

## COMPARATIVE ANALYSIS OF THE RESULTS OF GAS FLOW IN A LABYRINTH SEAL FOR A NUMERICAL AND THEORETICAL MODEL

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### Abstract

*Labyrinth seals are an important component of gas and steam turbines design. Many calculation models are available in order to determine the thermodynamic parameters of gas along the seal length: starting with models based on an analytical solution to the problem of gas flow through a seal to numerical models resulting in numerical data and visualizations of conditions inside the seal. In this article the results of calculations made using the Fanno model and the Saint-Venant's Principle with the  $k$ -epsilon model in Ansys Fluent are compared.*

**Keywords:** *labyrinth seals, Fanno curve, Saint-Venant's Principle, turbomachines, modelling*

### Introduction

Labyrinth seals are commonly used in turbomachines, such as generator, compressor and blower turbines. They have been designed for and integrated into turbomachines for the purpose of dampening a liquid leak between two spaces: one in which there is a high pressure of the working medium, and the other with a lower one. They are placed between the rotor shaft and the housing. The seals are used when the unit shaft is rotating with a high speed. They perform important functions in the design and operation of machines. Several problems must be highlighted, though. One of them is the issue of the labyrinth wear and the resultant reduction in the turbine efficiency. Another one is the lack of possibility of controlling the seal operation and specifying its overhaul date.

A labyrinth seal consists of connected tooth. Its tightness depends on the number of flow contractions, density of the seal teeth, height of clearance and volume of chambers between the teeth. In an ideal sealing the flow is turbulent just downstream of the seal and completely decelerated upstream of another contraction. A simplified model of a labyrinth seal is shown in Fig. 1.

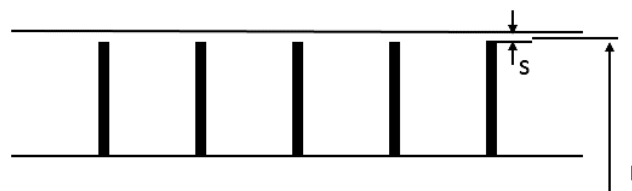


Fig. 1. Semi-full labyrinth seal

In a labyrinth seal the value of clearances between rotating and fixed elements is constant. Therefore, the surface area of those clearances should be equal for all teeth:

$$A = 2\pi r s \quad (1)$$

where:

- r – average clearance radius,
- A – clearance cross section,
- s – clearance height.

### Labyrinth seal calculation methods

Knowing the input parameters of the seal operation, i.e. gas pressure upstream of the seal, pressure downstream of the seal, number of flow contractions and the seal geometry, the thermodynamic parameters of gas along the labyrinth length can be calculated in the following manner:

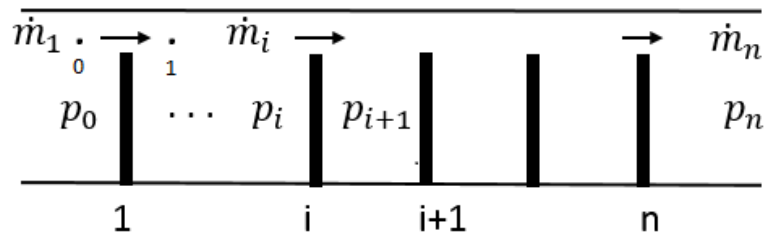


Fig. 2. Seal model used for calculations

Let us assume that the pressure drop in subsequent clearances is linear and calculate it using the following formula:

$$\Delta p = \frac{p_0 - p_n}{n} \quad (2)$$

where:

n – number of seal teeth.

The seal clearance surface area is determined in accordance with formula (1). Specific volume of gas and entropy  $s_0$  were derived from charts. Thus all the parameters enabling the specification of the 0 point location on chart h – s (Figure 3) were determined.

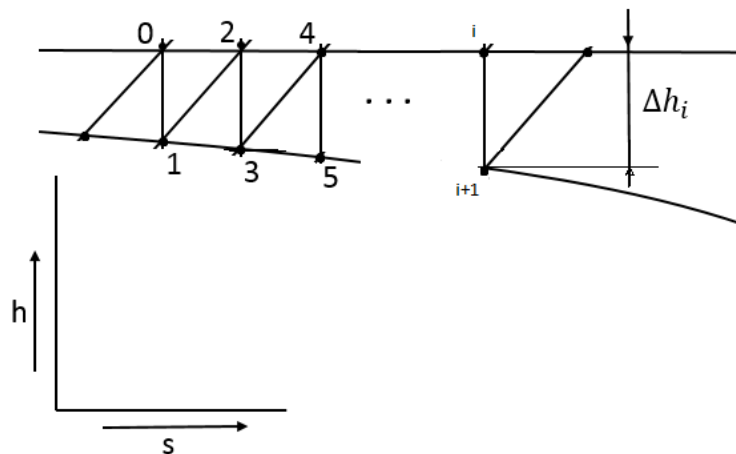


Fig. 3. Fanno curve in seal

In order to determine the change in steam parameters during an isentropic expansion process between points (0-1) the following formula should be used:

$$\frac{c_i}{v_i} = \frac{\dot{m}_i}{A_i} = K = \text{idem} \quad (3)$$

Gas velocity at the segment outlet is determined using the following formula:

$$c_i = v_i \frac{\dot{m}_i}{A_i} \quad (4)$$

Enthalpy reduction in the seal segment:

$$\Delta h_i = \frac{c_i^2}{2} \quad (5)$$

Considering operational and geometric parameters of the seal we obtain:

$$\Delta h_i = v_i^2 \frac{\dot{m}_i^2}{2A_i^2} \quad (6)$$

Considering the enthalpy reduction in the first segment of the seal in the  $h-s$  diagram we obtain the location of point 1 – the outlet from the first segment. The assumption that the conversion between points 1 – 2 is an isobaric deceleration of the stream ( $p_1 = p_2$ ) guarantees the conversion of kinetic energy into heat through deceleration, as a result of friction combined with the lack of heat exchange with the surroundings. During this conversion process, steam enthalpy increases to the value equal to the value prior to the medium entering the seal.

$$h_2 = h_1 + \frac{c_2^2}{2} = h_0 \quad (7)$$

Prior to each subsequent contraction it is assumed that the medium has a constant value of enthalpy equal to the initial enthalpy. Gas temperature is read from steam charts. In view of the fact that mass deceleration takes place during an isobaric process and considering the initially calculated pressure distribution and the presented method for performing calculations, the location of all points on the  $h-s$  diagram will be determined.

The main issue with labyrinth seals is the determination of the size of a mass stream flowing through individual teeth of the seal. Most calculation models use the Saint-Venant's equation for this purpose:

$$\dot{m}_i = A \sqrt{2g \frac{\kappa}{\kappa - 1} \left[ \left( \frac{p_{i+1}}{p_i} \right)^{\frac{2}{\kappa}} - \left( \frac{p_{i+1}}{p_i} \right)^{\frac{\kappa-1}{\kappa}} \right] \frac{p_i}{v_i}} \quad (8)$$

Using the given calculation model we obtain differences in gas mass streams in the individual clearances of the seal. It means that the pressure distribution in subsequent segments of the seal

is unclear and the pressure distribution calculations must be corrected. The method for performing those corrections is described in article [5.].

Currently, the computing power of computers enables quick Computational Fluid Dynamic calculations with a high level of accuracy. They constitute an alternative to analytical methods. Fluent software included in the Ansys package contains a calculation model based on a turbulent flow k – epsilon standard. The K-epsilon model is the most common of all turbulent models that ensures sufficient calculation accuracy. It is based on transport equations. The equation for k turbulence kinetic energy:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \epsilon - Y_M + S_k \quad (9)$$

and energy dissipation  $\epsilon$ :

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_i}(\rho \epsilon u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] + C_{1\epsilon} \frac{\epsilon}{k} (G_k + C_{3\epsilon} G_b) - C_{2\epsilon} \rho \frac{\epsilon^2}{k} + S_\epsilon \quad (10)$$

Where:

$\mu_t$ - turbulence viscosity,

$S$  – strain tensor.

$G_k$  represents the velocity gradient included in the turbulence kinetic energy, while  $Y_M$  represents fluctuations resulting from turbulence changes.  $S_\epsilon$  and  $S_k$  are initial conditions.  $C_{1\epsilon}$ ,  $C_{2\epsilon}$  and  $C_\mu$  are experimentally determined constants for the K – Epsilon model and they amount to:  $C_{1\epsilon} = 1,44$ ,  $C_{2\epsilon} = 1.92$  and  $C_\mu = 0,09$ , respectively.

$\sigma_k$ ,  $\sigma_\epsilon$  are experimentally determined Prandtl turbulence numbers for the turbulence kinetic energy and they amount to  $\sigma_k = 1,0$ ,  $\sigma_\epsilon = 1,3$ , respectively.

## Simulations

The model assumed for the calculations involves the seal length of 200 mm. There are 20 flow contractions (teeth) along the seal length, Figure 4. The seal pitch equals  $c=10$  mm, and the distance between teeth amounts to  $b=9$  mm. The tooth thickness is  $d=1$  mm. It is assumed that the average clearance of the seal amounts to 0,5 mm. The following input parameters of the seal operation were assumed: temperature  $T_0$  and pressure  $p_0$  of gas upstream of the seal inlet equal 308 K and 4,5 bar respectively, while the assumed gas pressure at the seal outlet equals 3 bars. Analytical calculations assume maximum gas velocity at the moment the gas flows through a clearance. In a chamber between clearances gas is completely decelerated as a result of the isobaric expansion process. Computer simulations were performed in accordance with the k-epsilon turbulent model.

Calculations were performed for the following parameters:

Number of seal teeth	n	20	
Seal length	L	200	mm
Tooth height	h	9,5	mm
Seal pitch	c	10	mm
Chamber width	B	9	mm
Tooth thickness	d	1,0	mm
Nominal seal clearance	S	0,5	mm

Shaft diameter	H	150	mm
Gas pressure upstream of seal	$p_0$	4,5	bar
Gas temperature upstream of seal	$T_0$	308	K
Gas pressure downstream of seal	$p_k$	3,0	bar

The seal geometry is presented in Figure 4.

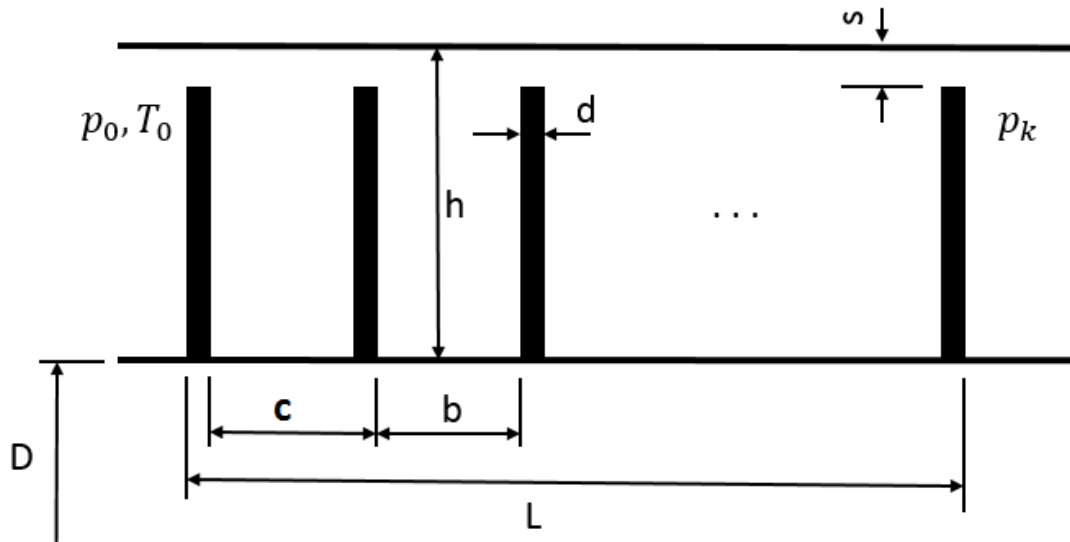


Fig. 4. Geometry of labyrinth seal used for calculations. In the figure:  $b$  – chamber width,  $D$  – shaft diameter,  $d$  – tooth thickness,  $h$  – tooth height,  $L$  – seal length,  $p$  – seal pitch,  $p_0$  – gas pressure upstream of seal,  $p_k$  – gas pressure downstream of seal,  $s$  – nominal seal clearance,  $T_0$  – gas temperature downstream of seal

A calculation mesh presented in Figure 5 was used for the calculations. The mesh contains 1 284 700 elements. At crucial points, i.e. seal clearances, the mesh is more dense.

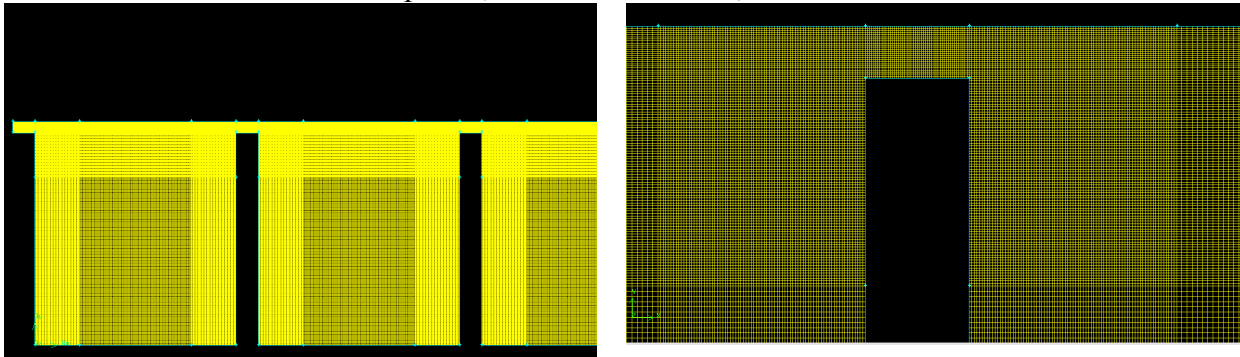


Fig. 5. Calculation mesh for labyrinth seal

Figure 6 shows a diagram representing pressure distribution along the seal length. The results obtained through numerical calculations using CFD modelling are highlighted in blue, while the results obtained through an analytical model are highlighted in orange. The analytical model was described in item [4]. There is a slight difference in the obtained results caused by the fact that in the analytical model we obtain averaged pressure values. In the case of CFD calculations we obtain a calculated pressure values profile in the mesh nodes along the entire width of a clearance. The pressure profile is presented in Figure 6. The diagram shows average values for the results obtained with the CFD method.

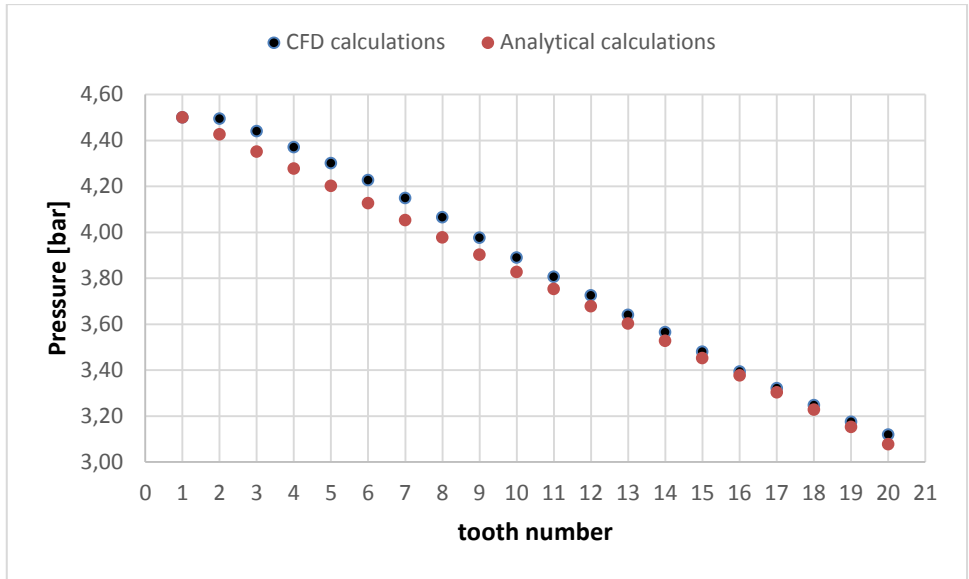


Fig. 6. Comparison of distribution of pressures obtained using analytical and numerical K-epsilon models

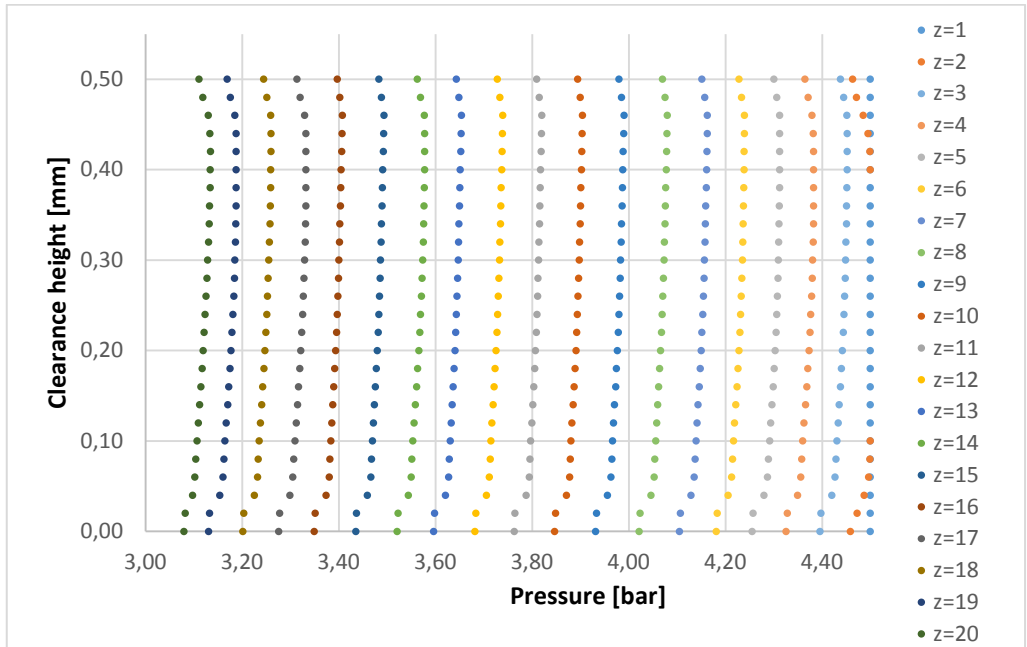


Fig. 7. Diagram of pressure profiles in individual tooth of seal

Figure 7 shows pressure distribution in individual seal teeth. As expected, pressure values in the profile reach their maximum values within approximately 0,05 mm from the frame. These profiles have a similar shape in all clearances of the seal.

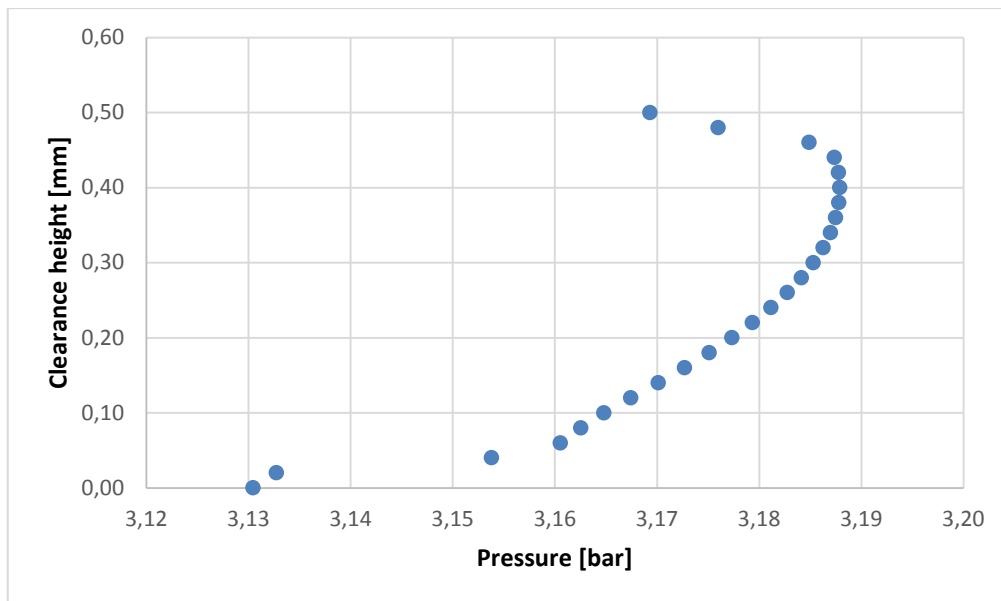


Fig. 8. Gas pressure profile for *i*-th tooth

Figure 8 shows a pressure profile in the cross section of the seal *i*-th clearance. The pressure value in the clearance is increasing while moving in the direction of the frame. It means that friction between the seal and gas reaches its maximum values by the walls, having maximum values near the tooth.

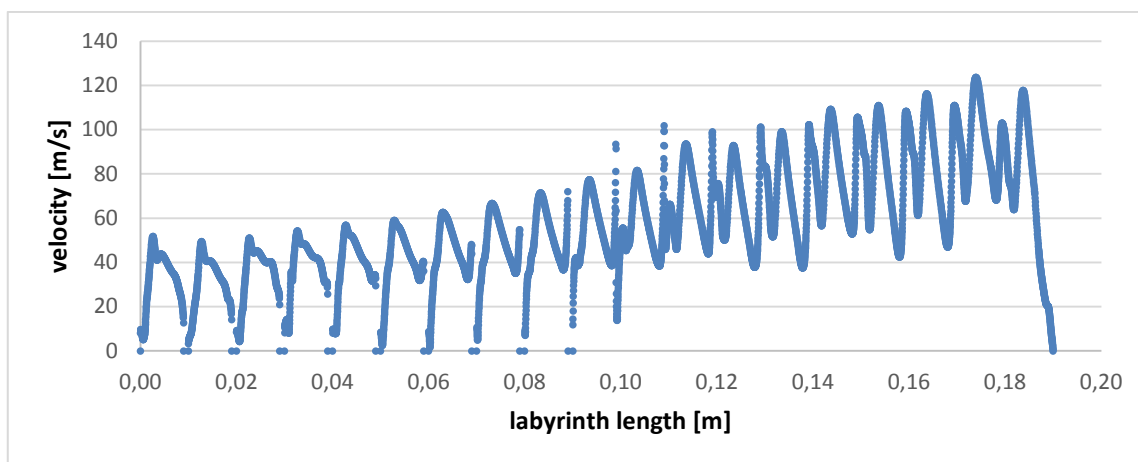


Fig. 9. Diagram of gas velocity dependence in the function of seal length

Figure 9 shows a diagram of gas velocity versus the seal length. Cyclic repetition of locally maximum velocity peaks may be observed. It results from the fact that the medium velocity is maximum when flowing through a clearance and gas is decelerated in the chamber between the seal teeth.



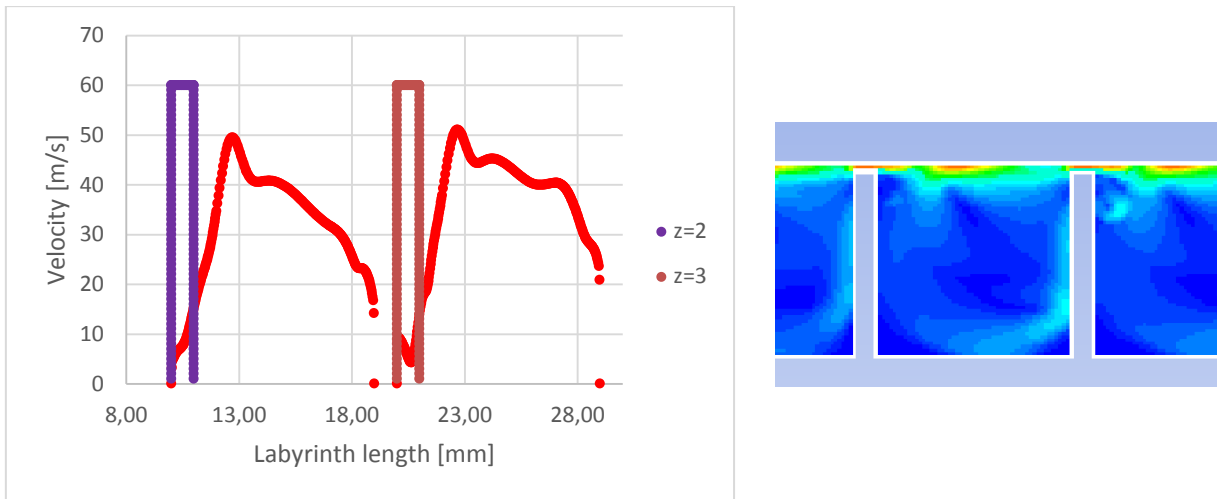


Fig. 10. Diagram of gas velocity dependence for the second and third tooth

The velocity diagram for the second and third tooth of the seal is given in Figure 10. The velocity profile in the mid of the seal clearance is highlighted in red, while the tooth thickness is highlighted in orange and purple. It can be observed that maximum velocities are reached by gas just downstream of the tooth. Local increments in gas velocity downstream of the clearance may also be observed – Figure 10.

The advantage of the CFD calculations over analytical calculations is the possibility of visualization. Figures 11 and 12 show pressure distribution and the velocity field along the length of the labyrinth seal. Such simulations make it possible to present the solution to the problem thanks to a diversified colour map which facilitates the understanding of the challenges faced. It is possible to select calculation mesh nodes in such a manner that more results are obtained in areas requiring special attention. Analytical calculations do not provide for this, as they provide average values.

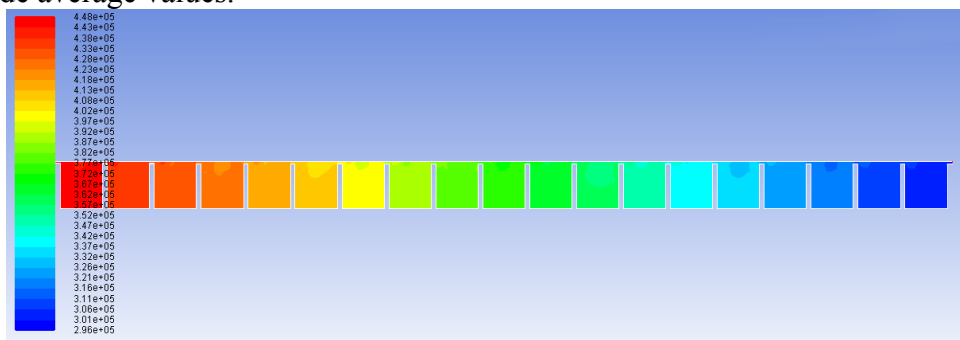


Fig. 11. Pressure distribution in labyrinth seal

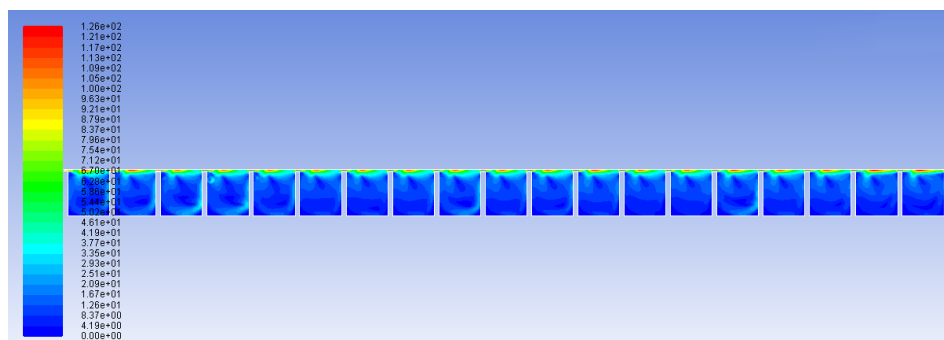


Fig. 12. Gas velocity field in labyrinth seal



## Summary

There are many calculation models available for labyrinth seals. All of them provide similar results. Theoretical models provide only the most important information, while CFD calculations also enable the visualization of the obtained results. They provide information on the gas thermodynamic parameters at each point of the labyrinth seal. Because of the computers computing power increase it is possible to model the operation of more and more complex systems.

The results obtained using both calculation models are similar. The k-epsilon model provides more information than the analytical model. It means that depending on the required precision of results we may use both models interchangeably. It also proves the correctness of both models.

## References

- [1.] Chmielniak T.; Turbiny ciepłe: podstawy teoretyczne. Gliwice Wydawnictwo Politechniki Śląskiej 1998
- [2.] Dursen Eser, Jacob Y. Kazakia; Air Flow In Cavities of Labyrinth Seals, International Journal of Engineering Science, Vol 33, No. 15, pp. 2309 – 2326, 1995
- [3.] Joachimiak D.; Badanie uszczelnień labiryntowych z upustem; Ph. D. dissertation, Poznań University of Technology, Faculty of Machines and Transportation, Institute of Thermal Engineering Poznań 2013
- [4.] Kaszowski P.; Analiza pracy uszczelnień labiryntowych z upustem; Master's dissertation, Gdańsk University of Technology, Institute of Power Engineering and Industrial Instrumentation, Gdańsk 2011
- [5.] Kaszowski P., Dzida M., Krzyślak P.; Calculation of labyrinth seals with and without diagnostic extraction in fluid flow machines, Polish Maritime Research, No 4(80), 2013, Vol 20, pp. 34 - 38
- [6.] Kim Namhyo, Rhode David; Refined Turbulence Modeling for Swirl Velocity in Turbomachinery Seals, International Journal of Rotating Machinery, 9: 451 – 459, 2003
- [7.] Krzyślak P.; A method of diagnosing labyrinth seals in fluid-flow machines. Polish Maritime Research 3(57) 2008 Vol 15; pp. 38-41 10.2478/v10012-007-0081-2
- [8.] Perycz S.; Turbiny parowe i gazowe. Gdańsk Wydawnictwo Politechniki Gdańskiej 1988
- [9.] Piwowarski M., Kosowski K.; Uszczelnienia turbin cieplnych, Fundacja Promocji Przemysłu Okrętowego i Gospodarki Morskiej, Gdańsk 2009
- [10.] Schramm V., Denecke J., Kim S., Wittig S.; Shape Optimization of a Labyrinth Seal Applying the Simulated Annealing Method, International Journal of Rotating Machinery 10(5): 365 – 371, 2004
- [11.] Trüttnovsky K.; Berührungsfreie Dichtungen, Grundlagen und Anwendungen der Strömung durch Spalte und Labirynth. VDI – VERLAG bh DÜSSELDORF; Verlag des Vereins Deutscher Ingenieure 1964
- [12.] WANG Wei-zhe, LIU Ying-zheng, JIANG Pu-ning, CHEN Han-ping; Numerical Analysis of Leakage Flow Through Two Labyrinth Seals, Journal of Hydrodynamics, Ser.B, 2007, 19(1);107-112

