vec{H} \quad (1)$$

where  $\varepsilon_r$  is a scalar relative ferrite permittivity and  $\bar{\mu}$  have the following dyadic form:

$$\bar{\mu} = \mu_r (\vec{i}_\rho \vec{i}_\rho + \vec{i}_\phi \vec{i}_\phi) + j\mu_a (\vec{i}_\rho \vec{i}_\phi - \vec{i}_\phi \vec{i}_\rho) + \vec{i}_z \vec{i}_z \quad (2)$$

The components of electric and magnetic fields take form:

$$E_z = \sum_{m=-M}^M A_m J_m(k\rho) e^{jm\phi}$$

$$H_\rho = \sum_{m=-M}^M A_m \left( -\frac{Ym}{k\rho} J_m(k\rho) - \frac{Y\mu_a}{\mu_r} J'_m(k\rho) \right) e^{jm\phi}$$

$$H_\phi = \sum_{m=-M}^M -jA_m \left( \frac{\mu_a m}{\omega\mu_{eff}\mu_r\mu_0\rho} J_m(k\rho) + Y J'_m(k\rho) \right) e^{jm\phi}$$

where

$$k = \omega \sqrt{\mu_0 \mu_{eff} \varepsilon_0 \varepsilon_r}$$

$$\mu_{eff} = (\mu_r^2 - \mu_a^2) / \mu_r$$

$$Y = \sqrt{\varepsilon_r \varepsilon_0 / \mu_{eff} \mu_0}$$

and  $A_m$  are field unknown coefficients. The tensor parameters are calculated from [9]:

$$\mu_a = \frac{\gamma_f M_s f}{\gamma_f^2 H_i^2 - f^2}$$

$$\mu_r = 1 + \frac{\gamma_f^2 M_s H_i}{\gamma_f^2 H_i^2 - f^2}$$

where  $H_i$  is an internal bias magnetic field,  $M_s$  is a saturated magnetization,  $f$  is a frequency in MHz and  $\gamma_f = 35.2$  MHz/kA is gyromagnetic coefficient of ferrite.

Having the scattered field from the object derived, the relation between the total electric and total magnetic fields (the superposition of incident and scattered fields) calculated on the circular surface surrounding the object can be formulated as the impedance matrix, which describes the object. This way, in the further analysis, the object is treated as cylinder with circular cross-sections and can be easily matched with any external excitation as described in [7].

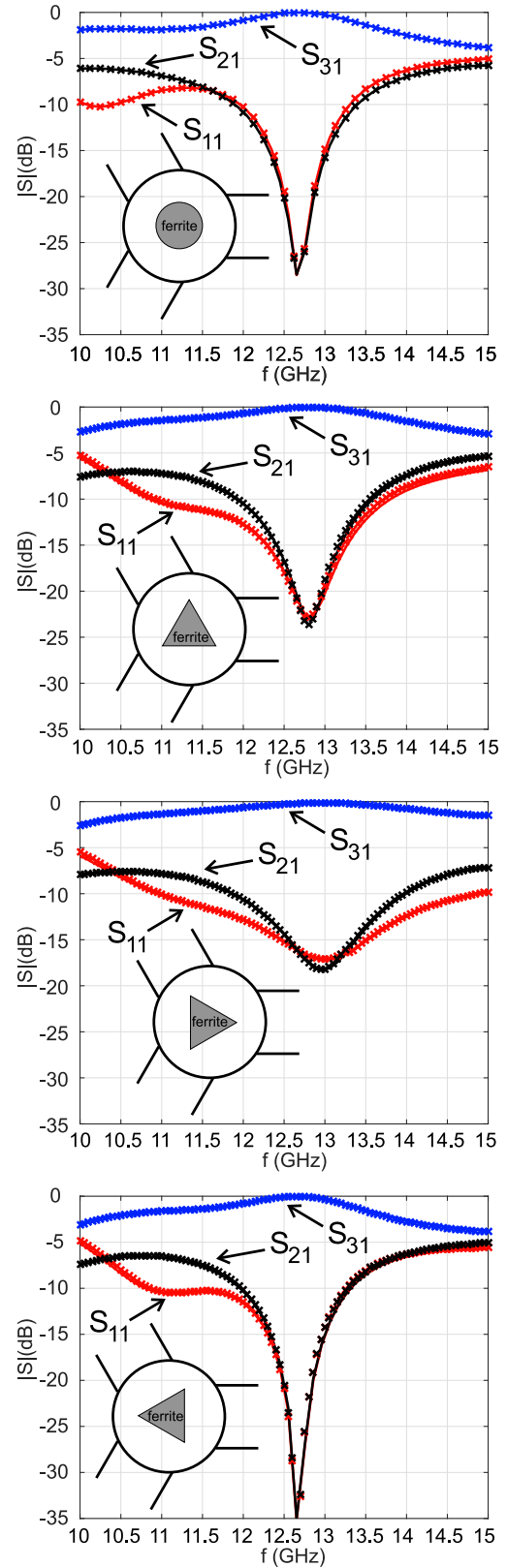


Fig. 3. Scattering parameters of the three-port waveguide junction (WR-75 waveguide) with circular (radius  $r = 3.2$  mm) and triangular ( $a = 8$  mm) ferrite cylinders with ferrite material parameters:  $\varepsilon_r = 9$ ,  $H_i = 20$  kA/m and  $M_s = 160$  kA/m. Solid line - this method; crosses - finite element method [12], [13]

### III. RESULTS

Several examples of electromagnetic wave scattering from ferrite post of arbitrary convex cross sections have been investigated. The first example is a circular cavity fed by three WR-75 waveguide ports of dimensions  $19.05 \times 9.525$  mm. The cavity has the same height as the waveguides and a radius  $R = 19.05/\sqrt{2}$  mm. Inside the cavity a ferrite post of triangular (equilateral) cross section of side dimension  $a = 8$  mm is placed. The post completely fills the height of the waveguide junction. The hypothetical ferrite material was used with parameters:  $\epsilon_r = 9$ , internal bias magnetic field intensity  $H_i = 20$  kA/m and saturated magnetization  $M_s = 160$  kA/m. The calculation of the scattering parameters of the junction have been performed for different post rotation with respect to waveguide junction and also for post with circular cross section of the same cross section area as the triangular post. The calculated scattering parameters of the junction are presented in Fig. 3. Due to the symmetry of the structure, only  $S_{11}$ ,  $S_{21}$  and  $S_{31}$  are shown. As can be observed from the results, the circulator operates around the frequency 12.7 GHz where the transmission occurs from port 1 to 3, from 3 to 2 and from 2 to 1. The advantage of utilization of triangular ferrite post instead of circular one is the possibility of additional tuning the structure by means of simple post rotation in the junction. Such property can be utilized in the manufactured device to compensate for material defects and improper dimensions or other mechanical inaccuracies occurred during manufacturing process of the structure.

The second example considers a triangular ferrite post, which can be realized with the use of self magnetized ferrite material [11], located in free space and illuminated perpendicularly by a plane wave. The calculated far field patterns for the case of three angles of excitation  $\phi = 0^\circ$ ,  $\phi = 120^\circ$  and  $\phi = 240^\circ$  are presented in Fig. 4. The ferrite material (hypothetical) parameters,  $\epsilon_r = 15$ ,  $M_s = 190$  kA/m and  $H_i = -15$  are assumed. As can be observed the ferrite post shifts of main lobe of the field pattern by  $60^\circ$  with respect to plane of excitation. Therefore, a space wave circulation was obtained similarly as in waveguide circulator form the first example.

Additionally, the post asymmetry allows to control the shape of scattered field by simple post rotation, which is not possible for cylindrical post with circular cross section. As shown in Fig. 5 the assumption of the proper post rotation angle allows to obtain the desired direction of main lobe between  $0^\circ$  and  $30^\circ$ . The ferrite material parameters,  $\epsilon_r = 8$ ,  $M_s = 225$  kA/m and  $H_i = 0$  are assumed.

### IV. CONCLUSION

A field matching technique has been utilized to investigate electromagnetic wave scattering in closed and open problems from cylindrical ferrite posts of arbitrary convex cross section. Unlike the circular post the rotation of objects with arbitrary shape allows for tuning the scattering parameters of the structures.

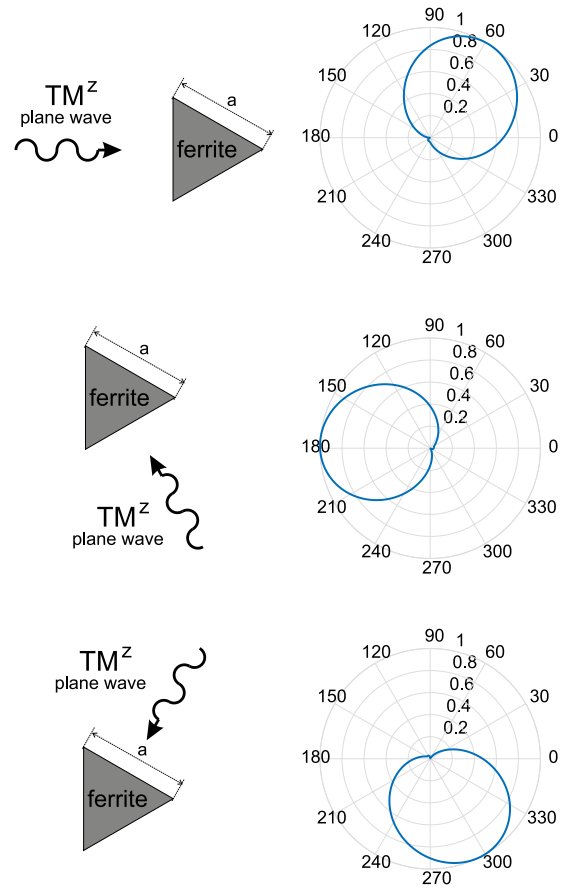


Fig. 4. Normalized energy characteristics  $|E_z|^2$  for  $TM^z$  scattering from ferrite cylinder with triangular cross section magnetized in  $+z$  directions. Ferrite post parameters:  $a = 0.15\lambda$  (equilateral triangle),  $\epsilon_r = 15$ ,  $H_i = -15$  kA/m,  $M_s = 190$  kA/m and  $f = 10$  GHz.

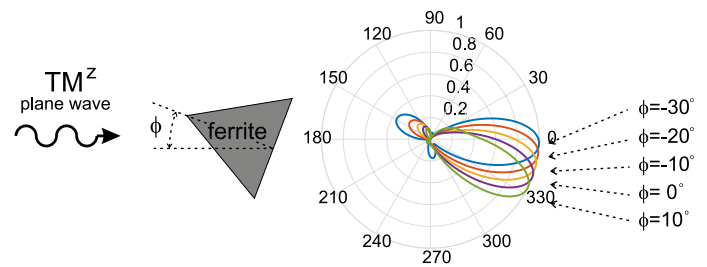


Fig. 5. Normalized energy characteristics  $|E_z|^2$  for  $TM^z$  scattering from ferrite cylinder with triangular cross section for different values of post rotation  $\phi$ . Ferrite post parameters:  $a = 0.5\lambda$ ,  $\epsilon_r = 8$ ,  $M_s = 225$  kA/m,  $H_i = 0$  and  $f = 10$  GHz.

### ACKNOWLEDGMENT

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