APPLICATION OF MAXIMUM LENGTH SEQUENCE IN SILENT SONAR

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Silent sonars are designed to reduce the distance over which their sounding pulses can be detected by intercept sonars. In order to meet this objective, we can use periodical sounding signals that have low power, a very long duration and wide spectrum. If used in the silent sonar's receiver, matched filtration ensures very good detection of motionless or slow moving targets. However, it is more difficult to detect echo signals of fast moving targets with Doppler effect causing significant error in target distance measurements. In an effort to find signals that can better resist Doppler effect, maximum length sequence was tested for its application in silent sonar. It has an elementary signal which includes linear frequency modulation. It was demonstrated that the signal produces much better results than those obtained with simple frequency modulation signals.

INTRODUCTION

The majority of today's submarines are equipped with sonars for detecting submarines, naval mines, remote operated vehicles (ROV), divers, navigation obstacles and other targets. All active sonars emit sounding pulses which can be detected by the enemy's on-board intercept system. Once a sounding signal is received, the enemy will know that our ship is there which puts it at risk. Sounding signal interception can be made more difficult with silent sonars [1], [2], [3]. While their target detection and positioning performance cannot be any worse than that of ordinary sonars of the same function, they must also ensure that intercept sonars will have difficulty detecting their signals. This is possible because sounding signals are known to the silent sonar but are unknown to the enemy's intercept system. By knowing the sounding signal, and the echo signal although with some limitation, we use matched filtration in the receiver and maximise the output signal to noise ratio and, by the same token, the range of the sonar. Because the sounding signal is not known to the intercept system, it must employ other types of detection whose output signal to noise ratio depends mainly on the power of the signals received. As a result, the lower the silent sonar's emitted power, the more difficult it becomes to intercept a sounding signal. The power of a signal can be reduced by proportionately increasing its duration at no harm to detection performance. This is because the output signal to noise ratio in the case of matched filtration is proportional to the signal's energy. It is also desirable for the sounding signal to be continuous and have a wide spectrum which makes it similar to acoustic noise and less likely to be detected. These criteria are met by periodical continuous signals with frequency modulation (FMCW), [1], [4]. With their narrow autocorrelation function, they ensure a very good range resolution of the sonar and reduce reverberation.

If designed for the signals, silent sonars will ensure the parameters desired for detecting motionless or slow moving targets. In the case of fast moving targets, Doppler effect deteriorates detection performance and causes significant distance error, [2], [3]. The distance error does not depend on the type of frequency modulation. What happens is that Doppler effect deteriorates detection performance in the case of linear frequency modulation (LFM) signals much more than it does in the case of signals with hyperbolic frequency modulation (HFM) [5].

The article will consider ways to reduce Doppler effect when measuring the distance to moving targets using the maximum length sequence (MLS) codes combined with frequency modulated sounding signals.

1. DOPPLER EFFECT IN LOW BAND MLS SIGNAL

It can be demonstrated, [2], [3], that the distance error measured by silent sonar which emits a continuous LFM or HFM signal of duration T, carrier frequency f_0 and spectrum width B is:

$$\Delta R = vT\frac{f_0}{B}, \tag{1}$$

where v is the speed at which the target is moving closer to or further away from the sonar's carrier.

To keep the emitted signal power significantly reduced while maintaining its energy, signal duration should be quite long. Because transmitting transducers have a limited transfer band, the actual quotient f_0/B ranges from 2 to 5. As a result, when velocity v is high of the order of 10 m/s and duration T=10 s the distance error will range from 200 m to 500 m and increases together with extended signal duration. If we want the error to stay within an acceptable value, signal duration should be of the order of 1 s or less which is in conflict with signal energy which we want to be high. As a consequence, the search is now for signals that are less sensitive to Doppler effect and have a narrow autocorrelation function.

Let is consider maximum length sequences as the basis for generating a sounding signal with the desired features. As we know, MLS is a pseudo-random sequence of $s(n)$ numbers +1 and -1 containing $L=2^N-1$ elements, [6]. The discreet spectrum of the sequence has constant values except the zero frequency line. Fig. 1 shows MLS L=63 long, and Fig. 2 shows its spectrum.

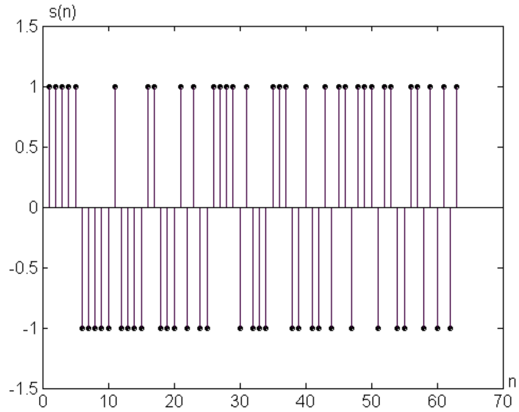

Fig.1. Maximum length sequence (L=63).

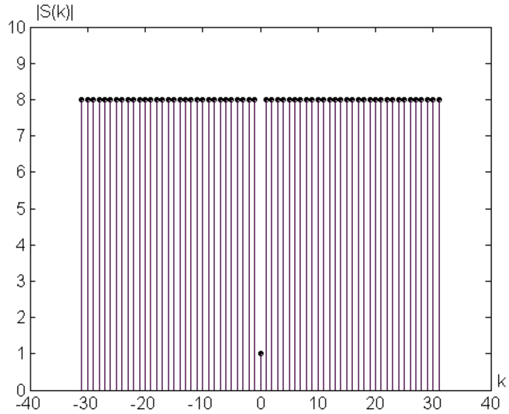

Fig.2 . MLS spectrum (L=63).

Fig. 3 shows the autocorrelation function determined for an L=63 sequence. Its maximal value is equal to L, and the other values do not exceed $\sqrt{L}$.
Fig. 4 gives the autocorrelation function calculated as, [7]:

$$r_{ss}(n) = \Im^{-1}\{|\Im[s(n)]^2\}, \tag{2}$$

which is equaivalent to the autocorrelation function of a periodical sequence of duration L. The function's maximum is equal to L, with the other values at $1/L$. This is the autocorrelation function we will be using in the article from now on.

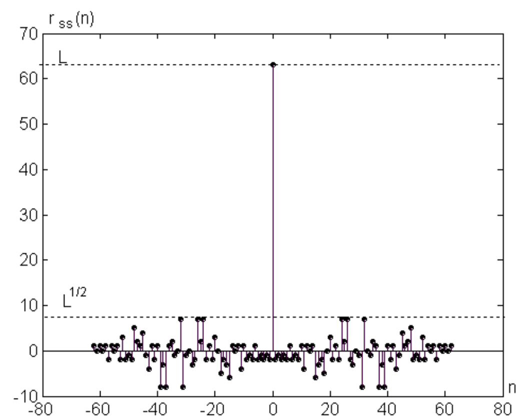

Fig.3. MLS autocorrelation function (L=63).

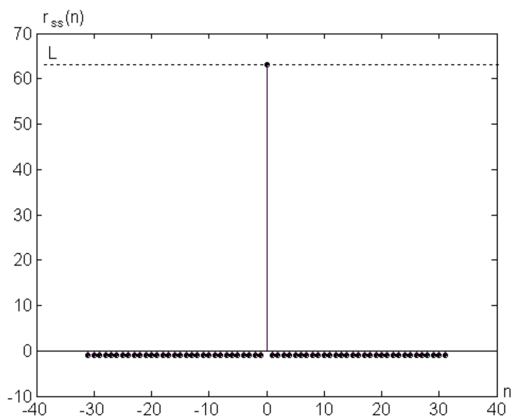

Fig.4. Cyclical MLS autocorrelation function (L=63).

A signal made from MLS, with short pulses similar to Dirac pulses replacing +1 and -1, is used to determine pulse responses of linear systems [6]. This particular application requires some modification of the signal because the sounding signal must be a narrow band signal. To obtain the sounding signal envelope *x(t)* we will first replace Dirac pulses with rectangular pulses of duration τ as shown in Fig. 5. The result is a sequence of rectangular pulses and its duration $T=L\tau$ is equal to the sounding signal's duration. In continuous time signal *x(t)* is a convolution of the MLS sequence of Dirac pulses *s(t)* and rectangular pulse which we will write down as:

$$x(t) = s(t) * \Pi(t/\tau) . \tag{3}$$

The signal autocorrelation function *x(t)* is equal to:

$$r_{xx}(t') = [s(t) * \Pi(t/\tau)] \otimes [s(t) * \Pi(t/\tau)] = r_{ss}(t') * r_{\Pi\Pi}(t') \ , \tag{4}$$

because the convolution operation alternates with the autocorrelation operation here marked as ⊗.

As you can see, the autocorrelation function is a convolution of the autocorrelation function of MLS sequence of Dirac pulses and the autocorrelation function of rectangular pulse. This is illustrated in Fig. 6 which shows the autocorrelation function for sequence *s(n)* from Fig.1. By comparing its autocorrelation function given in Fig. 4 with the autocorrelation function r_{xx} we can see that the basic parameters of both functions are identical. There is a major difference, however, in the width of the autocorrelation function which for function r_{xx} is 2τ, at the level of zero. The width of the autocorrelation function determines resolution which improves for shorter pulse τ duration.

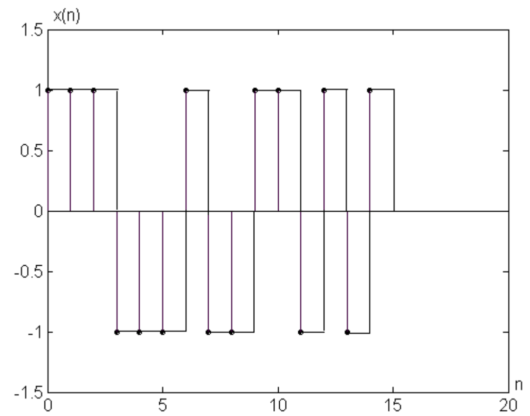

Fig.5. MLS sequence of rectangular pulses (L=15).

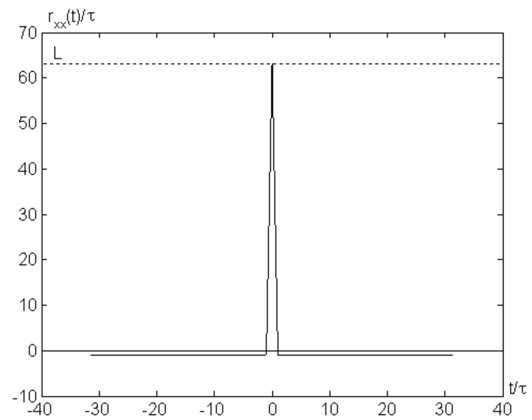

Fig.6. The autocorrelation function of an MLS sequence of rectangular pulses (L=15).

Let us now examine Doppler effect on the correlation function of the sounding signal envelope *x(t)* and the echo signal envelope *y(t)*. The function describes the signal envelopes at matched filter output. To this end let us assume that the echo signal reflects off a target moving towards the sonar at speed *v*. When signal emission first begins *x(t)* the target is R_0 away and the distance decreases with time by *vt*. The start of the signal reaches the target after time:

$$t_0' = R_0/(c+v), \tag{5}$$

where *c* is the velocity of acoustic wave in water. The return path is shorter by vt_0', and so the start of the signal covers the distance in time:

$$t_0'' = (R_0 - vt_0')/c\,. \tag{6}$$

The summaric delay is then:

$$t_0 = t_0'' + t_0' = 2R_0/(c+v)\,. \tag{7}$$

The end of the signal was emited after time *T* during which the target is at a distance of R_0-vT. As a consequence, the end of the signal reaches the target after time:

$$t_T^{'} = (R_0 - vT)/(c+v)\,, \tag{8}$$

and covers the return path in time:

$$t_T^{''} = (R_0 - vT - vt_T^{'})/c\,. \tag{9}$$

As a consequence, the end of the signal reaches the array of the sonar at the following moment of time:

$$t_T = T + t_T^{''} + t_T^{'} = t_0 + T\frac{c-v}{c+v}\,. \tag{10}$$

As can be concluded from the formula above, Doppler effect leads to time compression with a coefficient equal to:

$$d = \frac{c-v}{c+v}\,. \tag{11}$$

The echo signal duration is now dT and the same proportion is followed when duration of all τ pulses drops (or increases for a negative speed). As a result, the echo signal does not match the sounding signal which deteriorates the parameters of the correlation function r_{xy} of the sounding signal envelope $x(t)$ and echo signal envelope $y(t)$. The deterioration is clear when we compare the correlation function r_{xx} (Fig. 6) with correlation function r_{xy} (Fig.7) where the envelope $y(t)$ is the result of envelope time compression $x(t)$ with coefficient d=0.98 (v=15 m/s). The correlation function's maximum drops, there is a shift on the time scale and side lobe level is significantly increased. It can be demonstrated that these effects are in no relation to τ pulse duration. What they do depend on, however, are sequence length L and time compression coefficient d. This is shown in Fig. 8 with the correlation function calculated for L=127.

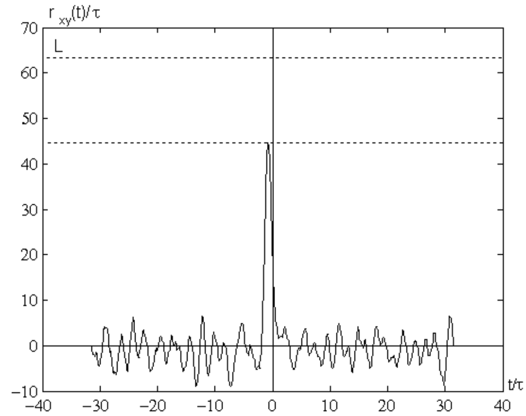

Fig.7. Correlation function of envelopes of sounding signal *x(t)* and echo signal *y(t)* (*L*=31).

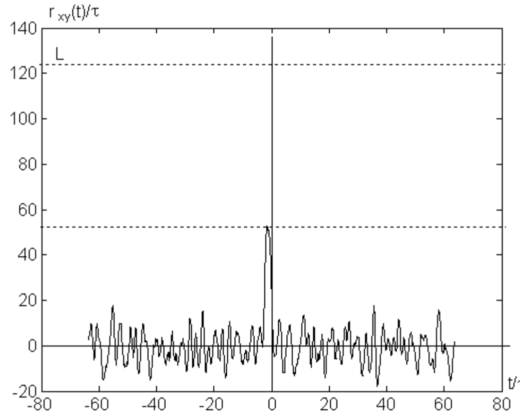

Fig.8. Correlation function of envelopes of sounding signal *x(t)* and echo signal *y(t)* (*L*=127).

Fig. 9 shows the relation w between the maximal value of the correlation function and the maximal height of lobes in the function of the order of N of maximum length sequence. The target's speed v is the parameter. The shift of the correlation function's maximum t_d is proportional to target speed v, as shown in Fig. 10. Numerical calculations have also shown that the shift is proportional to pulse duration τ and the number of elements of a maximum length sequence. They are described with the following empirical formula:

$$t_d \cong 1.08 \cdot \tau \frac{v}{c} L = 1.08 \frac{v}{c} T \,. \tag{12}$$

Hence, the distance error is:

$$\Delta R \cong \frac{1.08}{2} vT \tag{13}$$

As you can see from formula (1) it is $2f_0/B$ times smaller than the error in silent sonar using a sounding signal with frequency modulation.

Formula (12) gives a good approximation of the numerically determined shift of the correlation function's maximum for MLS which is $L<c/v$ long. When length L increases, the delay carries an even bigger error and refers approximately to the centre of the correlation function.

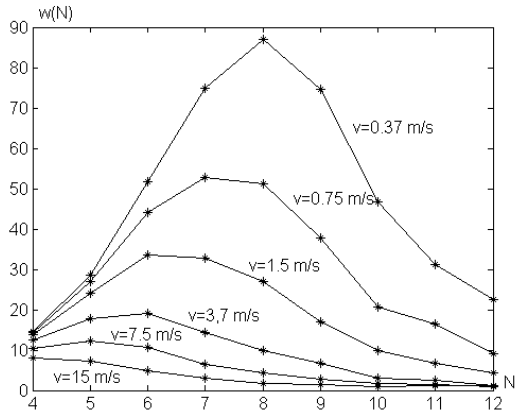

Fig.9. Relation between maximum of correlation function and side lobes level versus the MLS order *N* and target's speed *v*.

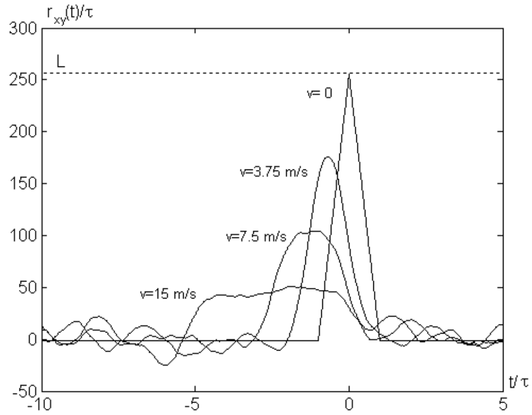

Fig.10. The amplitude and time shift of the correlation function's maximum t_d as function of target speed *v*.

When designing a silent sonar, the objectives should be to achieve:

- a long duration T to support a reduction in the power of the emitted signal while maintaining all of its energy,
- a small delay of the correlation function's maximum to minimise the distance error,
- a short duration of pulse τ to ensure good resolution,
- a high coefficient w to avoid small echo signals from being masked by big ones.

The analysis so far indicates an obvious fact which is that it is easier to meet all of the above requirements if speed v of the target under observation is low. As an example, for speed $v = 1.5$ m/s (average speed of underwater ROV and divers), the shift of the correlation function's maximum is $t_d \cong \tau \cdot 2^{N-10}$. If we assume that $N = 10$ ($L = 1023$), we obtain $t_d \cong \tau$ and $T = 1023\tau$. If we then assume that $\tau = 0.01$s we obtain a resolution $\delta R = c\tau/2$=7.5 m and the same distance error ΔR. The duration of the sounding signal is $T \cong 10$ s which allows a significant reduction in sounding signal power compared to the conventional pulse sonar. The power can be reduced $T/\tau \cong 1000$ times. Compared to sonars with FM sounding pulse, the gain is smaller and amounts from 20 to 100 depending on sounding pulse duration. The value of coefficient w can be read from Fig. 9 and amounts to about 10 which is satisfactory. By reducing the order of MLS to $N = 8$ three times, we increase the coefficient and decrease shift t_d four times. The distance error and duration T will not change, if we extend time τ four times. Resolution suffers as a result of the change now at $\delta R = 30$ m which is usually good enough for medium and long-range sonars. Please note that distance error of a FMCW sonar for $f_0/B = 2$ is $\Delta R = 30$ m (formula 1) and for $f_0/B = 5$ (practical size in the majority of sonars) we obtain $\Delta R = 75$ m.

When target speed exceeds $v \cong 5$ m/s using MLS does not yield fully satisfactory results. This is mainly caused by coefficient w being too small and generating a high level of the correlation function's side lobes, especially for bigger orders of MLS. For orders of $N = 5$ or $N = 6$, high values of τ are required before the desired duration T is achieved which deteriorates the sonar's resolution and increases distance error.

2. USE OF MLS CODES FOR FM SIGNALS KEYING

The MLS signal discussed above is a low band signal and as such it is not practical for silent sonar sounding signal. Because the signal's spectrum must be moved to higher frequencies, rectangular pulses of duration τ should be replaced with a narrow band signal of the desired carrier frequency. The simplest method to achieve this is to use the sinusoidal signal and the phase shift keying using MLS codes. The problem is, however, that the spectrum width of a signal with sinusoidal carrier frequency is equal to B=1/τ, which is very little. This is contrary to what the silent sonar was to ensure, i.e. a wide spectrum of the sounding signal. Because of this it makes good sense to use as a elementary signal with frequency modulation whose spectrum width does not depend on duration τ.

Elements of maximum length sequence and its matching low band signal shown in Fig. 5, have plus and minus signs which must be reflected in the parameters of the FM signal. When the sign changes, so do the phase or frequency. The "+" sign will be assigned an FM signal with frequency f_+ and the "–" sign will be assigned a signal with frequency f_-. If the spectrum width of both signals is B, the frequency is equal to $f_+ = f_0 + B/2$, and frequency - $f_- = f_0 - B/2$. This is how the spectrum width of the sounding signal is equal to 2B. Fig. 11 shows a normalised spectrum $Z(f)$ of FM signals in which $2B = 0.5 \cdot f_0$, where f_0 is the carrier frequency of the sounding signal. Fig. 12 shows the spectrum of FM signals keyframed by MLS codes. Fig. 13 shows a diagram of how the echo signal is processed in the receiver.

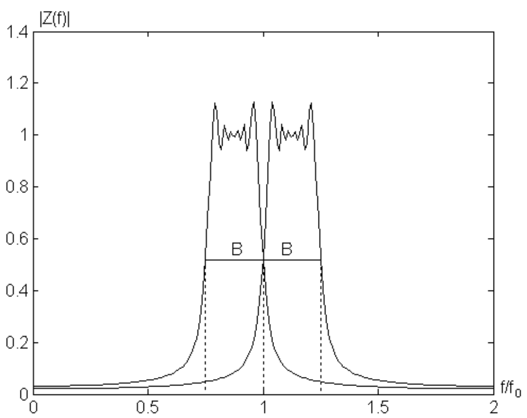

Fig.11. Spectra of FM signals, central frequencies f_- and f+ (L=123).

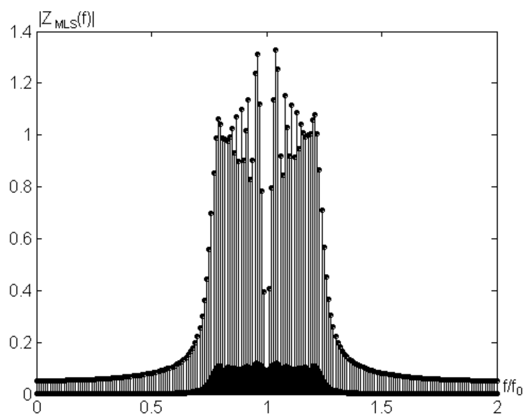

Fig.12. Spectra of FM signals keyframed by MLS codes (L=123).

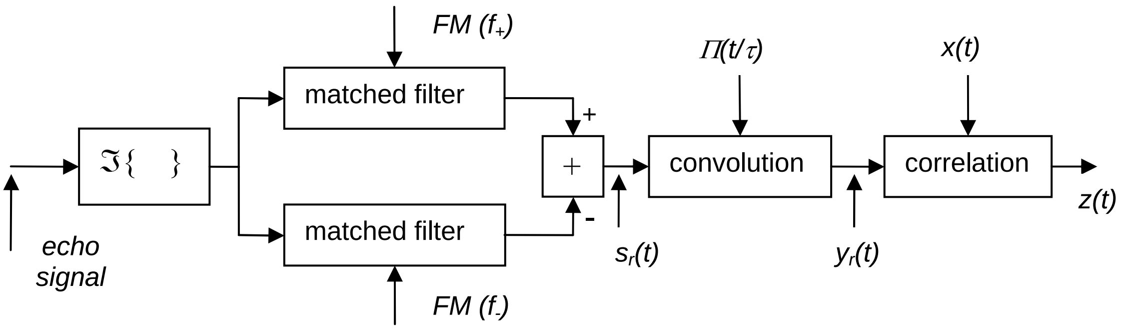

Fig.13. Diagram of echo signal processing in silent sonar receiver.

Once the Fourier transform of echo signal for duration T is determined, what follows is filtration matched to FM signals with duration τ, spectrum width B and mid frequencies f_- and f_+ . The result is a sequence of short pulses $s_r(n)$ matching the MLS sequence. They are shown in Fig. 14 for MLS of the order $N = 4$ (compare Fig. 5). When Doppler effect occurs, time compression ensures that the period over which the pulses are repeated is not τ. This leaves them uncorrelated with the original MLS sequence. To eliminate this adverse effect, a convolution is made of pulses $s_r(n)$ with the rectangular pulse of duration τ. What we obtain as a result is a low band signal $y_r(t)$ as shown in Fig. 15. When Doppler effect does not occur, the signal becomes deformed $x(t)$ as shown in Fig. 5.

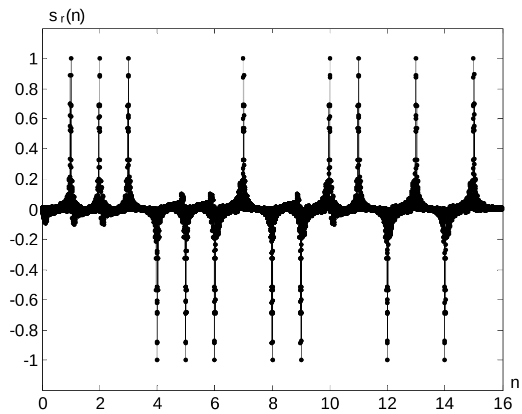

Fig.14. Signal after matched filtration.

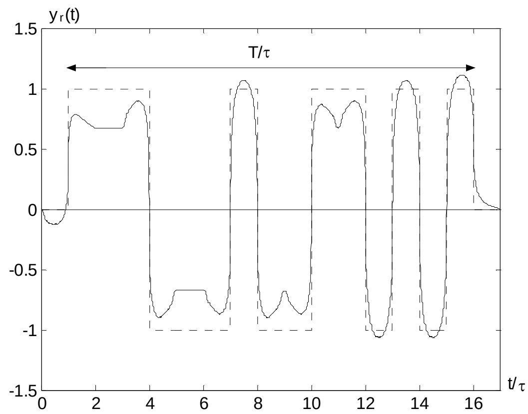

Fig.15. Echo signal envelope (--- $x(t)$).

Next, the correlation function is determined $z(t)$ of signal $y_r(t)$ with the model signal $x(t)$, described in formula (3). The properties of this function are like those described in detail in the previous section. In the case of our example, the correlation function without Doppler effect is shown in Fig. 16. As you can see, function $z(t)$ has parameters similar to the auto-correlation function r_{xx}. The theoretical level of side lobes, marked with the broken line in the Figure, is slightly exceeded due to the shape of function $y_r(t)$ as shown in Fig. 15. The level of side lobes can be reduced by narrowing the band width B of the FM signal and leaving mid frequencies f_- and f_+. This, however, deteriorates the shape of the sounding signal spectrum.

As the target moves, the maximum of the correlation function drops causing it to shift on the time axis and increasing the side lobe level just as was the case with the sounding signal envelope shown in Fig. 7 and Fig. 8. This is illustrated in Fig. 17 where the size of the signal $z(t)$ was normalised in relation to the maximal value of the correlation function in the absence of Doppler effect.

The shift of the correlation function maximum t_d is still described with formula (12), and for the data from Fig. 17 its numerically determined value is $t_d \cong 0.83\tau$.

To recapitulate we can say that the proposed type of modulation does not introduce any significant changes to Doppler effect resistance in relation to MLS envelopes.

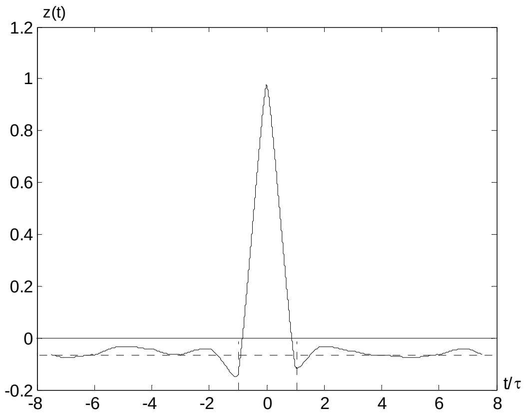

Fig.16. Signal without Doppler effect at receiver output.

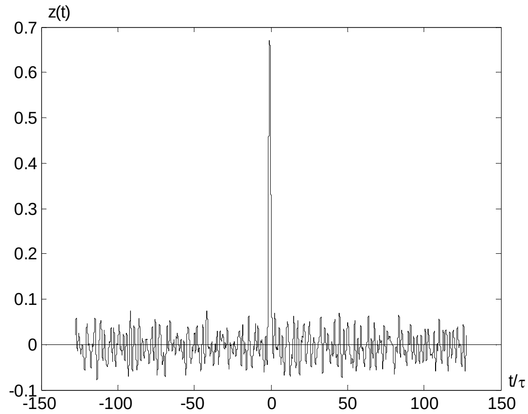

Fig.17. Receiver output signal for target velocity v = 4.5m/s, L=255.

3. NOISE

The noise in the system in question should be seen from two perspectives, namely noise produced by MLS as side lobes of the correlation function shown in Fig. 17 and as acoustic (or electric) noise at sonar receiver input. As for the first case, the conventional approach to detection would not be appropriate because this kind of noise is not additive. This is because its stochastic parameters depend on the size of the echo signal received. The problem is too complex to fit within the constraints of this article. This is why all we will do is determine the signal to acoustic noise ratio numerically for a non-Doppler effect case with a low level of side lobes amounting to 1/L.

Fig. 18 shows a signal with noise $z(t)$ for an LMS sequence $L = 255$ long, LFM signal spectrum width $B = 5$ kHz and pulse duration $\tau = 0.1$ s. Signal $z(t)$ was normalised in relation to its maximal value when no noise is present. The output signal to noise ratio SNR_o calculated as the quotient of the square of the function's maximal value $z(t)$ to noise variance has a mean value equal to $SNR_o \cong 54$. The input signal to noise ratio SNR_i which is the quotient of the echo signal power and noise variance at receiver input is $SNR_i = 0.02$. Hence, the improvement of the signal to noise ratio is equal to: $SNR_o/SNR_i \cong 2700$. Please note that the improvement cannot be attributed to MLS properties only. The fact that it is significant is also the result of FM signal matched filtration. This conclusion is confirmed by simulation results exemplified in Fig. 19. Compared to the signal from Fig. 18, signal duration τ was halved. As a result, the SNR_o dropped to about 35 leading to a drop in SNR_o/ SNR_i to about 1750.

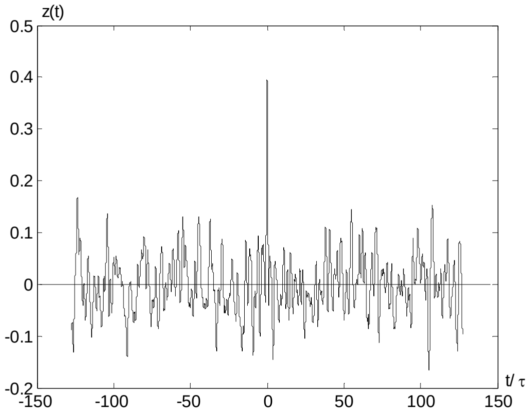

Fig.18. Signal with noise at matched filter output ($\tau = 0.1$ s).

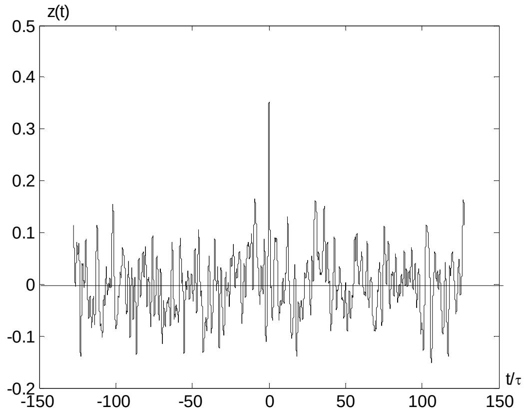

Fig.19. Signal with noise at matched filter output ($\tau = 0.05$ s).

3. CONCLUSIONS

If applied in silent sonar, maximum length sequence combined with FM sounding signals reduces error distance caused by Doppler effect unlike silent sonars emitting signals with linear or hyperbolic frequency modulation. In addition how big the error is does not depend on the width of the sounding signal spectrum ensuring more flexibility with sonar parameter selection. The downside of the proposed solution is the relatively high level of side lobes of the correlation function of the sounding signal and echo signal off the moving targets. The lobes deteriorate detection performance in a way similar to acoustic and electric noise. A separate analysis is required to identify detection performance in terms of detection and false alarm probability.

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