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Brief Literature Review and Classification System of Reliability Methods for Evaluating the Stability of Earth Slopes

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Abstract: The issue of slope stability is one of the most important and yet most difficult geotechnical problems. Assessing slope stability is particularly difficult because of the many uncertainties involved in the process. To take these uncertainties into account, probabilistic methods are used, and the reliability approach is adopted. There are many methods for reliability assessment of earth slope stability. However, there is no system that would organize all of these methods in an unambiguous way. In fact, these methods can be classified in different ways: by assignment to a deterministic classification of methods, by description of uncertainties of soil parameters, by level of reliability according to the theory of reliability, etc. The huge number of articles summarizing the research in this field, but in various "disordered" directions, certainly do not facilitate the understanding or ultimately the practical application of the reliability approach by the engineer. The paper proposes a universal classification system of reliability methods for evaluating the stability of earth slopes. This proposal is preceded by a brief literature review of both historical background and contemporary research on reliability analysis of earth slope stability.

1. Introduction

The assessment of slope stability in terms of reliability analysis is still an active research topic being investigated by many scientific centres around the world. In only the last dozen years or so, in the leading geological and geotechnical journals as well as proceedings summarising international conferences, several thousand publications have appeared on this subject. Research is being conducted in different directions covering a wide spectrum of problems related to the deterministic and probabilistic methods used, comparative analyses, modelling of uncertain soil parameters (random variable, random field), influence of randomness of various factors affecting slope stability, types of problem, acting loading, calculation procedures with the possibility of simplifying these calculations, etc. In the present form, such a huge number of articles, of which only some contribute to the development of the discipline, is a disorderly "multidirectional" collection. It seems obvious that it should be systematised, because even for most researchers specialising in this subject, the quantity of disordered information can lead to confusion and possibly even incorrect conclusions. Of course, such a collection still would not give the engineer a universal tool for work. On the contrary, it would consistently discourage them from reliably analysing slope stability. In fact, the unwillingness of engineers to apply reliability to slope analysis and other geotechnical problems results from the lack of basic knowledge of statistics and probability theory or from common misconceptions with respect to the requirements regarding probabilistic methods, especially from the insufficiency of convincing literature illustrating the implementation and benefits of such analyses. To facilitate putting the reliability approach to slope stability into practice, the applied methodologies and procedures should be simple, well-known to engineers and able to solve real slope problems. In order to achieve this, it is first necessary to set in order and systematise the

extensive literature on the subject, comprehend its particular aspects and then classify it. Also, some concepts and terms, not only those appearing in recent literature, should be clarified. Thus, a brief overview of earth slope stability reliability approaches starting from the historical background and moving to contemporary research is presented in this paper. In the latter, apart from the methods of analysis, particular attention is paid to the issues developed in the last dozen or so years such as system reliability and description of random soil properties, especially spatial variability. Some comments on applications of reliability approaches to slope stability analysis are discussed as well. The state of research in the subject presented in an orderly manner, as well as the explanation of a number of terms and concepts occurring here, will bring the engineer closer to understanding this difficult issue and will contribute to the wider application of the reliability approach in geotechnical practice.

This paper proposes a classification system of reliability methods for earth slope stability assessment that integrates deterministic slope stability methods, modelling of uncertain soil parameters and reliability level (commonly used in the structural safety analysis). Also, in the case of the most sophisticated approaches, further divisions related to improvement of computation efficiency are suggested.

2. Historical Background

In general, there are two types of analyses for evaluating the stability of slopes against failure: deterministic and probabilistic (reliability). In the first case, three basic approaches have been developed: the limit equilibrium method (LEM), the displacement-based finite element method (FEM) used by the strength reduction method (SRM) (also called shear strength reduction (SSR)) and limit analysis (LA) by lower and upper bound solutions theorems. The limit equilibrium method utilises an assumed slip surface and determines its static equilibrium, usually by discretising the assumed failing soil mass into slices. The forces are then summed for each slice, creating a statically determinate problem following some assumptions. By introducing the factor of safety for the entire sliding mass, global equilibrium is maintained for a system on the verge of failure. The LEM is the oldest technique for evaluating slope stability and the most commonly used in practice. However, it is restricted by its arbitrary choice of failure mechanism and by interslice forces. Several variants of this method have been proposed [1–6]. In the FEM by SRM used in evaluating slope stability, soil strength parameters continuously decrease until the first indications of failure appear. The safety factor is defined as the ratio of the real shear strength of the soil to the reduced shear strength. This method seems to be superior to the LEM because there is no need for the primary guess at determining the critical failure surface. In addition, this method does not require any assumptions about interslice forces. FEM analysis is a more rigorous and universal technique, but often less attractive due to its dependence on mesh density and the available computational capacity. The primary advantage of this method is that the critical slip surface is found automatically from the shear strain, which increases as the shear strength decreases. Unfortunately, other "slip" surfaces (i.e., local minima) are omitted. However, because of the high speed of modern computer systems, analysis by FEM is used today more often than before. The finite element method by strength reduction method was first proposed and applied to slope stability by Zienkiewicz et al. [7] and then used to assess slope stability in, among others, [8–10]. Limit analysis models soil as a material that is perfectly plastic and obeys an associated flow rule. This method employs a dichotomy of theorems to provide a solution: either upper bound or lower bound plasticity. The upper bound theorem of limit analysis is predominantly used in solving slope stability problems. Unfortunately, the application of LA is still limited, since most of the research findings are chart-based and prepared for particular cases, and there is no stability chart available to cover a wide range of different slope material properties, geometries, etc. The concept of limit analysis was proposed by Drucker and Prager [11] and was utilised in slope stability in [12–14] and others. A review of the three basic deterministic approaches of slope stability, including their shortcomings and possible errors, is discussed in [15].

In general, the probabilistic methods used in geotechnical engineering can be divided into two groups, depending on the description of the uncertainties of the soil parameters: the single random variable (SRV) approach and the random field (RF) approach. The former method is commonly used in practice due to its concise concepts and simplicity in analysis. The latter reliably analyses the spatial variability of the soil properties. Regarding slope stability, these methods make use both of the LEM and FEM and rarely LA. Interest in the analysis of slope stability using the reliability approach began over 50 years ago. The number of papers on this subject has increased considerably since 1975, when a second ICASP International Congress was held in Aachen. After that, the correction factor methods, the first-order second-moment (FOSM), second-order second-moment (SOSM) and Monte Carlo (MC) methods, dominated. However, the majority of probabilistic or reliability methods made use of traditional slope stability analysis techniques, i.e., LEM [16–25]. In the case of a highly nonlinear function of factor of safety, computations of the derivatives are impossible or inconvenient, thus rendering FOSM results inaccurate. In addition, different results can be obtained depending on how the limit state function is formulated. To avoid the main drawback of the FOSM and SOSM methods (in which the results unduly depend on the mean value), as observed in [26,27], the first-order reliability method (FORM) has begun to be widely used [28–31]. It is well known that FORM works only for slopes with a small probability of failure or a high reliability index. Otherwise, this method underestimates the probability of failure of slopes. A summary of research on probabilistic analysis of slope stability was given in the monograph [32]. A review of the literature on this topic can also be found in [24] or [33].

3. Contemporary Research

Recently, numerical methods have found particularly significant value in reliability analysis, mainly due to the rapid development of computerisation. Their use, however, was associated with further research problems. Some of them are briefly presented below and others are only mentioned.

Much of the literature on the reliability of earth slope stability concerns earthquakes or environmental loads (rainfall, temperature changes) [34–52]. Some studies concern slope failure modelling in three-dimensional analysis. However, research on this topic is omitted from this paper. Regardless, it appears that these methods fall within the reliability method classification system proposed in the paper.

3.1. Methodology

A significant change in approach to probabilistic slope analysis occurred with the use of the finite element method (FEM) for geotechnical problems. Different probabilistic methods related to FEM have been proposed, such as the perturbation method, the Neumann expansion method, the partial differential method, the spectral stochastic finite element method (SSFEM), etc. Along with the development of computers and software, the MC simulation became dominant because of its relative ease of application. This method is a conceptually simple tool for reliability analysis of slope stability regardless of the form of the performance function or the number of scenario failure events. It employs statistical averaging over random samples generated from the probability density function of the parameters to evaluate the probability of failure. It is the easiest to apply; however, its simulations are usually time consuming and computation demanding. The MC method is also robust to various deterministic analysis methods for slope stability analysis, such as LEM or FEM/FDM [30,53]. Ali et al. [54] combined finite element limit analysis with random fields to deliver a valuable tool for probabilistic analysis of stability problems. Using a two-layer soil slope as an example, they showed that their proposed approach performed much better than equivalent uniform meshes (used in deterministic analysis) in reducing the gap between the upper and lower bounds of probabilistic solutions. The random finite element/difference method (RFEM/RFDM) was developed to account for the effect of spatial variability [25,55]. This method combines random field theory with the

finite element/difference method in a Monte Carlo framework. However, it does not offer insight into the relative contributions of various uncertainties to the failure probability. This is of particular interest in engineering practice. The critical slip surface varies spatially and needs to be located for each random sample generated during the MCS. The RFEM/RFDM also suffers from a lack of efficiency at small probability levels, especially in spatially variable soils. A comparative analysis of different variants of probabilistic and reliability methods based on FEM and applied to slope reliability assessment problems was given in [55–58].

Unfortunately, in the case of complicated slopes, RFEM requires extensive computational effort. In order to improve the computation efficiency of slope reliability analysis, research has been conducted in two basic directions: simplifying the performance function of slope stability and reducing the sample size of Monte Carlo simulations. In the first case, the response surface method [59], Kriging methodology [60], surrogate models [61], artificial neural network [62], support vector machine [63] or genetic algorithms [64] are usually utilised. The latter includes advanced sampling techniques such as subset simulation (SS), importance sampling (IS), etc. For example, the importance sampling probability density function is much closer to the failure region; thus, fewer realisations are needed for slopes with a low probability of failure. On this basis, Wang et al. [53] incorporated the SS method and Ching et al. [65] the IS method into the LEM. Li et al. [66] proposed the deaggregation approach, whereas Li et al. [67] combined MCS with both the LEM (in a first step) and the FEM. Hung et al. [68] combined method SS with the RFEM and Li et al. [69] proposed an advanced MCS method called generalised subset simulation (GSS). In GSS, the system failure event is decomposed into a number of scenario failure events through fault tree analysis, and the system failure probability and scenario failure probabilities are calculated by a single run of GSS. Thus, repeatedly performing the simulation for different failure events becomes unnecessary. This is of special importance when the number of slope failure events is large. An efficient machine learning (ML)-aided stochastic reliability analysis technique for spatially variable slopes was introduced in [70]. An attempt to classify the finite element method for slope stability analysis by comparing random fields was made in [71,72].

Recently, the material point method (MPM), which is a variant of the finite element method, has been applied in the reliability analysis of earth slopes. This method has been shown to be a robust spatial discretisation method for simulating multiphase interactions. It has distinct advantages in solving extremely large deformation problems. Wang et al. [73] proposed the random material point method (RMPM), which combines random field theory and the MPM. It differs from the random finite-element method (RFEM) by assigning random field (cell) values to material points that are free to move relative to the computational grid rather than to Gauss points in a conventional finite-element mesh. The authors demonstrated the method for an idealised slope in a strain softening soil. The occurrence and evolution of various slope failure modes during large deformation in spatially variable soils using Monte Carlo simulation combined with the MPM was investigated in [74].

Most of the research regarding the reliability analysis of slope stability (in the case of simplifying the performance function) has been devoted to the RSM. The fundamental concept of this method is the approximation of the relationship between the factor of safety and soil model parameters (such as cohesion and friction angle) by a simple and explicit polynomial function (approximation model) or by artificial neural networks. In problems of slope stability, nonlinear problems dominate. Therefore, the quadratic response surface method is most popular for more complicated system reliability analyses. The RSM was introduced into the reliability analysis of slope stability, with implicit performance functions, by Wong [75]. Several papers have been devoted to this problem [55,59,76–83]. The RSM may be applicable to some relatively simple cases, but such an approximation may not be possible for complicated multilayered slopes. The stratified response surface method was proposed in [84] in order to tackle the complexity problem. In this method, the number of stratified response surfaces is related to the number of soil layers that have

uncertain soil parameters. A review of RSM can be found in [85]. A comparative analysis on the computational efficiency and accuracy of four commonly-used RSM approaches was performed in [67] for several cases of slope reliability problems with or without taking the spatial variability of soil into account.

In the past few decades, the point estimate method (PEM) proposed by Rosenblueth [86] has found increasing use in many engineering fields. There are some limitations to this proposal for multiple variables, but a number of researchers have modified the procedure to optimise computational time and accuracy in these cases [87,88]. In fact, PEM has become one of the most popular methods in practical geotechnical reliability analysis. Many authors have applied this method for probabilistic analysis of slope design, indicating its efficiency and accuracy [89–93].

In recent years, various modifications of the "classical" reliability methods as well as their combinations have been developed. For example, it is worth mentioning a novel simplified method, the so-called quantile-based first-order second-moment method (QFOSM). This method has a level of simplicity close to the FOSM and yet has accuracy close to the FORM. It does not require performing large number of performance function evaluations (as in MCS, IS) nor operating in the standard normal space (as in FORM/SORM). The search for the critical quantile position is a one-dimensional root finding problem, which can be readily implemented [94]. A second-order orthogonal experimental design, SOED-based RSM, was proposed in [78]. The SOED is constructed by changing the length of star points, and the main characteristic of the SOED is that the design matrix is diagonal. The so-called combined response surface Method (CRSM), a simple, straightforward hybrid computational procedure incorporating the response surface method (RSM) linked with standard Monte Carlo (MC) simulations and the point estimate method (PEM), was proposed by Winkelmann et al. [95]. In order to address the multicollinearity existing in slope reliability analysis, a modified version of the RSM utilising the least absolute shrinkage and selection operator (Lasso), ridge regression, elastic net regression and stepwise regression was given in [96]. A novel multidimensional cloud model coupled with connection numbers theory for evaluation of slope stability was proposed in [97]. The cloud model is a tool for depicting fuzziness and randomness of evaluation indicators in a unified way. A modified one-dimensional conditional Markov chain model describing stratigraphic boundary uncertainty and MCS was introduced by [98].

3.2. System Reliability

There may be many possible slip surfaces in a slope reliability analysis. However, two basic types of failure probability should be distinguished: the failure probability of the slope along an individual slip surface and the overall failure probability of the slope. The critical deterministic surface is one of many failure surfaces for which the safety factor is minimal and the reliability index is calculated. However, this index is not necessarily the minimum value. The minimum reliability index occurs on some critical probabilistic surface that does not, in general, coincide with the critical deterministic one. Locating the critical probabilistic surface may require additional computations. The critical slip surface with the minimum safety factor varies spatially when spatial variability is considered. Identification of that surface among a large number of potential slip surfaces is an elementary step in slope stability analysis. In real soil slopes, various slope failure modes (i.e., slip surfaces) are caused by stratification (i.e., layered soils). In general, there may exist many potential failure modes or slip surfaces. Thus, it is justified to analyse the slope reliability problem rather in the framework of system reliability. In such a case, each potential slip surface is a component, and the critical slip surface is the weakest component. System failure takes place when a landslide occurs along the critical slip surface. Slope reliability analysis is then defined as a system reliability analysis. It is obvious that the overall failure probability is greater than the failure probability of any individual potential slip surface because of system effects. The difference depends on the correlation between the failure probabilities of the different slip surfaces, associated with various slip surfaces, for which no general

formulation is available. Although a slope may have many potential slip surfaces, its system failure probability is managed by only a few critical slip surfaces, which are called representative slip surfaces (RSS). The system reliability can be conveniently calculated based on these slip surfaces if the RSS are identified. An important aspect of reliability slope stability analysis is identifying the RSS. The failure probability of a system is generally more difficult to calculate than that of a single mode. First, instead of calculating the system failure probability directly, several researchers assessed the bounds of system failure probability [99,100]. However, these bounds could be wide when the single-mode failure probabilities are all large. The system reliability of slope stability has found much greater use along with the increasing development of simulation methods. These methods have allowed system failure probability to be predicted more accurately. For example, Ching et al. [65] applied the IS method, Huang et al. [101] combined the MCS method with the FEM, Wang et al. [53] used the SS and Zhang et al. [36] utilised the Hassan and Wolff method, which identifies the most critical slip surface of soil slopes based on sensitivity analysis, to assess the system reliability of a slope. It is also worth mentioning the papers [84] and [31], where a local search method and a barrier method for identification of RSS were developed, respectively. A system reliability analysis approach applied for layered soil slopes based on multivariate adaptive regression splines and MCS was given in [102]. A relatively new method to identify the RSS of arbitrary shapes based on the shear strength reduction method, for system reliability of soil slopes, was proposed in [103]. In that paper, a Kriging-based response surface to approximate the deterministic slope stability model was used. An efficient RSM-based MCS for risk assessment considering multiple failure surfaces was proposed in [104], whereas its extension by the effects of the stratigraphic boundary uncertainty was given in [105]. Duan et al. [106], viewing the task of representative slip surfaces as a multi-design-point identification problem, proposed a barrier-based optimisation method based on the shear strength reduction method to identify the RSS of arbitrary shapes.

3.3. Uncertainties of Soil Parameters

The uncertainties of the soil properties are a major contributor to the uncertainty in the stability of slopes. They comprise both a random measurement error (a result of a statistical uncertainty in the expected values and the effects of bias) and real spatial variability (variations in the trend of the parameters). Research on the influence of systematic error of soil parameters on slope stability has been carried out almost from the beginning of the application of probabilistic methods [24,26,93,107–111]. The copula theory to evaluate slope reliability in the presence of incomplete probability information was given in [112–114]. Application of a coupled Markov chain model that considered geological uncertainty for slope reliability was introduced in [115]. That model was also used in [116], where the inherent variability of the soil parameters was included. Among the available extensive literature on slope analysis, a significant part concerns hypothetical cases, usually simple slopes with a single soil layer or with two soil layers. The spatial variability of soil properties is one of the main sources of geotechnical uncertainty and significantly affects the reliability of slope stability. Thus, it is important that the spatially varying soil properties should be modelled appropriately. However, in such a case, the computing time consumed can be significantly longer. Ignoring this variability leads to an overestimation of the variance of the slope safety factor that can result either under- or overestimating the probability of slope failure [25,53,117,118]. Research on the influence of spatial variability of soil properties on the probability of slope failure has already begun; it was addressed by Vanmarcke [20], and a significant contribution was made in [119]. The effect of the spatial variability of soil properties on slope stability has been studied using the LEM in [24,53,120,121] and using the RFEM in [55,76,101,122,123]. The influence of spatially correlated soil properties on the probability of failure was analysed in [124]. The cross-correlation among Markov random fields leading to a generic approach for modelling multivariate cross-correlated geotechnical random fields based on vine cop-

ulas was proposed in [114]. Griffith et al. [55] showed that the probability of failure of slopes computed by RFEM may differ significantly from that obtained using a random field and the LEM. The RSM was used for this purpose in [59]. Application of the RFEM requires an understanding the relationships between correlation length, mesh coarseness, and the necessary number of simulations. Fundamental aspects in the employment of random fields in the numerical analysis of geotechnical structures were investigated in [125]. Application of Bayesian analysis to account for spatially variable soil in the reliability slope approach was proposed in [126]. Simplified approaches to incorporating the spatial variability of soil properties in the reliability analysis of slope stability should also be noted. Li et al. [127] used equivalent homogeneous random soil parameters in the form of single random variables applied to the RFDM, whereas Liu et al. [128] utilised a simplified framework based on the multiple response surface method (MRSM) and Monte Carlo simulation. It is also worth mentioning a simplified approach [129], proposing a method that combines the advantages of the sequential search and bisection search of the strength reduction method (SRM).

4. Applications

Use of reliability slope stability analysis in practical applications is still limited. Most engineers are not familiar with probabilistic concepts. The methods used here are unknown or seem to be too difficult for them. Even if they are not, the data needed for a detailed statistical evaluation of the soil properties is usually unavailable. There are too few papers presenting applications of the reliability approach in a comprehensible and simple way.

Practical spreadsheet techniques in the Microsoft Excel software for a probabilistic slope stability analysis that obtains the same reliability index as FORM, based on Janbu's generalised procedure of slices, were proposed by Low and Tang [28]. Those authors also modified spreadsheets by including cases with correlated non-normals and explicit and implicit performance functions [130]. The same technique was applied to the generalised Morgenstern–Price method of slices [29]. Furthermore, a spreadsheet approach based on Monte Carlo simulation and using Bishop's method, including spatial variability of the input variables, was proposed in [24]. Wang et al. [53] implemented an advanced MCS method called subset simulation. Low and Phoon [131] illustrated geotechnical reliability-based analyses, among others for a soil slope, using the Excel spreadsheet platform. They focused their attention on practical procedures available in commercial FORM/SORM packages and suggested such reliability-based analyses to fill a complementary role to the Eurocode 7 design approach. An iterative algorithm for FORM analysis involving correlated non-normal variables and their spatial variability was proposed in [123]. The authors demonstrated its usefulness for practicing engineers for geotechnical reliability analysis where deterministic software is used. A logical framework for the load and resistance factor design (LRFD) of slopes based on reliability analysis was proposed in [110]. MCS has been adopted into commercial software packages such as SVSlope, FLAC, PLAXIS and GEO5 [64]. In addition, Cao et al. [111] presented a practical approach of reliability slope stability analysis that implements an advanced MCS method called "subset simulation" in a spreadsheet environment.

There are also simplified probabilistic slope stability design charts. For example, the chart for purely cohesive soil [25] and for cohesive–frictional soil [109].

5. Classification System

In the case of rock slopes, the slope stability probability classifications (SSPC) proposed by Hack et al. [132] is commonly used. It is based on a three-step approach and on the probabilistic assessment of independently different failure mechanisms in a slope. It is also known as the modified Hack classification system [133] and was applied in, among others, [134,135]. Unfortunately, there is still no such system for the reliability stability methods of earth slopes. Thus, a classification proposal is presented below.

In a structural safety analysis, there are different levels of reliability, depending on the importance of the structure, grouped under four basic levels [136]:

- level 1—partial factor approach—employs only one "characteristic" value of each uncertainty parameter;
- level 2—estimates two values of each uncertainty parameter, usually the mean value, standard deviation, and the correlation between these parameters;
- level 3—best estimate of the probability of failure—knowledge of the join distribution of all uncertain parameters is required;
- level 4—reliability methods appropriate for structures of major economic importance, taking into account the structures' economic value, including the consequences of their failure.

The level 2 reliability methods are included in limit state design codes [137]. Methods of level 2 include a range of approximate or iterative procedures such as the perturbation method, FOSM, SOSM, FORM, SORM, PEM, etc. Methods of level 3, in the strict sense, require determining the mathematically exact probability of structural failure as a result of integrating the joint probability density function of random variables. In the case of slope stability, they can only be used for simplified, idealised cases. However, in a broader sense, level 3 methods require estimates of all probabilistic measures. The Monte Carlo simulation method has become the dominant procedure here as a result of the rapid development of computer techniques that have taken place in recent years. In order to improve the efficiency of this method while maintaining the accuracy of calculations, various reduction techniques have been developed (e.g., stratified sampling, Latin Hypercube simulation, importance sampling and Russian roulette and splitting). The response surface method and the methods of artificial neural networks have also grown in popularity in the analysis of the reliability of slopes. Both methods allow all probabilistic measures to be estimated and can also be qualified to the level 3 reliability method.

In the deterministic approach to slope stability, three basic approaches—the limit equilibrium method (LEM), the displacement-based finite/different element method (FEM/DEM), and limit analysis (LA)—have been developed. Probabilistic methods used in geotechnical engineering are commonly divided into two groups, depending on the description of the uncertainties of the soil parameters: the single random variable (SRV) approach and the random field (RF) approach.

The proposal of a classification of reliability methods of slope stability is presented the Table 1. It combines deterministic methods of slope stability with random modelling of the soil medium and includes levels of reliability.

Table 1. Classification of reliability methods of earth slope stability.

Random Soil Model	SRV			RF		
Deterministic Method Reliability Level	LEM	FEM/DEM/MPM	LA	LEM	FEM/DEM	LA
1	SRVLEM1	SRVFEM1	SRVLA1	RFLEM1	-	-
2	SRVLEM2	SRVFEM2	SRVLA2	RFLEM2	RFFEM2	RFLA2
3	SRVLEM3	SRVFEM3	SRVLA3	RFLEM3	RFFEM3	RFLA3

The abbreviations in Table 1 refer to the random soil model, deterministic slope stability method and reliability level. For example: SRVLEM1—soil modelled as a single random variable (SRV), limit equilibrium method (LEM) and Level 1 reliability method; RFFEM3—soil modelled as a random field, finite/different element method and Level 3 reliability method; etc.

Currently, the FEM/DEM is the predominant deterministic method of stability assessment, soil is usually modelled as a random field and level 3 reliability methods are usually applied. Most of the research focus here is on computation efficiency. Thus, the RFFEM3 methods can be divided into two groups depending on how the performance function of slope stability is simplified and on the reduction techniques (Figure 1).
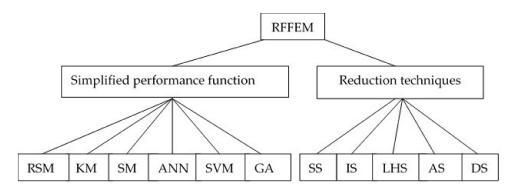
Figure 1. Division of RFFEM3 methods. RSM—response surface method. Improved RSM methods: KM—Kriging methodology, SM—surrogate models, ANN—artificial neural network, SVM—support vector machine, GA—genetic algorithms, SS—subset simulation, IS—importance sampling, LHS—Latin Hypercube sampling, AS—adaptive sampling, DS—directional simulation.

Slopes can be subjected both to static (gravity) and dynamic (earthquake, waves) loads as well as environmental loads (rainfall, temperature changes). Also, slope failure modelling can be carried out in a two-dimensional or three-dimensional analysis. However, these factors should not affect the proposed classification system of reliability methods. Instead, new subdivisions could be introduced on their basis.

6. Conclusions

In reviewing the reliability methods for evaluating earth slope stability, several important conclusions emerged. First of all, this issue has probably been the most published (as a geotechnical problem in terms of reliability) both because of its importance and level of difficulty. Many methods of probabilistic analysis have been developed, and thanks to the rapid progress of computer techniques, there has also been rapid development and acceleration of reliability analysis methods. It was also important to improve the method of ground investigation and to introduce the spatial variation of soil parameters into the calculation model. Unfortunately, the papers on reliability analysis of slopes actually cover basically all possible aspects, while the journals or proceedings in which they are published are not thematically assigned. Thus an attempt was made to organise the huge number of publications by assigning them to several research directions. In this paper, a review of the literature on reliability assessment of earth slope stability is presented in a fairly concise manner.

Generally, the engineer is quite conservative when it comes to applying new methods, especially when they are not simple—and such are probabilistic methods and reliability analysis. The terms and concepts explained in this paper as well as the general characteristics of the methods used should help the engineer to understand the benefits of the reliability approach to geotechnical problems and contribute to its application in practice.

Many probabilistic methods of slope stability assessment are known, and they can be divided according to different criteria. However, the classification of these methods for earth slopes has not yet been developed. The disorder that occurs as a result may make it difficult or discouraging for an engineer to read publications. The classification system proposed in this article organises these methods and will contribute to a better perception of this difficult issue.

This literature review shows that although reliability methods are extremely advanced and the computational possibilities almost unlimited, in practice, the simplest methods are usually used. This is mainly due to the fact that engineers are not familiar with probabilistic concepts; thus, it is difficult to incorporate them into practice.

The author is aware of the fact that the literature review included in the paper is incomplete. Certainly, a number of publications unavailable to him were omitted here. First of all, in accordance with the title of the work, the study area was narrowed thematically only to earth slopes. Environmental impacts, e.g., those caused by earthquakes, excessive

precipitation or temperature changes, were also ignored. However, the reliability methods of slope stability analysis used in these cases are included in the proposed computing classification system. It is also possible that among the great number of recent proposals, there are methods that do not fit into the presented classification system, and it could be necessary to modify it accordingly.

References

1. Fellenius, W. Calculations of the Stability of Earth Dams. In Proceedings of the Second Congress of Large Dams, Washington, DC, USA, 7–12 September 1936; Volume 4, pp. 445–463.
2. Bishop, A.W. The use of the slip circle in the stability analysis of slopes. *Geotechnique* **1955**, *5*, 7–17. [CrossRef]
3. Janbu, N. *Slope Stability Computation, Embankment- Dam Engineering: Casagrande Volume*; John Wiley & Sons: New York, NY, USA, 1973; pp. 47–86.
4. Nonveiller, E. The stability analysis of slopes with a slide surface of general shape. In Proceedings of the 6th International Conference on Soil Mechanics and Foundation Engineering, Montreal, QC, Canada, 8–15 September 1965; Volume 2, pp. 522–525.
5. Spencer, E. A method of analysis of the stability of embankments assuming parallel inter-slice forces. *Geotechnique* **1967**, *17*, 11–26. [CrossRef]
6. Morgenstern, N.R.; Price, V.E. The Analysis of the Stability of General Slip Surfaces. *Geotechnique* **1965**, *15*, 77–93. [CrossRef]
7. Zienkiewicz, O.C.; Humpheson, C.; Lewis, R.W. Associated and Non-Associated Visco-Plasticity and Plasticity in Soil Mechanics. *Geotechnique* **1975**, *25*, 671–689. [CrossRef]
8. Dawson, E.M.; Roth, W.H.; Drescher, A. Slope Stability Analysis by Strength Reduction. *Geotechnique* **1999**, *49*, 835–840. [CrossRef]
9. Griffiths, D.V.; Lane, P.A. Slope stability analysis by finite elements. *Geotechnics* **1999**, *49*, 387–403. [CrossRef]
10. Cheng, Y.M. Location of critical failure surface and some further studies on slope stability analysis. *Comput. Geotech.* **2003**, *30*, 255–267. [CrossRef]
11. Drucker, D.C.; Prager, W. Soil mechanics and plastic analysis or limit design. *Q. Appl. Math.* **1952**, *10*, 157–165. [CrossRef]
12. Chen, W.F.; Giger, M.W.; Fang, H.Y. On the limit analysis of stability of slopes. *Soils Found.* **1969**, *9*, 23–32. [CrossRef]
13. Donald, I.B.; Chen, Z.Y. Slope stability analysis by the upper bound approach: Fundamentals and methods. *Can. Geotech. J.* **1997**, *34*, 853–862. [CrossRef]
14. Michalowski, R.L. Slope stability analysis: A kinematical approach. *Géotechnique* **1995**, *45*, 283–293. [CrossRef]
15. Przewłócki, J. Some comments on slope stability evaluation. Part I: Deterministic analysis. *Inżynieria Morska Geotech.* **2004**, *2*, 141–149.
16. Wu, T.H.; Kraft, L.M. Safety Analysis of Slopes. *J. Soil Mech. Found. Div.* **1970**, *96*, 609–630. [CrossRef]
17. Cornell, C.A. First-order uncertainty analysis of soil deformation and stability. In Proceedings of the 1st International Conference Applications of Statistics and Probability in Soil and Structural, Hong Kong, 13–16 September 1971; pp. 129–144. Available online: https://trid.trb.org/view/128568 (accessed on 16 July 1974).
18. Alonso, E.E. Risk analysis of slopes and its application to slopes in Canadian sensitive clays. *Geotechique* **1976**, *26*, 453–472. [CrossRef]
19. Tang, W.H.; Yucemen, M.S.; Ang, A.H.S. Probability based short term design of soil slopes. *Can. Geotech. J.* **1976**, *13*, 201–215. [CrossRef]
20. Vanmarcke, E.H. Reliability of earth slopes. *J. Geotech. Eng.* **1977**, *103*, 1227–1246.
21. Chowdhury, R.N.; Grivas, D. Probabilistic model of progressive failure of slopes. *J. Geot. Eng.* **1982**, *108*, 803–917. [CrossRef]
22. Tobutt, D.C. Monte Carlo simulation methods for slope stability. *Comput. Geosci.* **1982**, *8*, 199–209. [CrossRef]
23. Chowdhury, R.N.; Tang, W.H.; Sidi, I. Reliability model of progressive slope failure. *Géotechnique* **1987**, *37*, 467–481. [CrossRef]
24. El-Ramly, H.; Morgenstern, N.R.; Cruden, D.M. Probabilistic slope stability analysis for practice. *Can. Geotech.* **2002**, *39*, 665–683. [CrossRef]
25. Griffiths, D.V.; Fenton, G.A. Probabilistic Slope Stability Analysis by Finite Elements. *J. Geotech. Geoenviron. Eng.* **2004**, *130*, 507–518. [CrossRef]
26. Christian, J.T.; Ladd, C.C.; Baecher, G.B. Reliability applied to slope stability analysis. *J. Geot. Eng.* **1994**, *120*, 2180–2207. [CrossRef]
27. Hassan, A.M.; Wolff, T.F. Search algorithm for minimum reliability index of earth slopes. *J. Geotech. Geoenviron. Eng.* **1999**, *125*, 301–308. [CrossRef]
28. Low, B.K.; Tang, W.H. Probabilistic Slope Analysis Using Janbu's Generalized Procedure of Slices. *Comput. Geotech.* **1997**, *21*, 121–142. [CrossRef]

29.	Low, B.K.; Gilbert, R.B.; Wright, S.G. Slope reliability analysis using generalized method of slices. *J. Geotech. Geoenviron. Eng.* **1998**, *124*, 350–362. [CrossRef]

30.	Cho, S.E. Effects of spatial variability of soil properties on slope stability. *Eng. Geol.* **2007**, *92*, 97–109. [CrossRef]

31.	Cho, S.E. First-order reliability analysis of slope considering multiple failure modes. *Eng. Geol.* **2013**, *154*, 98–105. [CrossRef]

32.	Knabe, W.; Przewłócki, J. *Probabilistic Slope Stability Analysis*; Institute of Hydro-Engineering of Polish Academy of Sciences: Gdańsk, Poland, 1990.

33.	Przewłócki, J. Some comments on slope stability evaluation. Part II: Probabilistic analysis. *Inżynieria Morska Geotech.* **2004**, *3*, 141–149.

34.	Cornell, C.A. Reliability-based earthquake-resistant design—The future. In Proceedings of the Eleventh World Conference on Earthquake Engineering, Acapulco, Mexico, 23–28 June 1966.

35.	Refice, A.; Capolongo, D. Probabilistic modeling of uncertainties in earthquake-induced landslide hazard assessment. *Comput. Geosci.* **2002**, *28*, 735–749. [CrossRef]

36.	Zhang, J.; Huang, H.W.; Juang, C.H.; Li, D.Q. Extension of Hassan and Wolff method for system reliability analysis of soil slopes. *Eng. Geol.* **2013**, *160*, 81–88. [CrossRef]

37.	Zhang, L.L.; Zhang, L.M.; Tang, W.H. Rainfall-induced slope failure considering variability of soil properties. *Geotechnique* **2006**, *55*, 215–220. [CrossRef]

38.	Zhang, J.; Huang, H.W.; Zhang, L.M.; Zhu, H.H.; Shi, B. Probabilistic prediction of rainfall-induced slope failure using a mechanics-based model. *Eng. Geol.* **2014**, *168*, 129–140. [CrossRef]

39.	Bray, J.D.; Travasarou, T. Simplified Procedure for Estimating Earthquake-Induced Deviatoric Slope Displacements. *J. Geotech. Geoenvironmental Eng.* **2007**, *133*, 381–392. [CrossRef]

40.	Wasowski, J.; Keefer, D.K.; Lee, C.T. Toward the next generation of research on earthquake-induced landslides: Current issues and future challenges. *Eng. Geol.* **2011**, *122*, 1–8. [CrossRef]

41.	Rathje, E.M.; Saygili, G. Estimating Fully Probabilistic Seismic Sliding Displacements of Slopes from a Pseudoprobabilistic Approach. *J. Geotech. Geoenviron. Eng.* **2011**, *137*, 208–217. [CrossRef]

42.	Rathje, E.M.; Wang, Y.; Stafford, P.J.; Antonakos, G.; Saygili, G. Probabilistic assessment of the seismic performance of earth slopes. *Bull. Earthq. Eng.* **2014**, *12*, 1071–1090. [CrossRef]

43.	Santoso, A.M.; Phoon, K.K.; Quek, S.T. Effects of soil spatial variability on rainfall-induced landslides. *Comput. Struct.* **2011**, *89*, 893–900. [CrossRef]

44.	Chiu, C.F.; Yan, W.M.; Yuen, K.V. Reliability analysis of soil-water characteristics curve and its application to slope stability analysis. *Eng. Geol.* **2012**, *135*, 83–91. [CrossRef]

45.	Tan, X.H.; Hu, N.; Li, D.; Shen, M.F.; Hou, X.L. Time-Variant Reliability Analysis of Unsaturated Soil Slopes Under Rainfall. *Geotech. Geol. Eng.* **2013**, *31*, 319–327. [CrossRef]

46.	Lee, D.; Lai, M.; WU, J.; Chi, Y.; Ko, W.; Lee, B. Slope management criteria for Alishan Highway based on database of heavy rainfall-induced slope failures. *Eng. Geol.* **2013**, *162*, 97–107. [CrossRef]

47.	Dou, H.Q.; Han, T.C.; Gong, X.N.; Qiu, Z.Y.; Li, Z.N. Effects of the spatial variability of permeability on rainfall-induced landslides. *Eng. Geol.* **2015**, *192*, 92–100. [CrossRef]

48.	Du, W.; Wang, G. A one-step Newmark displacement model for probabilistic seismic slope displacement hazard analysis. *Eng. Geol.* **2016**, *205*, 12–23. [CrossRef]

49.	Vega, J.A.; Hidalgo, C.A. Quantitative risk assessment of landslides triggered by earthquakes and rainfall based on direct costs of urban buildings. *Geomorphology* **2016**, *273*, 217–235. [CrossRef]

50.	Xiao, J.; Gong, W.; Martin, J.R.; Shen, M.; Luo, Z. Probabilistic seismic stability analysis of slope at a given site in a specified exposure time. *Eng. Geol.* **2016**, *212*, 53–62. [CrossRef]

51.	Xiong, M.; Huang, Y. Stochastic seismic response and dynamic reliability analysis of slopes: A review. *Soil Dyn. Earthq. Eng.* **2017**, *100*, 458–464. [CrossRef]

52.	Pang, R.; Xu, B.; Kong, X.; Zhou, Y.; Zou, D. Seismic performance evaluation of high CFRD slopes subjected to near-fault ground motions based on generalized probability density evolution method. *Eng. Geol.* **2018**, *246*, 391–401. [CrossRef]

53.	Wang, Y.; Cao, Z.; Au, S.K. Practical reliability analysis of slope stability by advanced Monte Carlo simulations in a spreadsheet. *Can. Geotech. J.* **2011**, *48*, 162–172. [CrossRef]

54.	Ali, A.; Lyamin, A.V.; Huang, J.; Li, J.H.; Cassidy, M.J.; Sloan, S.W. Probabilistic stability assessment using adaptive limit analysis and random fields. *Acta Geotech.* **2017**, *12*, 937–948. [CrossRef]

55.	Griffiths, D.V.; Huang, J.; Fenton, G.A. Comparison of slope reliability method of analysis. In Proceedings of the Geoflorida Conference Advances in Analysis, Modeling and Design, West Palm Beach, FL, USA, 20–24 February 2010; pp. 1952–1961.

56.	Farah, K.; Ltifi, M.; Hassis, H. A Study of Probabilistic FEMs for a Slope Reliability Analysis Using the Stress Fields. *Open Civ. Eng. J.* **2015**, *9*, 196–206. [CrossRef]

57.	Johari, A.; Fooladi, H. Comparative study of stochastic slope stability analysis based on conditional and unconditional random field. *Comput. Geotech.* **2020**, *125*, 103707. [CrossRef]

58.	Liu, S.Y.; Shao, L.T.; Li, H.J. Slope stability analysis using the limit equilibrium method and two finite element methods. *Comput. Geotech.* **2015**, *63*, 291–298. [CrossRef]

59. Li, L.; Chu, X.S. Multiple response surfaces for slope reliability analysis. *Int. J. Numer. Anal. Methods Geomech.* **2015**, *39*, 175–192. [CrossRef]
60. Zhang, J.; Huang, H.W.; Phoon, K.K. Application of the Kriging-based response surface method to the system reliability of soil slopes. *J. Geotech. Geoenviron. Eng.* **2013**, *139*, 651–655. [CrossRef]
61. Kang, F.; Qing Xu, Q.; Li, J. Slope reliability analysis using surrogate models via new support vector machines with swarm intelligence. *Appl. Math. Model.* **2016**, *40*, 6105–6120. [CrossRef]
62. Cho, S.E. Probabilistic stability analyses of slopes using the ANN-based response surface. *Comput. Geotech.* **2009**, *36*, 787–797. [CrossRef]
63. Kang, F.; Li, J.S.; Li, J.J. System reliability analysis of slopes using least squares support vector machines with particle swarm optimization. *Neurocomputing* **2016**, *209*, 46–56. [CrossRef]
64. Tun, Y.W.; Pedroso, D.M.; Scheuermann, A.; Williams, D.J. Probabilistic reliability analysis of multiple slopes with genetic algorithms. *Comput. Geotech.* **2016**, *77*, 68–76. [CrossRef]
65. Ching, J.; Phoon, K.-K.; Hu, Y.-G. Efficient Evaluation of Reliability for Slopes with Circular Slip Surfaces Using Importance Sampling. *J. Geotech. Geoenviron. Eng.* **2009**, *135*, 768–777. [CrossRef]
66. Li, L.; Wang, Y.; Cao, Z. Probabilistic slope stability analysis by risk aggregation. *Eng. Geol.* **2014**, *176*, 57–65. [CrossRef]
67. Li, D.Q.; Xiao, T.; Cao, Z.J.; Phoon, K.K.; Zhou, C.B. Efficient and consistent reliability analysis of soil slope stability using both limit equilibrium analysis and finite element analysis. *Appl. Math. Model.* **2016**, *40*, 5216–5229. [CrossRef]
68. Huang, J.; Fenton, G.; Griffiths, D.V.; Li, D.; Zhou, C. On the efficient estimation of small failure probability in slopes. *Landslides* **2016**, *14*, 491–498. [CrossRef]
69. Li, D.Q.; Yang, Z.H.; Cao, Z.J.; Au, S.K.; Phoon, K.K. System reliability analysis of slope stability using generalized Subset Simulation. *Appl. Math. Model.* **2017**, *46*, 650–664. [CrossRef]
70. He, X.; Xu, H.; Sabetamal, H.; Sheng, D. Machine learning aided stochastic reliability analysis of spatially variable slopes. *Comput. Geotech.* **2020**, *126*, 103711. [CrossRef]
71. Dyson, A.P.; Tolooiyan, A. Prediction and classification for finite element slope stability analysis by random field comparison. *Comput. Geotech.* **2019**, *109*, 117–129. [CrossRef]
72. Dyson, A.P.; Tolooiyan, A. Comparative Approaches to Probabilistic Finite Element Methods for Slope Stability Analysis. *Simul. Model. Pract. Theory* **2020**, *100*, 102061. [CrossRef]
73. Wang, B.; Feng, X.; Pan, P.; Li, S. Slope failure analysis using the material point method. *Yanshilixue Yu Gongcheng Xuebao Chin. J. Rock Mech. Eng.* **2017**, *36*, 2146–2155. [CrossRef]
74. Liu, X.; Wang, Y.; Li, D.Q. Investigation of slope failure mode evolution during large deformation in spatially variable soils by random limit equilibrium and material point methods. *Comput. Geotech.* **2019**, *111*, 301–312. [CrossRef]
75. Wong, F.S. Slope reliability and response surface method. *J. Geotech. Eng.* **1985**, *111*, 32–53. [CrossRef]
76. Xu, B.; Low, B.K. Probabilistic stability analyses of embankments based on finite-element method. *J. Geotech. Geoenviron. Eng.* **2006**, *132*, 1444–1454. [CrossRef]
77. Li, D.Q.; Zheng, D.; Cao, Z.J.; Tang, X.S.; Phoon, K.K. Response Surface method fo slope stability: Review and comparison. *Eng. Geolog.* **2016**, *203*, 3–14. [CrossRef]
78. Hu, B.; Su, G.S.; Jiang, J.; Xiao, Y. Gaussian process-based response surface method for slope reliability analysis. *Advances in Civ. Eng.* **2019**, *2019*. [CrossRef]
79. Li, D.; Chen, Y.; Lu, W.; Zhou, C. Stochastic response surface method for reliability analysis of rock slopes involving correlated non-normal variables. *Comput. Geotech.* **2011**, *38*, 58–68. [CrossRef]
80. Liu, Z.; Choi, J.C.; Nadim, F.; Lacaasse, S. Reliability analysis of sensitive clay slope with the response surface method. In Proceedings of the GeoShanghai 2018 International Conference: Geoenvironment and Geohazard, Shanghai, China, 27–30 May 2018; pp. 63–72. [CrossRef]
81. Zhang, J.; Zhang, L.M.; Tang, W.H. New methods for system reliability analysis of soil slopes. *Can. Geotech. J.* **2011**, *48*, 1138–1148. [CrossRef]
82. Tan, X.H.; Shen, M.F.; Hou, X.L.; Li, D.; Hu, N. Response Surface Method of Reliability Analysis and its Application in Slope Stability Analysis. *Geotech. Geol. Eng.* **2013**, *31*, 1011–1025. [CrossRef]
83. Zhou, X.P.; Huang, X.C. Reliability analysis of slopes using UD-based response surface methods combined with LASSO. *Eng. Geol.* **2018**, *233*, 111–123. [CrossRef]
84. Ji, J.; Low, B.K. Stratified Response Surfaces for System Probabilistic Evaluation of Slopes. *J. Geotech. Geoenviron. Eng.* **2012**, *138*, 1398–1406. [CrossRef]
85. Banaki, R.; Ahmad, F.; Tabarroki, M.; Yahaya, A.S. Probabilistic Analyses of Slopes: A State of the Art Review—Inpressco. *Int. J. Curr. Eng. Technol.* **2013**, *3*, 58–63.
86. Rosenblueth, E.Y.-I. Point estimates for probability moments. *Proc. Nat. Acad. Sci. USA* **1975**, *72*, 3812–3814. [CrossRef]
87. Harr, M.E. Probabilistic estimates for multivariate analyses. *Appl. Math. Model.* **1989**, *13*, 313–318. [CrossRef]
88. Hong, H.P. An efficient point estimate method for probabilistic analysis. *Reliab. Eng. Syst. Saf.* **1998**, *59*, 261–267. [CrossRef]
89. Baecher, G.B.; Christian, J.T. *Reliability and Statistics in Geotechnical Engineering*; John Wiley & Sons: Chichester, UK, 2003.
90. Przewłócki, J. *Problems of Stochastic Soil Mechanics—Reliability Analysis*; Dolnośląskie Wydawnictwo Edukacyjne: Wrocław, Poland, 2006.

91. Gibson, W. Probabilistic methods for slope analysis and design. *Aust. Geomech.* **2011**, *46*, 1–11.
92. Wang, J.P.; Huang, D. Rosen Point: A Microsoft Excel-based program for the Rosenblueth point estimate method and an application in slope stability analysis. *Comput. Geosci.* **2012**, *48*, 239–243. [CrossRef]
93. Ahmadabadi, M.; Poisel, R. Assessment of the application of point estimate methods in the probabilistic stability analysis of slopes. *Comput. Geotech.* **2015**, *69*, 540–550. [CrossRef]
94. Yang, Z.; Ching, J. A novel simplified geotechnical reliability analysis method. *Appl. Math. Model.* **2019**, *74*, 337–349. [CrossRef]
95. Winkelmann, K.; Zabuski, L.; Przewłócki, J.; Górski, J. Reliability-Based Stability Analysis of a Baltic Cliff by the Combined Response Surface Method. *Geotech. Geol. Eng.* **2020**, *38*, 5549–5563. [CrossRef]
96. Zhang, T.; Zhou, X.P.; Liu, X.F. Reliability analysis of slopes using the improved stochastic response surface methods with multicollinearity. *Eng. Geol.* **2020**, *271*, 105617. [CrossRef]
97. Wang, M.; Wang, X.; Liu, Q.; Shen, F.; Jin, J. A novel multi-dimensional cloud model coupled with connection numbers theory for evaluation of slope stability. *Appl. Math. Model.* **2020**, *77*, 426–438. [CrossRef]
98. Liu, L.L.; Cheng, Y.M.; Pan, Q.J.; Dias, D. Incorporating stratigraphic boundary uncertainty into reliability analysis of slopes in spatially variable soils using one-dimensional conditional Markov chain model. *Comput. Geotech.* **2020**, *118*, 103321. [CrossRef]
99. Oka, Y.; Wu, T.H. System reliability of slope stability. *J. Geot. Eng.* **1990**, *116*, 1185–1189. [CrossRef]
100. Chowdhury, R.N.; Xu, D.W. Geotechnical system reliability of slopes. *Reliab. Eng. Syst. Saf.* **1995**, *47*, 141–151. [CrossRef]
101. Huang, J.; Griffiths, D.V.; Fenton, G.A. System reliability of slopes by RFEM. *Soils Found.* **2010**, *50*, 343–353. [CrossRef]
102. Liu, L.L.; Cheng, Y.M. Efficient system reliability analysis of soil slopes using multivariate adaptive regression splines-based. *Comput. Geotech.* **2016**, *79*, 41–54. [CrossRef]
103. Ma, J.Z.; Zhang, J.; Huang, H.W.; Zhang, L.L.; Huang, J.S. Identification of representative slip surfaces for reliability analysis of soil slopes based on shear strength reduction. *Comput. Geotech.* **2017**, *85*, 199–206. [CrossRef]
104. Zhang, J.; Huang, H.W. Risk assessment of slope failure considering multiple slip surfaces. *Comput. Geotech.* **2016**, *74*, 188–195. [CrossRef]
105. Liu, L.L.; Cheng, Y.M.; Zhang, S.H. Conditional random field reliability analysis of a cohesion-frictional slope. *Comput. Geotech.* **2017**, *82*, 173–186. [CrossRef]
106. Duan, X.; Zhang, J.; Huang, H.; Zeng, P.; Zhang, L. System reliability analysis of soil slopes through constrained optimization. *Landslides* **2020**, *18*, 655–666. [CrossRef]
107. Malkawi, A.I.; Hassan, W.F.; Abdulla, F.A. Uncertainty and reliability analysis applied to slope stability. *Struct. Saf.* **2000**, *22*, 161–187. [CrossRef]
108. Wang, Y.; Cao, Z.; Au, S.K. Efficient Monte Carlo Simulation of parameter sensitivity in probabilistic slope stability analysis. *Comput. Geotech.* **2010**, *37*, 1015–1022. [CrossRef]
109. Javankhoshdel, S.; Bathurst, R.J. cohesive and cohesive-frictional (c-ɸ) soils. *Can. Geotech. J.* **2014**, *51*, 1033–1045. [CrossRef]
110. Salgado, R.; Kim, D. Reliability Analysis of Load and Resistance Factor Design of Slopes. *J. Geotech. Geoenviron. Eng.* **2014**, *140*, 57–73. [CrossRef]
111. Cao, Z.; Wang, Y.; Li, D. *Probabilistic Approaches for Geotechnical Site Characterization and Slope Stability Analysis*; Springer: Berlin/Heidelberg, Germany, 2017.
112. Wu, X.Z. Probabilistic slope stability analysis by a copula-based sampling method. *Comput. Geosci.* **2013**, *17*, 739–755. [CrossRef]
113. Tang, X.S.; Li, D.Q.; Zhou, C.B.; Phoon, K.K. Copula-based approaches for evaluating slope reliability under incomplete probability information. *Struct. Saf.* **2015**, *52*, 90–99. [CrossRef]
114. Tang, X.S.; Wang, M.X.; Li, D.Q. Modeling multivariate cross-correlated geotechnical random fields using vine copulas for slope reliability analysis. *Comput. Geotech.* **2020**, *127*, 103784. [CrossRef]
115. Li, D.Q.; Qi, X.H.; Cao, Z.J.; Tang, X.S.; Phoon, K.K.; Zhou, C.B. Evaluating slope stability uncertainty using coupled Markov chain. *Comput. Geotech.* **2016**, *73*, 72–80. [CrossRef]
116. Deng, Z.P.; Li, D.Q.; Qi, X.H.; Cao, Z.J.; Phoon, K.K. Reliability evaluation of slope considering geological uncertainty and inherent variability of soil parameters. *Comput. Geotech.* **2017**, *92*, 121–131. [CrossRef]
117. Jha, S.K.; Ching, J. Simplified reliability method for spatially variable undrained engineered slopes. *Soils Found.* **2013**, *53*, 708–719. [CrossRef]
118. Jiang, S.H.; Li, D.Q.; Zhang, L.M.; Zhou, C.B. Slope reliability analysis considering spatially variable shear strength parameters using a non-intrusive stochastic finite element method. *Eng. Geol.* **2014**, *168*, 120–128. [CrossRef]
119. Li, K.S.; Lumb, P. Probabilistic design of slopes. *Can. Geotech. J.* **1987**, *24*, 520–531. [CrossRef]
120. Low, B.K.; Lacasse, S.; Nadim, F. Slope reliability analysis accounting for spatial variation. *Georisk* **2007**, *1*, 177–189. [CrossRef]
121. Suchomel, R.; Mašín, D. Comparison of different probabilistic methods for predicting stability of a slope in spatially variable c–u soil. *Comput. Geotech.* **2010**, *37*, 132–140. [CrossRef]
122. Griffiths, D.V.; Huang, J.; Fenton, G.A. Influence of spatial variability on slope reliability using 2-D random field. *J. Geotech. Geoenviron. Eng.* **2009**, *135*, 1367–1378. [CrossRef]
123. Ji, J.; Zhang, C.; Gao, Y.; Kodikara, J. Effect of 2D spatial variability on slope reliability: A simplified FORM analysis. *Geosci. Front.* **2018**, *9*, 1631–1638. [CrossRef]
124. Kim, J.M.; Sitar, N. Reliability approach to slope stability analysis with spatially correlated soil properties. *Soils Found.* **2013**, *53*, 1–10. [CrossRef]

125. Mahmoudi, E.; Schmüdderich, C.; Hölter, R.; Zhao, C.; Wichtmann, T.; König, M. Stochastic field simulation of slope stability problems: Improvement and reduction of computational effort. *Comput. Methods Appl. Mech. Eng.* **2020**, *369*, 113167. [CrossRef]
126. Jiang, S.H.; Papaioannou, I.; Straub, D. Bayesian updating of slope reliability in spatially variable soils with in-situ measurements. *Eng. Geol.* **2018**, *239*, 310–320. [CrossRef]
127. Li, X.Y.; Zhang, L.M.; Gao, L.; Zhu, H. Simplified slope reliability analysis considering spatial soil variability. *Eng. Geol.* **2017**, *216*, 90–97. [CrossRef]
128. Liu, L.; Deng, Z.; Zhang, S.; Cheng, Y. Simplified framework for system reliability analysis of slopes in spatially variable soils. *Eng. Geol.* **2018**, *239*, 330–343. [CrossRef]
129. Luo, X.; Cheng, T.; Li, X.; Zhou, J. Slope safety factor search strategy for multiple sample points for reliability analysis. *Eng. Geol.* **2012**, *129–130*, 27–37. [CrossRef]
130. Low, B.K.; Tang, W.H. Efficient spreadsheet algorithm for first-order reliability method. *J. Eng. Mech.* **2007**, *133*, 1378–1387. [CrossRef]
131. Low, B.K.; Phoon, K.K. Reliability-based design and its complementary role to Eurocode 7 design approach. *Comput. Geotech.* **2015**, *65*, 30–44. [CrossRef]
132. Hack, R.; Price, D.; Rengers, N. A new approach to rock slope stability—A probability classification (SSPC). *Bull. Eng. Geol. Environ.* **2003**, *62*, 167–184. [CrossRef]
133. Lindsay, P.; Campbell, R.N.; Fergusson, D.A.; Gillard, G.R.; Moore, T.A. Slope stability probability classification, Waikato Coal Measures, New Zealand. *Int. J. Coal Geol.* **2001**, *45*, 127–145. [CrossRef]
134. Heidelbe, B. Assessment of rock slope stability by probabilistic Slope e Stability Probability Classification m method along highway cut slopes in Adilcevaz-Bitlis (Turkey). *J. Mt. Sci.* **2016**, *13*, 1893–1909.
135. Ersöz, T.; Topal, T. Assessment of rock slope stability with the effects of weathering and excavation by comparing deterministic methods and slope stability probability classification (SSPC). *Environ. Earth Sci.* **2018**, *77*, 1–18. [CrossRef]
136. Melchers, R.E. *Structural Reliability Analysis and Prediction*; John Wiley and Sons: New York, NY, USA, 1999.
137. Shuppener, B. Eurocode 7: Geotechnical Design-Part 1: General Rules-Its Implementation in the European Member States. In Proceedings of the 14th European Conference on Soil Mechanics and Geotechnical Engineering, Madrid, Spain, 24–27 September 2007; Volume 2, pp. 279–289.