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Arching of Railway Turnouts by Analytical Design Method

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The paper involved the issue of arching of the railway turnouts. This is an issue which is given relatively less attention to scientific and research activities. Reference has been made to the book by Wladyslaw Rzepka, under the title "Curved turnouts in plan and profile", which has been used in Poland for more than 50 years as the main source of information relating to the turnouts on the curve. The book is a compilation of elaborations describing the contemporary state of knowledge being to a large extent a display of German achievements of the forties and fifties in the twentieth century. It has been pointed out that the theory accumulated in the book was adapted to the contemporary calculation needs. The major drawback in the given solutions seems to be ambiguity in the adopted reference system. In the present study an analytical approach is made to the subject matter, and the adopted system of coordinates is connected to the initial position of the turnout being arched. Three possible turnout arching variants have been analyzed to determine some universal mathematical relations. They describe the coordinates of the main track end and the diverging one, the value of the circular arc radius of the diverging track and equations of both the tracks. The analytical record also proves useful for authentic applications. It should be noted that it may be particularly beneficial to use the analytical method to design connections of parallel tracks located in a circular arc (using curved turnouts).

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1. INTRODUCTION

The issue of railway turnouts has been discussed in many publications [eg. 1-6]. This applies mainly to the problem of turnouts for high speed trains and the issue of dynamic interaction between train and turnout. On the other hand, much less attention is devoted to issues related to the arching of turnouts. This does not mean that in many countries the subject-matter of turnout arching is neglected. The interest taken in this sphere can be indicated by some publications [7-11].

In Poland for more than 50 years the fundamental, widely acknowledged source of information dealing with the problems of curved turnouts is paper [12]. As one should judge on the basis of the bibliography included in the book, it is a compilation of articles presenting the state of knowledge of that time, which was to a large extent an outcome of achievements of the forties and fifties in the twentieth century. The author gives authentic German publications [13-18] and, moreover, he makes references to some articles in Bahningenieur, Eisenbahnbau, Eisenbahn and Organ für die Fortschritte des Eisenbahnwesens.

Paper [12] is in fact an exceptional work relating to the analyzed problem in a complex way, including a survey of a large number of cases. Every situation has its own geometric scheme which provides a basis for determining an appropriate symbolic registration. The paper deals, among other things, with the following problems:

- Method of turnout arching,
- Connection of a straight track with a track situated in circular arc using curved turnout,
- Connection of tracks situated in circular arc using curved turnouts,
- Connection of curved turnouts with curved track intersections and curved crossovers,
- Double connection of tracks situated in arc,
- Curved turnouts with straight frogs,
- Curved turnouts on transition curves,
- Curved turnouts laid in tracks with the cant.

It should be noted that papers concerning the above subject, which appeared in Poland some later years, in principle, did not introduce anything novel but treated the elaboration [12] as an example to be followed and referred to. This relates to the last edition of the book [19] and, for example, paper [20] of the year 2015.

While analyzing paper [12] it is possible to note a characteristic regularity. The particular theoretical relations are determined on the basis of geometrical patterns by the application of their fundamental properties (circles, right-angled triangles, similarity of triangles). However, the use of trigonometric functions in the obtained formulae is avoided. In this connection one can arrive at an obvious conclusion that all the elaborated theory was adapted to the calculation technique of the time when the basic operations were carried out by a mechanical arithmometer or a slide rule.

To tell the truth since that time there survived also numerous simplifications that have been used up to now. In this way at the very beginning of the paper [12] it has been noted that the proper position of the track is determined by measuring the theoretical arrow of the rise of arch chord at a required place. A scheme for the determination of arrow *f* in circular arc is shown in Fig. 1 where the following denotations are used: *R* – radius of circular arc, *c* – half the length of chord.

By the application of the elementary relation $c^2 + (R - f)^2 = R^2$ one can derive an exact equation for the value of arrow in the form of $f = R - \sqrt{R^2 - c^2}$, however, in paper [12] an approximate formula $f = \frac{c^2}{2\,R}$ is given for this case. The applied simplification has at present no justification, but it must have looked in a different way taking into account the calculation possibilities of that time (the square root could not be calculated by using slide rule).

In deriving the basic geometric relationships a presentation has been made, among other things, of the procedure for finding the coordinates of the circumference centre in relation to the measuring line. In the next chapter, in order to illustrate the adopted procedure, the way of determining the radius of the diverging track in the curved turnout for a chosen variant is analyzed.

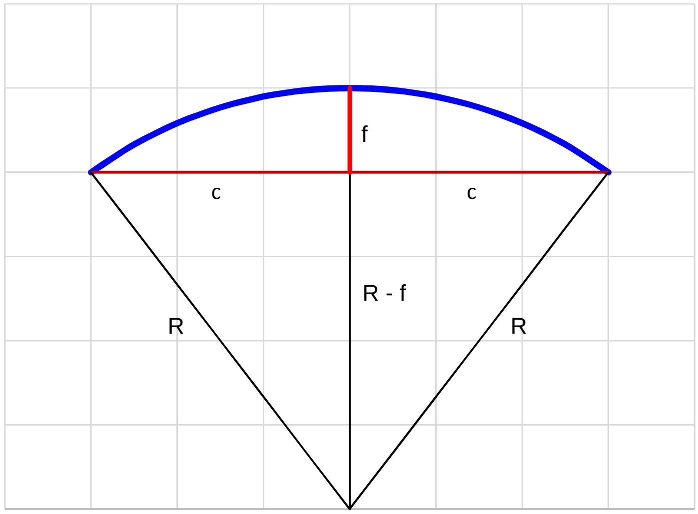

Fig. 1. Scheme for the determination of arrow *f* in the circular arc

The necessity of finding possibly the simplest theoretical relations has distorted the transparency of the entire work. When reading the book one comes to the conclusion that it is unnecessarily complicated and all the attempts could have been made much simpler. It all resulted in assuming a common opinion that the book [12], however, very valuable, is of little accessibility to the reader and may some of them simply discourage from taking interest in to. It is likely that this was the reason why the turnout arching formation has not become an object of special attention in scientific investigation in spite of the fact that in the range of calculation possibilities there has been made a tremendous progress during this time.

2. TRADITIONAL METHOD OF DETERMINING THE DIVERGING TRACK RADIUS IN A CURVED TURNOUT

A traditional method used in paper [12] to derive a formula for the radius of a diverging track of a one-sided curved turnout is shown in Fig. 2.

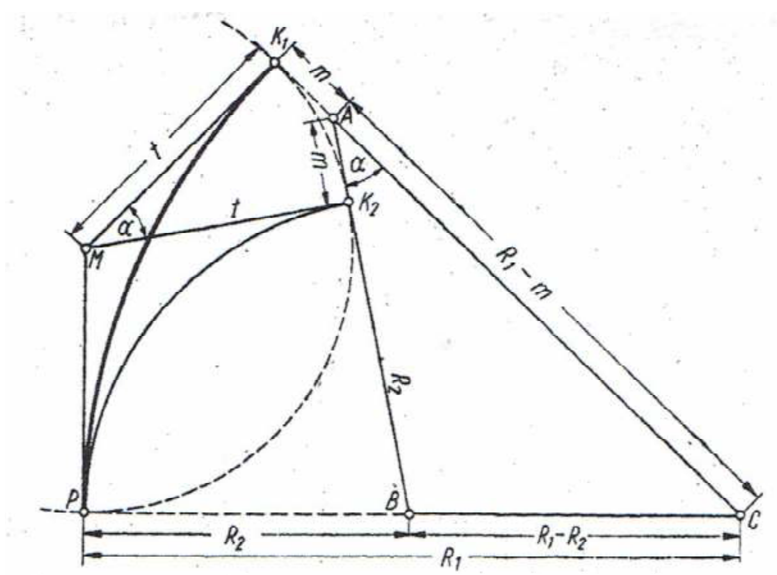

Fig. 2. A scheme for the determination of the diverging track radius in the one-sided curved turnout (according to paper [12])

In bending the main track of an ordinary points (the standard one) into a curved track of a given radius R_1, the diverging track of a turnout changes its shape simultaneously to form an arc of radius R_2 the magnitude of which depends on radius R_1 of the main track and radius *R* of the ordinary turnout. Values *α*, *t* and *m* used in Fig. 2. Characterize the ordinary turnout and do not undergo a change in arc formation. They are connected with one another by relations:

$$t = R\tan\frac{\alpha}{2} \tag{2-1}$$

$$m = \frac{t^2}{R} \tag{2-2}$$

Tangents to the circle of a radius equal to *t* projecting out of the turnout centre, drawn at the end points of turnout – K_1 of the main track, and K_2 of the diverging track – create triangle *ABC* outside the circumference of the circle. The sides of the triangle are, $\overline{AB} = R_2 + m$, $\overline{AC} = R_1 - m$, $\overline{BC} = R_1 - R_2$. To determine radius R_2 advantage is taken of a known formula for calculating the angle of a triangle when all its sides are given, namely

$$\tan\frac{\alpha}{2} = \sqrt{\frac{(p-b)\,(p-c)}{p\,(p-a)}} \tag{2-3}$$

where *a* denotes a side that is lying opposite angle α, while *p* indicates half of the triangle's circumference.

In the case analyzed, $= R_1 - R_2$, $b = R_1 - m$, $c = R_2 + m$, hence the triangle's circumference $2\,p = a + b + c = 2\,R_1$, which means $p = R_1$. Thus it follows that $p - a = R_2$, $-b = m$, $p - c = R_1 - R_2 - m$. After substituting the above to Equation (2-3) it is possible to obtain the relation

$$\tan\frac{\alpha}{2} = \sqrt{\frac{m\,(R_1 - R_2 - m)}{R_1\,R_2}} \tag{2-4}$$

Making use of equations (2-1), (2-2) and (2-4) and some transformations one can obtain the equation

$$\frac{m}{t^2} = \frac{R_1 - R_2 - m}{R_1\,R_2}$$

which provide the formula for magnitude R_2.

$$R_2 = \frac{R\,R_1 - t^2}{R + R_1} \tag{2-5}$$

3. ANALYTICAL APPROACH TO THE PROBLEM

It appears that the main drawback in the equations presented in paper [12] is the ambiguity in approval of the frame of reference. The coordinates' systems are most often connected with the centre of a particular circular arc whose position in the calculation procedure may alter together with the change of the values of geometric parameters. In connection with the above the position of a particular point does not result from function relation $y(x)$, but it is determined individually and, if necessary, a correction follows. Equations of circular arc are not used. Its determination is only related to coordination points: the starting and the end one and between them use is made of geodetic methods to determine the arc of a given radius.

Paper [21] proposes a new analytical approach to the turnout arching subject. With such an approach to the problem, the crucial question is the adoption of an appropriately established system of coordinates. This can be done in a very simple way. The only point which does not change its position during the arching of the (ordinary) turnout, is the outset of the turnout subjected to arching. It should become the starting point of the adopted system of coordinates. The axes of the coordinates system will be determined by the outset position of the curved turnout (Fig. 3).

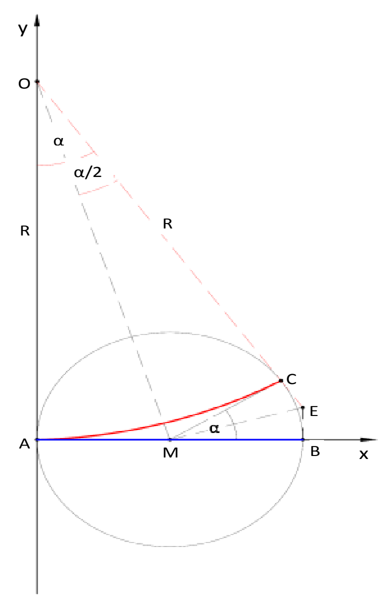

Fig. 3. Diagram of an ordinary turnout prior to its arc formation

In the turnout of angle 1 : *n* and radius *R* of the circular arc in diverging track the central angle is

$$\propto = \operatorname{atan}\frac{1}{n} \tag{3-1}$$

while the value of the turnout tangent is

$$t = \overline{AM} = \overline{MB} = \overline{MC} = R\tan\frac{\propto}{2} \tag{3-2}$$

The coordinates of the end of the diverging track which is situated on the turnout leading circle, are as follows:

$$x_C = t\,(1 + \cos\propto)\,, \qquad y_C = t\sin\propto$$

whereas the diverging track equation is

$$y = R - \sqrt{R^2 - x^2}\,, \quad x \in \langle 0, x_C\rangle \tag{3-3}$$

In the arc formation, values $\propto$, t and $\overline{BC}$ remain the same as in the basic turnout. Thus, the triangle of turnout angle MBC is unchanged (similar to point *A*), but its position alters (it turns around point *M*). Ends of the basic track and the diverging one should be situated on the turnout leading circle whose centre coincides with the centre of the basic turnout being arched (i.e. point *M*, Fig. 3).

4. VARIANTS OF THE TUNOUT ARC-FORMATION

In general it is necessary to take into consideration three arching variants of the turnout:

- One-sided arching towards the inside of the geometric system, where the arc of the main track of radius R_1 is consistent with the arc of the diverging track in the basic track of radius *R* (Variant I, Fig. 4),
- Both-sided arching where the arc of the basic track is directed opposite to the arc of the diverging track of the basic turnout, with the condition $R_1 > R$ that is binding (Variant II, Fig. 6),
- Single-sided arching outside the geometric system, where the arc of the basic track is directed opposite to the arc of the diverging track in the basic turnout, with condition $R_1 < R$ being in force (Variant III, Fig. 8).

Each variant of the turnout arching is characterized by the following procedures:

- Determination of coordinates of the main track end (point K_1),
- Determination of coordinates of the end of the diverging track (point K_2),
- Determination of the arc radius R_2 of the diverging track of the turnout,
- Determination of both the track equations.

4.1 Arching of the Turnout – Variant I

In variant I the arching of the turnout, the arc of a main track is in conformity with the diverging track arc of the ordinary turnout (Fig. 4). The central angle of the main track arc results from the relation

$$t = R_1 \tan\frac{\propto_1}{2}$$

Hence, it follows that

$$\propto_1 = 2\operatorname{atan}\frac{t}{R_1} \tag{4-1}$$

where value t follows from relation (3-2).

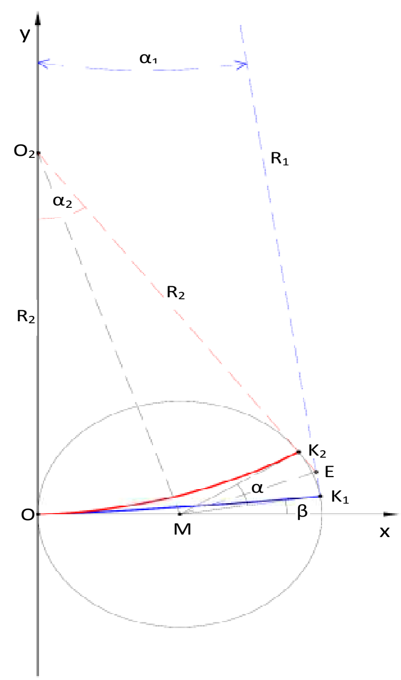

Fig. 4. A scheme of an ordinary turnout arching in variant I

Point K_1 lies on the straight line of formula

$$y = R_1 - \cot\propto_1 x$$

at distance R_1 from the centre of arc $O_1(0, R_1)$. A set of equations is obtained:

$$\begin{cases} y_{K1} = R_1 - \cot \propto_1 \ x_{K1} \\ (x_{K1} - x_{O1})^2 + (y_{K1} - y_{O1})^2 = R_1^2 \end{cases} \tag{4-2}$$

From the above equations (4-2) it is possible to determine the coordinates of point K_1.

$$x_{K1} = \frac{R_1}{\sqrt{1+(\cot \propto_1)^2}} \tag{4-3}$$

$$y_{K1} = \left[1 - \frac{\cot \propto_1}{\sqrt{1+(\cot \propto_1)^2}}\right] R_1 \tag{4-4}$$

In the determination of the coordinates of point K_2 , angle $\beta = \propto_1$ (Fig. 4). Point K_2 lies on the straight line of equation

$$y = \tan(\propto + \beta)\ (x - t)$$

at distance t from point $M(t, 0)$. Hence, the following set of equations is obtained:

$$\begin{cases} y_{K2} = \tan(\propto + \beta)\ (x_{K2} - t) \\ (x_{K2} - x_M)^2 + (y_{K2} - y_M)^2 = t^2 \end{cases} \tag{4-5}$$

The obtained formulae provide coordinates of point K_2.

$$x_{K2} = t + \frac{t}{\sqrt{1+[\tan(\propto+\beta)]^2}} \tag{4-6}$$

$$y_{K2} = \frac{\tan(\propto+\beta)}{\sqrt{1+[\tan(\propto+\beta)]^2}}\ t \tag{4-7}$$

To determine the arc radius R_2 of the turnout diverging track advantage is taken of the fact that point K_2 (the coordinates of which are already known) lies at distance R_2 from the centre of arc $O_2(0, R_2)$. The following formula is obtained

$$(x_{K2} - x_{O2})^2 + (y_{K2} - y_{O2})^2 = R_2^2$$

which indicates that

$$R_2 = \frac{x_{K2}^2 + y_{K2}^2}{2\ y_{K2}} \tag{4-8}$$

Now it is possible to take down the equations of both the tracks:

– the main track

$$y = R_1 - \sqrt{R_1^2 - x^2} \quad , \ x \in \langle 0, x_{K1} \rangle \tag{4-9}$$

– the diverging track

$$y = R_2 - \sqrt{R_2^2 - x^2} \quad , \ x \in \langle 0, x_{K2} \rangle \tag{4-10}$$

Fig. 5 presents examples of diagrams of horizontal ordinates of the main track and the diverging one for variant I of arching the turnout 1: 12 – 500.

4.2 Arching the Turnout – Variant II

In variant II the turnout arching the arc of the main track is directed opposite the arc of the diverging track in the basic turnout, where condition $R_1 > R$ (Fig. 6) is in force. As in variant I the central angle of the arc of the main track follows from relation (4-1).

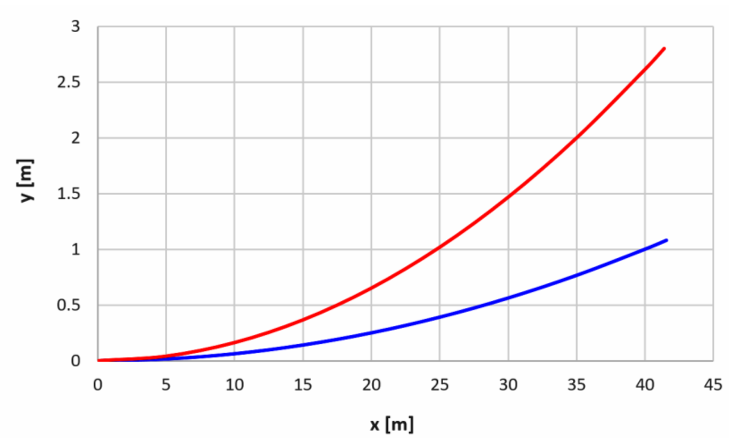

Fig. 5. Diagrams of horizontal ordinates of the main track (blue colour) and the diverging one (red colour) for variant I while arching the turnout 1 : 12 – 500 (R_1 = 800 m, R_2 = 307.360 m, using different scales)

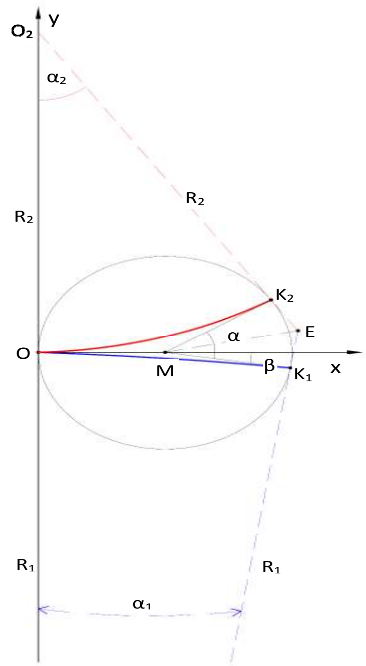

Fig. 6. A scheme of an ordinary turnout arching in variant II

Point K_1 lies on the straight line of equation

$$y = -R_1 + \cot \propto_1 x$$

at a distance R_1 from the centre of arc $O_1(0, -R_1)$. Thus it is possible to obtain the set of equations:

$$\begin{cases} y_{K1} = -R_1 + \cot \propto_1 \ x_{K1} \\ (x_{K1} - x_{O1})^2 + (y_{K1} + y_{O1})^2 = R_1^2 \end{cases} \quad (4\text{-}11)$$

by means of which one can determine the coordinates of point K_1. As it appears abscissa x_{K1} follows from equation (4-3), while ordinate y_{K1} from the relation

$$y_{K1} = -\left[1 - \frac{\cot \propto_1}{\sqrt{1+(\cot \propto_1)^2}}\right] R_1 \quad (4\text{-}12)$$

The coordinates of point K_2 are determined, as in Variant I, by formulae (4-6) and (4-7), where angle $\beta = -\propto_1$ (Fig. 6). The arc radius R_2 of the turnout diverging track is obtained from equation (4-8). The main track equation is as follows:

$$y = -R_1 + \sqrt{R_1^2 - x^2} \quad , \quad x \in \langle 0, x_{K1} \rangle \quad (4\text{-}13)$$

whereas the diverging track equation has the form of formula (4-10). Fig. 7 gives examples of diagrams of horizontal ordinates that appear in the main track and the diverging one for variant II of arching the turnout 1 : 12 – 500 when $R_1 > R$.

4.3 Arching the Turnout – Variant III

In variant III of the turnout arching, the arc of the main track is directed opposite to the arc of the diverging track in the basic turnout, where the condition $R_1 < R$ (Fig. 8) is in force. As in some earlier variants the central angle of the main track arc results from the relation (4-1). It is easy to prove that the coordinates of point K_1 are the same as the ones of variant II, described by equations (4-3) and (4-12). The coordinates of point K_2 are determined, as in variants I and II, by equations (4-6) and (4-7), where angle $\beta = -\propto_1$.

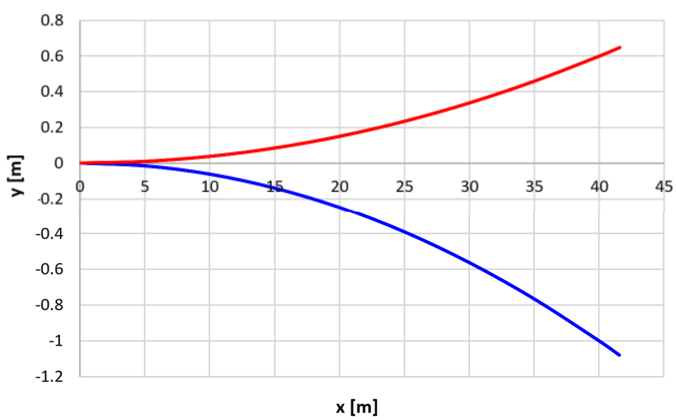

Fig. 7. Diagrams of horizontal ordinates of the main track (blue colour) and the diverging one (red colour) for variant II while arching the turnout 1 : 12 – 500 (R_1 = 800 m, R_2 = 1334.775 m, using different scales)

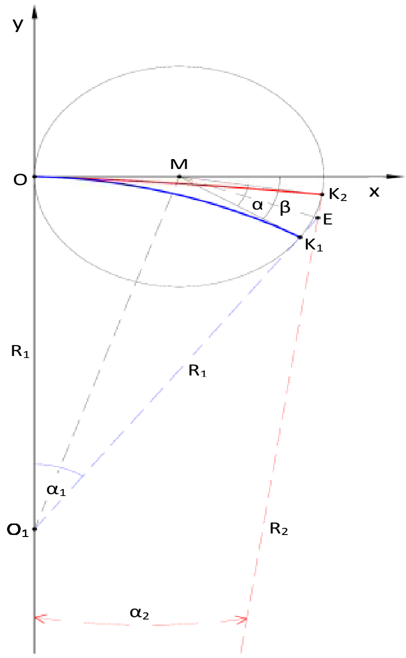

Fig. 8. A scheme of an ordinary turnout arching in variant III

For the reason that $y_{K2} < 0$, the radius of arc R_2 of the diverging track in the turnout results from the equation

$$(x_{K2} - x_{O2})^2 + (-y_{K2} - y_{O2})^2 = R_2^2$$

From the above equation it follows that

$$R_2 = -\frac{x_{K2}^2 + y_{K2}^2}{2\, y_{K2}} \qquad (4\text{-}14)$$

The equation for the ordinary track has the form of formula (4-13), while the diverging track is expressed by

$$y = -R_2 + \sqrt{R_2^2 - x^2} \quad , x \in \langle 0, x_{K2} \rangle \qquad (4\text{-}15)$$

Fig. 9 illustrates examples of diagrams of horizontal ordinates of a main and diverging track for variant III of arching turnout 1 : 12 – 500 when $R_1 < R$.

4.4 Comparison of the Calculation Equations

The presented analysis of the turnouts arching indicates that most of the equations worked out for variant I are valid in variants II and III, and the variations that are found, consist in the change of the sign in some expressions. Thus, there arises a possibility of making a list of some universal equations including all the variants under consideration.

– fundamental values

$$\propto = \operatorname{atan}\frac{1}{n} \qquad t = R \tan\frac{\propto}{2} \qquad \propto_1 = 2 \operatorname{atan}\frac{t}{R_1}$$

– coordinates of point K_1

$$x_{K1} = \frac{R_1}{\sqrt{1 + (\cot \propto_1)^2}}$$

$$y_{K1} = \pm\left[1 - \frac{\cot \propto_1}{\sqrt{1 + (\cot \propto_1)^2}}\right] R_1$$

(+) variant I, (−) variants II, III

– coordinates of point K_2

$$x_{K2} = t + \frac{t}{\sqrt{1 + [\tan(\propto + \beta)]^2}}$$

$$y_{K2} = \frac{\tan(\propto + \beta)}{\sqrt{1 + [\tan(\propto + \beta)]^2}}\, t$$

$\beta = \pm\propto_1$ (+) variant I, (−) variants II, III

– radius of arc R_2 in diverging track of the arched turnout

$$R_2 = \pm\frac{x_{K2}^2 + y_{K2}^2}{2\, y_{K2}}$$

(+) variants I, II, (−) variant III

– equation of the main track of curved turnout

$$y = \pm\left(R_1 - \sqrt{R_1^2 - x^2}\right), \quad x \in \langle 0, x_{K1} \rangle$$

(+) variant I, (−) variants II, III

– equation of the diverging track of curved turnout

$$y = \pm\left(R_2 - \sqrt{R_2^2 - x^2}\right), \quad x \in \langle 0, x_{K2} \rangle$$

(+) variants I, II, (−) variant III

The presented theory of the turnout arching is undoubtedly more transparent than a particular procedure described in book [12]. The analytical notation used offers much greater possibilities for some specific applications.

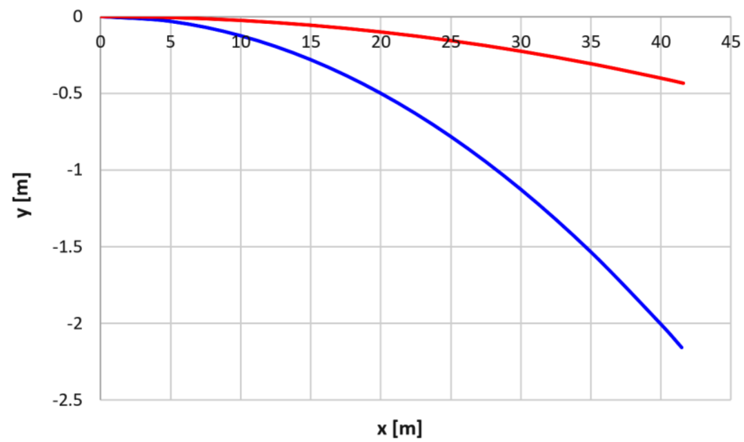

Fig. 9. Diagrams of horizontal ordinates of the main track (blue colour) and the diverging one (red colour) for variant III while arching the turnout 1 : 12 – 500 (R_1 = 400 m, R_2 = 2004.325 m, using different scales)

5. VERIFICATION ATTEMPT

It appears that it is possible to verify the presented turnout arching method. In paper [12] there has been worked out a general formula for arc radius R_2 used in the diverging track of curved turnout the value of which is dependent on radius R_1 of the main track and radius R of the ordinary turnout. The formula is presented in the form of

$$R_2 = \frac{R\,R_1 \pm t^2}{R \mp R_1} \tag{5-1}$$

Formula (5-1) causes some interpretative difficulties and undoubtedly for this reason the author of paper [12] makes use of a simplified formula in his calculation examples noted further on:

$$R_2 = \frac{R\,R_1}{R \mp R_1} \tag{5-2}$$

The author gives an explanation for it as a less effective than term t^2, which is much smaller than term $R\,R_1$. It might be of some interest to know that formula (5-2) is binding in British regulations up to now [22].

If one makes an assumption that values R and R_1 are positive, then formula (5-1) involves variant I, for which the solution of the problem, described by equation (2-5), has been given at point 2, and variant III where

$$R_2 = \frac{R\,R_1 + t^2}{R - R_1} \tag{5-3}$$

does not include variant II for which the equation, as it might appear, was incorrectly derived and should assume the following form:

$$R_2 = -\frac{R\,R_1 + t^2}{R - R_1} \tag{5-4}$$

However, one can avoid making problems mentioned above on assuming the following agreement on signs relating to the occurrence of the arc radii (in the coordinates system used in the paper):

$+(R)$ for a positive ordinate of the center of the circular arc (in Variants I and II),

$-(R)$ for a negative ordinate of the center of the circular arc (in Variant III).

Having made such an assumption, the use of equation (2-5), binding variant I, can be extended to all arching turnout variants under consideration.

On this basis it is possible to compare the compatibilities of radius R_2 determined by an analytical method, with an adequate value obtained by formula (2-5). It is worthy of mention at this place that the forms of the formulae applied are quite different in both the techniques. It is assumed the basic turnout 1 : 12 – 500 for which $\propto = 0.083141$ rad and $t = 20.797$ m. In consequence of the performed calculations the following values of R_2 can be obtained. In the analytical method they are positive, but equation (2-5) makes it necessary to keep to a certain agreement on the signs to be used.

– Variant I $R_1 = 800$ m (Fig. 5)

- in analytical method:

$\propto_1 = 0.051982$ rad, $x_{K1} = 41.566$ m,

$y_{K1} = 1.081$ m, β = 0.051982 rad

$x_{K2} = 41.405$ m, $y_{K2} = 2.802$ m,

$$R_2 = \frac{41.405^2 + 2.802^2}{2 * 2.802} = 307.360 \text{ m}$$

- by using formula (2-5):

$$R_2 = \frac{500 * 800 - 20.797^2}{500 + 800} = 307.360 \text{ m}$$

– Variant II $R_1 = 800$ m (Fig. 7)

- in analytical method:

$\propto_1 = 0.051982$ rad, $x_{K1} = 41.566$ m,

$y_{K1} = -1.081$ m, β = −0.051982

$x_{K2} = 41.584$ m, $y_{K2} = 0.648$ m,

$$R_2 = \frac{41.584^2 + 0.648^2}{2 * 0.648} = 1334.775 \text{ m}$$

- by the application of equation (2-5):

$$R_2 = \frac{500 * (-800) - 20.797^2}{500 + (-800)} = 1334.775 \text{ m}$$

– Variant III $R_1 = 400$ m (Fig. 9)

- using the analytical method:

$\propto_1 = 0.103893$ rad, $x_{K1} = 41.482$ m,

$y_{K1} = -2.157$ m, β = −0.103893 rad

$x_{K2} = 41.590$ m, $y_{K2} = -0.432$ m,

$$R_2 = \frac{41.590^2 + 0.432^2}{2 * 0.432} = 2004.325 \text{ m}$$

- by the use of formula (2-5):

$$R_2 = \frac{500 * (-400) - 20.797^2}{500 + (-400)} = -2004.325 \text{ m}$$

Owing to a full conformity obtained in the calculations it is possible to state that there has been noted a mutual verification of the compared solutions relating to radius R_2 of the diverging track of the curved turnout. However, a mere knowledge of the radius is not sufficient. It also makes it necessary to have the coordinates of both the track ends, which allows the equations to be written in an analytical way together with a justification of their application range. The method presented in paper [12] does not offer such possibilities.

6. CONCLUSION

The paper involved the issue of arching of the railway turnouts. This is an issue which is given relatively less attention to scientific and research activities. Therefore, an attempt was made to a new, analytical approach to this problem.

In Poland for more than 50 years the fundamental commonly used elaboration dealing with the problem of curved turnouts is book [12]. As one should judge on the basis of the biographical items enclosed in it the book is a compilation of work presenting the state of current knowledge which was the result of German achievements of the 40s and the 50s of the twentieth century.

In fact it is an exceptional work relating to the analyzed problem in a complex way. The book is equipped with an analysis of a large number of occurring cases. The few elaborations that were published in the next years did not include any problems under consideration but only treated the book [12] as a unique one and only made some references to it.

While reading the book [12] one arrives at an irresistible conclusion that the whole theory proposed, was adapted to the contemporary calculation possibilities. The necessity to find possibly the largest number of theoretical relationships distorted, however, the transparency of carrying out the entire course of proceeding. Reading the book one has the impression that it was unnecessarily complicated and all the problems could have been approached in a much simpler way.

It seems that the main drawback of the solutions presented in the discussed book is the ambiguity about the adoption of the frame of reference. In the paper an analytical approach is adopted to the question where the primary issue is the assumption of an appropriately determined system of coordinates. The system has been linked with the initial position of the turnout being curved.

Attention has been concentrated on three possible variants of a turnout arching providing them with universal mathematical relations. They describe the coordinates of the ends of the main track and the diverging one, value of radius of the diverging track arc and equations of both the tracks. This has significantly contributed to the extension of the turnout arching theory as against the one given in book [12]. The applied analytical record also gives much greater possibilities for authentic applications. It should be noted that it may be particularly beneficial to use the analytical method to design connections of parallel tracks located in a circular arc (using curved turnouts).

To prove the correctness of the obtained theoretical relations use has been made of a formula, originating from classic theory, for the value of the radius of diverging track arc. The formula is given an appropriate interpretation. As a result of obtaining full calculation conformity, it is possible to say that in this respect there has taken place a mutual verification of the solutions being compared.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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