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Digital Processing of Frequency–Pulse Signal in Measurement System

D. Świsulski, E. Pawłowski and M. Dorozhovets

Abstract The work presents the issue of the use of multichannel measurement systems of sensors processing input value to impulse signal frequency. The frequency impulse signal obtained from such sensors is often required to be processed at the same time with a voltage signal which is obtained from other sensors used in the same measurement system. In such case, it is usually necessary to sample the output signals from all sensors in the same, predetermined points in time. Sampling voltage signal by means of A/D converters is practically possible in any selected time points, and the sampling frequency–pulse signal FPS requires special algorithms. The authors present the algorithms of digital signal processing pulse frequency offline and online modes, providing the acquisition of samples at certain evenly distributed points over time.

Keywords Multi-channel measurements system · Converter with frequency output · Frequency-pulse signal · Analog-to-digital conversion · Instantaneous frequency · Signal reconstruction

1 Introduction

Modern measurement systems use various digital signal processing algorithms. For this purpose, all the measured values of the input system must be converted to their digital representations by suitable sensors and A/D converters, for which the most commonly used is an indirect voltage signal. This is due to the fact that now commonly available are integrated A/D converters processing only analogue voltage signal.

Often used instead of the voltage signal is the frequency signal [1, 2], which is easily and accurately processed into the digital form by means of metering systems and has a high resistance to interference, which greatly facilitates the transmission over long distances. The frequency signal can be obtained using integrated voltage-to-frequency converters VFC [3] or using sensors with frequency output [4, 5]. In multichannel measurement systems, both types of signals are often used simultaneously: voltage and frequency [6]. The structure of the measurement system under consideration is shown in Fig. 1.

During the acquisition of analogue signals $x(t)$, defined at any time in the time interval of observation, they are processed on the next channel on the N strings of numbers $\{D_k[0], D_k[1], D_k[2], \ldots, D_k[N-1]\}$. They represent, respectively, the instantaneous values of analogue signals at regular intervals and are placed in the memory M of the measurement system. To maintain proper timing relationship between all the input quantities, analogue-to-digital processing in each channel should be carried out at the same time instants, typically evenly spaced in time, as required by DSP algorithms currently used (e.g. FFT, Hilbert transformat, etc.).

If the signal is an intermediate voltage $u(t)$ produced by sensor type x/u, then the processing to digital form is implemented as standard in the A/D converter, and the

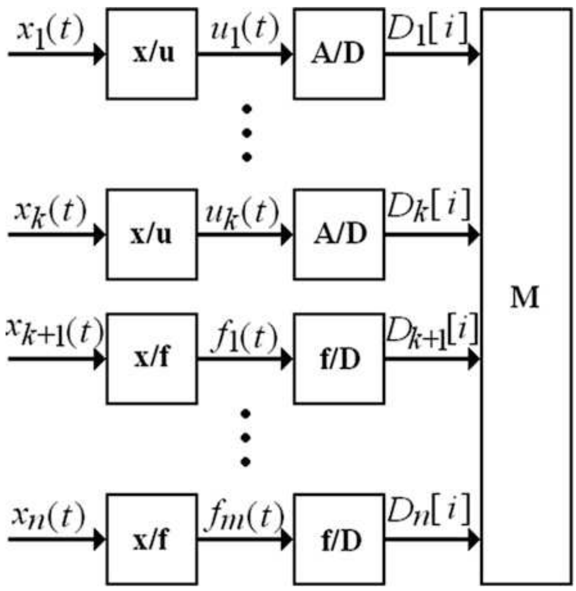


Fig. 1 Multichannel measuring system with tracks that use voltage signals u_1, … u_k and frequency signal f_1, … f_m

moment sampling can be chosen almost arbitrarily. A significant problem occurs when the intermediate signal is a variable in time frequency $f(t)$, since the pulses of the output frequency signal of the sensor type x/f are generated with a different time interval. This time depends on the mean value of the input $x(t)$ at the time of the previous pulse [7]. Therefore, to obtain the frequency signal, samples uniformly distributed at certain time instants require special methods other than voltage circuit.

2 Frequency–Pulse Signal

The sensor frequency output is a converter of the instant value of the measured quantity $x(t)$ to the instantaneous value of the frequency $f(t)$ with a coefficient of proportionality known as sensitivity of the sensor S:

$$f(t) = Sx(t). \tag{1}$$

The frequency f is a parameter of periodic signal $y(t)$ present at the output of the sensor. In practice, it is usually a voltage signal, which may be of any shape: sinusoidal, triangular, rectangular, etc. [7]. In such case, the instantaneous values of the signal $y(t)$ do not reflect the actual values of the measured value $x(t)$, but they are a particular form of a periodic-input function $F(\varphi)$, describing the shape of the signal $y(t)$. The argument of the function $F(\varphi)$ is a variable during the phase $\varphi(t)$ of the signal $y(t)$:

$$y(t) = F(\varphi(t)). \tag{2}$$

If the frequency of the output of the sensor is constant f = const, then the phase $\varphi(t)$ of this signal is a linear function of time:

$$\varphi(t) = 2\,\pi ft + \theta, \tag{3}$$

where Θ is the initial phase. Otherwise, when the frequency $f(t)$ of the signal changes, the phase signal $\varphi(t)$ is described by an integral relationship:

$$\varphi(t) = 2\pi \int_0^t f(t)\mathrm{d}t + \theta. \tag{4}$$

Regardless of the actual shape of the signal $y(t)$ described by Eq. (2), in measuring systems, processing frequency to form a numerical includes only the selected characteristic of the states of this signal, usually timely rising or falling slope, which is marked by periods of the signal. Therefore, in practice, the signal $y(t)$ can be considered as a series of pulses Dirac δ shown in Fig. 2, appearing at moments t_i separated from each other by intervals T_i, corresponding to the following equal increments of the signal phase $\Delta\varphi$ equal to the period of 2π:

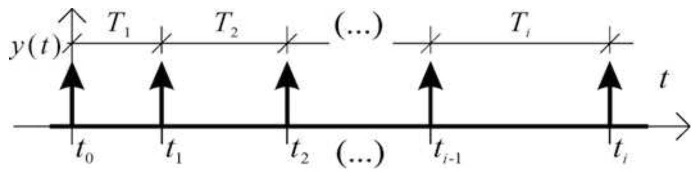


Fig. 2 The idealised frequency–pulse signal

$$y(t) = \sum_{i=-\infty}^{+\infty} \delta(t - t_i), \quad t_i - t_{i-1} = T_i. \tag{5}$$

The signal $y(t)$ shown in Fig. 2, described by the relation (5), whose instantaneous frequency $f(t)$ is proportional to the value of the processed $x(t)$ according to Eq. (1), will be called as the frequency–pulse signal FPS. The fundamental problem is to convert instantaneous frequency $f(t)$ of the signal FPS $y(t)$ (5) to its digital representation of $D[i]$ (Fig. 1). In measurement technique, having respectively taken into account the transformed Eq. (4), the instantaneous frequency $f(t)$ of the signal is determined by the derivative of the phase φ of this signal with respect to time t [8]:

$$f(t) = \frac{1}{2\pi} \frac{\mathrm{d}\varphi}{\mathrm{d}t}. \tag{6}$$

Phase $\varphi(t)$ for a frequency–pulse signal FPS (5) is a continuous function of time, but the function F describing the shape of the signal $y(t)$ (2) is not a one to one: different values of the phase $\varphi(t)$ correspond to the same signal values $y(t)$. In the present signal, FPS (5) information is not accessible about the increase of the phase φ of the signal between the successive pulses. For such a signal, one cannot determine arbitrarily small increments of phase φ, and so one cannot also determine the frequency f on the basis of the derivative (6) for any time t. It is also clear that for the frequency–pulse signal FPS (5) we can determine only the increases of phase equal to a multiple of the period 2π. So in order to determine the frequency f of the pulse signal (5) the derivative (6) should be replaced by the quotient of the growth of phase $\Delta\varphi$ and time gain Δt [9]:

$$f(t) = \frac{1}{2\pi} \frac{\Delta\varphi}{\Delta t}. \tag{7}$$

Pulse at time t_i (rys. 2) is the incremental phase of the signal FPS (5) by an angle $\Delta\varphi = 2\pi$ relative to the pulse at time t_{i-1}, in block f/D implementing sampling pulse frequency signal $f(t)$ (Fig. 1); successive time intervals T_i are measured digitally via filling them with impulses T_{ref} of reference frequency f_{ref}, which allows to determine the next sampling frequency f_i:

$$f_i = \frac{1}{2\pi}\frac{\Delta\varphi}{\Delta t} = \frac{1}{2\pi}\frac{2\pi}{t_i - t_{i-1}} = \frac{1}{T_i} = \frac{1}{K_i T_{\mathrm{ref}}} = \frac{f_{\mathrm{ref}}}{K_i}, \qquad (8)$$

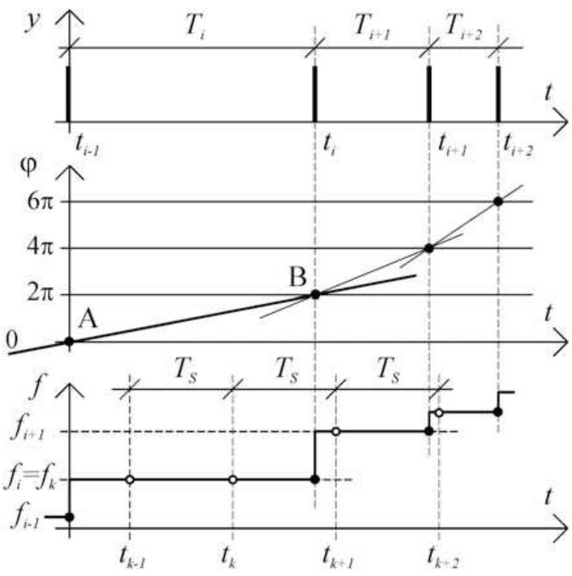
where K_i is the number of impulses of reference frequency f_{ref}, which have been counted in time T_i. The resultant quantization error can be analysed by simulation methods [9]. If the input value $x(t)$ varies with time, the more frequencies f_i are obtained at times t_i and unevenly distributed over time (Fig. 2). Besides, the frequency value f_i is not an instantaneous value, being an average value for the time T_i, and therefore does not assign it to the time t_i, but properly chosen time t_i^* being within the interval T_i [9]. For an unknown form of $x(t)$ it is not known where the moments of time t_i^* can be reasonably attributed to the value of f_i. Since the location of points (t_i^*, f_i) is not known, it is impossible unfortunately to approximate the value of $f(t)$ for any time t. The frequency–pulse signal FPS has, however, the additional advantageous property: all the pulses occur exactly at those moments t_i, in which the phase angle of the signal growth $\Delta\varphi = 2\pi$ is equal to the period of the signal $y(t)$. As a result, points $(t_i, 2\pi i)$ allow clearly to approximate the course of the instantaneous phase $\varphi(t)$ of the signal (5), and after calculation of the derivative (6) also the course of the instantaneous frequency $f(t)$.

The methods of processing a frequency–pulse signal FPS can be divided into two groups, depending on the position of the time for which the measurement result is determined. When measuring in offline mode, first time t_i of all pulses are remembered and then one determines the value measured in moments of measurement adopted by the position of the pulse of both preceding and occurring after the moment of the measurement. In online test, the measured value is determined only on the basis of the position of the pulse preceding the moment of measurement.

3 Offline Processing

While converting the frequency–pulse signal FPS in offline mode in M system memory (Fig. 1), the location of all subsequent pulses at times t_i, distant in time o $T_{i,}$ has been remembered, which allows approximating the course of instantaneous values of the phase $\varphi(t)$ of the signal and after differentiation (6), reconstituting the instantaneous frequency $f(t)$ [10]. For extremely low-frequency signals, it is often sufficient to assume that the frequency $f(t)$ is constant in successive time intervals T_i; according to the formula (3), the phase of the signal $\varphi(t)$ can be interpolated in this range with a straight line. The procedure is shown in Fig. 3. Two points A and B define a straight line $\varphi(t) = a_0 + a_1 t$ approximating phase of the signal in the time interval of (t_{i-1}, t_i), in which the signal phase growth occurred $\Delta\varphi = 2\pi$. After simple transformations, we calculate $a_0 = 0$ and $a_1 = 2\pi/T_i$, and taking into account (6) after setting the derivative we get $f(t) = f_i = 1/T_i$ for $t \in (t_{i-1}, t_i)$. Following an analogous procedure to the next interval T_i we get a line approximating the course

Fig. 3 The phase linear approximation of the frequency–pulse signal



of $f(t)$ in accordance with accepted at the beginning of assumptions, which will determine the frequency f_k at times $t_k = kT_S$ evenly distributed dug the period T_S of evenly sampling:

$$f_k = f(t_k) = f(kT_S) = 1/T_i, \quad t_k \in (t_{i-1}, t_i). \tag{9}$$

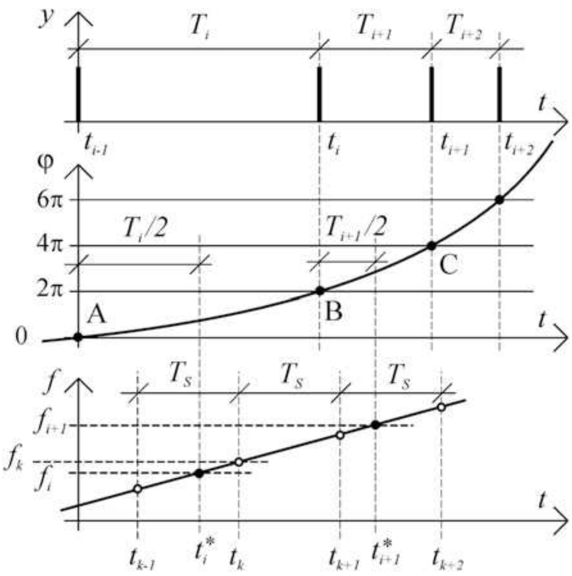
For signals changing faster, one can assume a linear change in frequency as a function of $f(t)$, and then one should approximate the course of the signal phase with a polynomial of the second degree $\varphi(t) = a_0 + a_1t + a_2t^2$. This requires determination of the coordinates of three points A, B and C (Fig. 4) corresponding to the phase increments $\varphi(t)$ of the signal by further multiple 2π at times t_{i-1}, t_i, t_{i+1}. The three moments of time determine the position of three successive signal pulses $y(t)$, away from each other by times T_i and T_{i+1}.

Points (0, 0), (T_i, 2π) and ($T_i + T_{i-1}$, 4π) enable the arrangement of a system of three equations with three unknowns a_0, a_1 and a_2. After solving this system of equations one receives dependency of the instantaneous phase value $\varphi(t)$, and after differentiation (6) we obtain the polynomial interpolating instantaneous frequency $f(t)$ (1) of the signal $y(t)$ in the time interval of (t_{i-1}, t_{i+1}), based on the two neighbouring inter-pulse times T_i, T_{i+1}:

$$f(t) = \frac{1}{T_i + T_{i+1}}\left[\frac{T_{i+1}}{T_i} - \frac{T_i}{T_{i+1}} + 2 + 2\left(\frac{1}{T_{i+1}} - \frac{1}{T_i}\right)t\right]. \tag{10}$$

Substituting Eq. (10) $f(t_i^* = 1/T_i)$ one can show that for the linear frequency change its mean value is equal to the instantaneous value of exactly half of the time

Fig. 4 The second degree phase approximation of the frequency–pulse signal



interval T_i. Therefore, it is reasonable to interpolate in the time interval from the time $t_i^* = t_{i-1} + T_i/2$ to time $t_{i+1}^* = t_i + T_{i+1}/2$, for which the instantaneous frequency takes appropriate values $f_i = 1/T_i$ and $f_{i+1} = 1/T_{i+1}$. Following points t_i^*, f_i:

$$\begin{aligned} t_i^* &= t_{i-1} + T_i/2 = \sum_{j=1}^{i-1} T_j + T_i/2 \\ f_i &= 1/T_i \end{aligned} \tag{11}$$

mark the following sections of a broken line, which allows the download of frequency f_k at times $t_k = kT_S$ equally distant in even time sampling period T_S:

$$\begin{aligned} f_k &= f(t_k) = \frac{(t_k - t_i^*)f_{i+1} + (t_{i+1}^* - t_k)f_i}{t_{i+1}^* - t_i^*} \\ t_k &= kT_S, \quad t_k \in (t_i^*, t_{i+1}^*). \end{aligned} \tag{12}$$

In justified cases, one can also approximate the phase $\varphi(t)$ of the pulse signal FPS with a higher order polynomial.

4 Online Processing

During online mode, to determine the value of the measured value at and time t_k on the basis of the pulse signal FPS, one can use only the position of the use preceding the moment t_k, which requires the use of extrapolation instead of interpolation used in offline mode. The easiest way of processing the pulse frequency signal lies in the fact that the value of the signal $f_k = f_i$ at any time t_k is calculated (8) from the length of the last inter-pulse range T_i preceding time t_k (Fig. 5).

If the value of the processed signal $x(t)$ changes during the measurement, it also changes the interval T_i between pulses FPS. Thus, for a longer period T_i of pulse signal FPS, the longer is the time between the time $t_i^* = t_{i-1} + T_i/2$ which is assigned to the frequency $f_i = 1/T_i$, and time t_k for which one should extrapolate the values of the signal f_k. This means that the value obtained by extrapolating f_k may differ significantly from that which was actually sampled at the time t_k. In such case, the value of f_k at any time t_k can be calculated from the formula (13) using extrapolation of two adjacent inter-pulse ranges T_{i-1}, T_i preceding the time t_k, assuming a linear variation in frequency and given that the frequency f_i resulting from the measurement interval T_i is equal to the instantaneous frequency at the middle point of the interval t_i^* (11) (Fig. 6):

$$\begin{aligned} f_k &= f_{i-1} + \frac{(f_i - f_{i-1})\left(t_k - \left(\sum_{j=1}^{i-2} T_j + \frac{1}{2}T_{i-1}\right)\right)}{\left(\sum_{j=1}^{i-1} T_j + \frac{1}{2}T_i\right) - \left(\sum_{j=1}^{i-2} T_j + \frac{1}{2}T_{i-1}\right)} \\ &= \frac{1}{T_{i-1}} + \frac{2\left(\frac{1}{T_i} - \frac{1}{T_{i-1}}\right)\left(t_k - \sum_{j=1}^{i-2} T_j - \frac{1}{2}T_{i-1}\right)}{T_{i-1} + T_i}. \end{aligned} \tag{13}$$

The frequency determined on the basis of the last two periods may differ materially from its current value. Figure 7 shows the error distribution of the

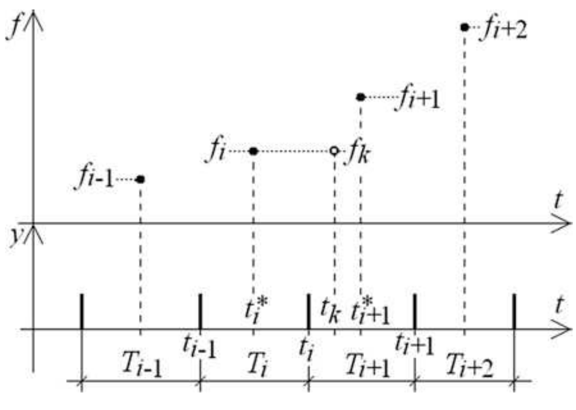


Fig. 5 Online frequency determination in interval between the last inter-pulse range

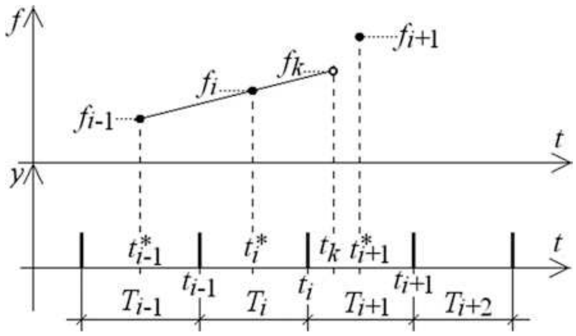


Fig. 6 Online frequency extrapolation of the last two inter-pulse ranges T_{i-1} and T_i

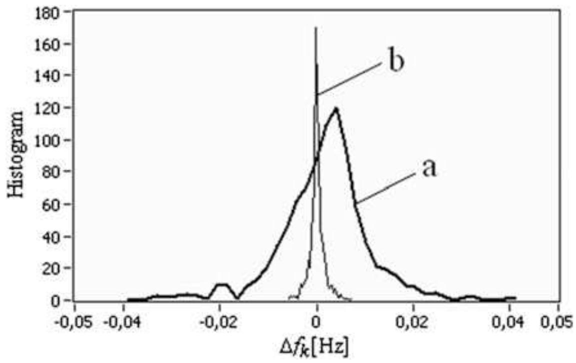


Fig. 7 Histograms of errors Δf_k for f_0 = 20 Hz, a = 2 Hz/s, s_{max} = 0.001 Hz and T_S = 100 ms; **a** frequency determination from last period, **b** frequency determination from two last periods

measured frequency, assuming a linear change of frequency $f = f_0 + a{\cdot}t + s$ for f_0 = 20 Hz, a = 2 Hz/s, s_{max} = 0.001 Hz and the even sampling period T_S = 100 ms, where s is the noise (s simulates the noise of the measured signal, but also processing errors). Error values were calculated as the difference between the frequency f_k' obtained from the measurement and value f_k accepted as true at the time $t_k : \Delta f_k = f_k' - f_k$.

If the noise level will be greater, errors measured using an extrapolation based on the last two periods may be even greater than a single measurement period. As an example, Fig. 8 shows the distributions of errors obtained for the same parameters as in Fig. 7, but with a noise value of s_{max} = 0.1 Hz.

A similar analysis as for the linear frequency changes can be made to change the sine. Assuming that during measurement of the measured quantity changes as a function of time in a sinusoidal manner with frequency f_s, with the superimposed noise s:

$$f(t) = f_0 + f_m \cdot \sin(2\pi f_s t) + s, \qquad (14)$$

Fig. 8 Histograms of errors Δf_k for f_0 = 20 Hz, a = 2 Hz/s, s_{max} = 0.1 Hz and T_S = 100 ms; **a** frequency determination from last period, **b** frequency determination from two last periods

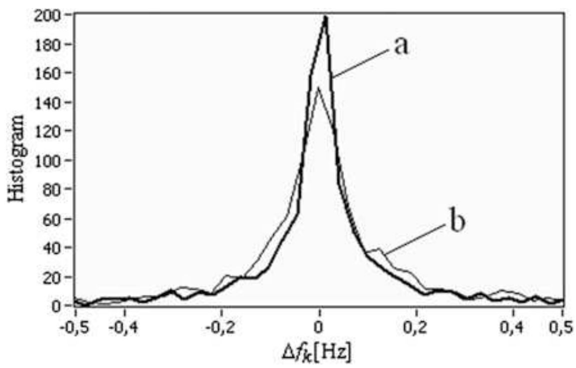
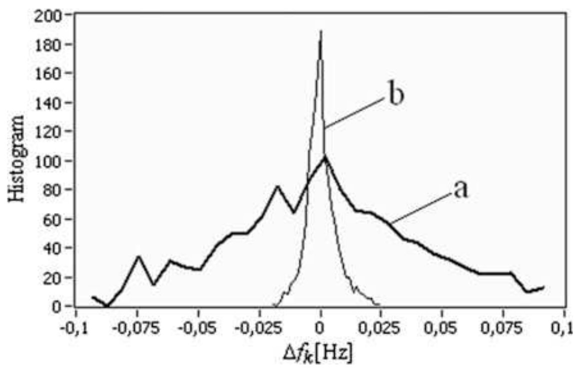


Fig. 9 Histograms of errors Δf_k distribution for f_0 = 20 Hz, f_m = 2 Hz, f_s = 0.3 Hz, s_{max} = 0.001 Hz and T_S = 20 ms; **a** frequency determination from last period, **b** frequency determination from two last periods



where

f_0 the constant pulse signal frequency $f(t)$ and
f_m amplitude changes in the frequency of the pulse signal $f(t)$.

Figure 9 shows a distribution of errors with the sinusoidal pulse signal frequency change for f_0 = 20 Hz, f_m = 2 Hz, f_s = 0.3 Hz, s_{max} = 0.001 Hz and an even sampling period of T_S = 20 ms.

Figure 10 shows the error distribution obtained for the same parameters as in Fig. 9, but for s_{max} = 0.1 Hz.

The frequency f_k determined based on the last two ranges T_{i-1}, T_i (13) may also differ significantly from its current level if it does not vary in a manner similar to a linear, but in a random manner. In such case, a more favourable method may be the one in which another earlier interval T_{i-2} is used. The additional frequency f_{i-2} is used to determine the difference (see Figs. 11 and 12):

$$\Delta f_{i-2} = \left| f_{i-2} - f'_{i-2} \right|, \tag{15}$$

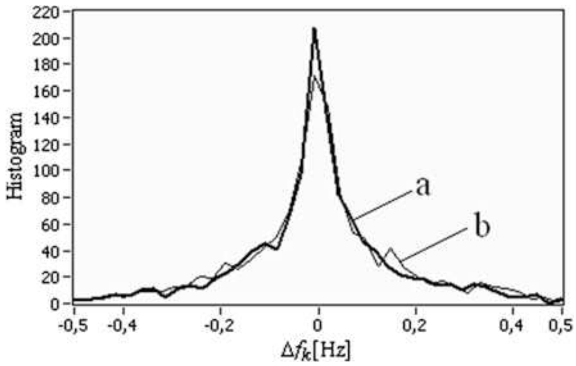


Fig. 10 Histograms of errors Δf_k distribution for $f_0 = 20$ Hz, $f_m = 2$ Hz, $f_s = 0.3$ Hz, $s_{\max} = 0.1$ Hz and $T_S = 20$ ms; **a** frequency determination from last period, **b** frequency determination from two last periods

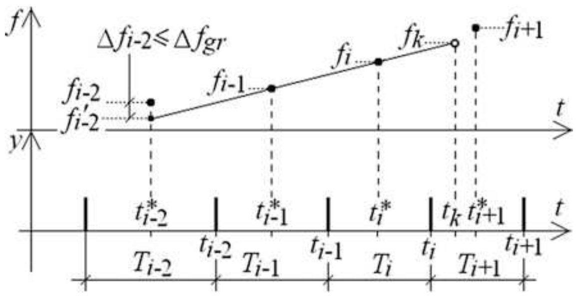


Fig. 11 Online frequency determination of the last three ranges for $\Delta f_{i-2} \leq \Delta f_{\mathrm{gr}}$

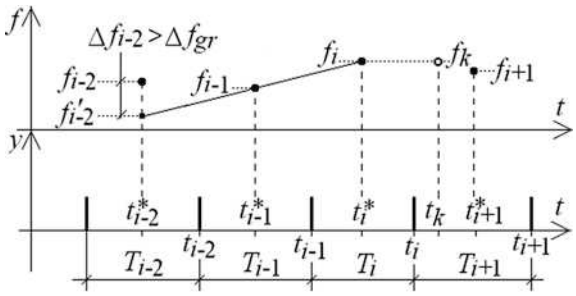


Fig. 12 Online frequency determination of the last three ranges for $\Delta f_{i-2} > \Delta \mathrm{f}_{\mathrm{gr}}$

where

f_{i-2} frequency value determined from the interval T_{i-2},
f'_{i-2} frequency value in the middle of the range of T_{i-2} determined by extrapolation of the ranges T_{i-1} and T_i.

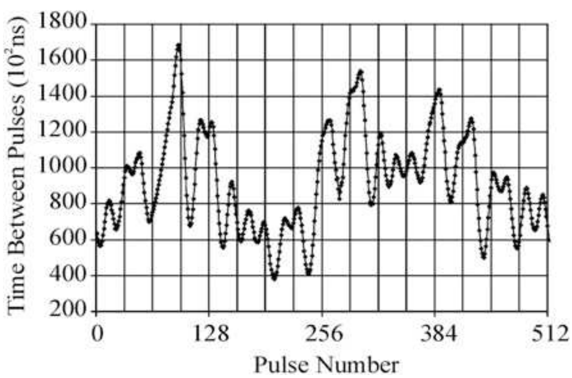
Obtained from the formula (15) value Δf_{i-2} is compared with the established experimentally limit value Δf_{gr}. For $\Delta f_{i-2} \leq \Delta f_{\mathrm{gr}}$ as a result of the measurement is taken the frequency f_k determined by extrapolation of the intervals T_{i-1} and T_i (Fig. 11). If $\Delta f_{i-2} > \Delta f_{gr}$ as a result of the measurement, the frequency f_i value is assumed to be determined from the interval T_i (Fig. 12).

For the conditions shown in Figs. 7, 8, 9 and 10 measurement simulations were repeated. For the proposed method, the error distributions were obtained corresponding to the distribution b in Figs. 7 and 9 and a distribution similar to that in Figs. 8 and 10. This confirms the validity of the adopted considerations. For the measured signal with a small change dynamics and low noise, frequency is determined by extrapolating on the basis of the last two periods. With high content of noise frequency, value is determined from the last period. The simulations performed adopted experimentally determined limit value of $\Delta f_{gr}/f_{i-2} = 0.2\%$. It has been obtained by determining the error values as a function of limit value.

5 Practical Application

For the practical verification of the algorithms, combined were the single-phase electric motor of low power with incremental rotary pulse encoder generating 512 pulses per revolution of the shaft. For measuring and recording the times T_i a measuring card was used (CTM-PER, company KEITHLEY) co-working with an IBM PC class computer. The measuring card has a 28-bit counter that counts the reference signal of 10 MHz, the FIFO memory provides meter reading “on the fly”, and for measuring the next time T_i in the range from 0.1 ms to 26.8 s and a transfer of results into the computer memory. Figure 13 shows the value of 512 times T_i recorded in one revolution of the motor shaft, which is then converted to the rotation speed and assigned in accordance with (11) times t_i^* lying in the middle of T_i giving a sample distributed unevenly over time. For greater clarity, Fig. 14 only shows the position of the selected part of pulses for i = 234, … 268, covering part of the graph in Fig. 13 for the times t_i = 19.9, … 22.9 ms. The total measurement time was 47.3 ms (one full rotation of the motor shaft). Uniform sampling period T_S = 92.6 ms (sampling rate approx. 10,800 samples/s) was adopted in such a way, so as to obtain the same number of 512 samples. Using the algorithm for offline (12) 512 signal values were calculated at times $t_k = kT_s$ evenly distributed over time

Fig. 13 Intervals T_i for one revolution of the motor shaft



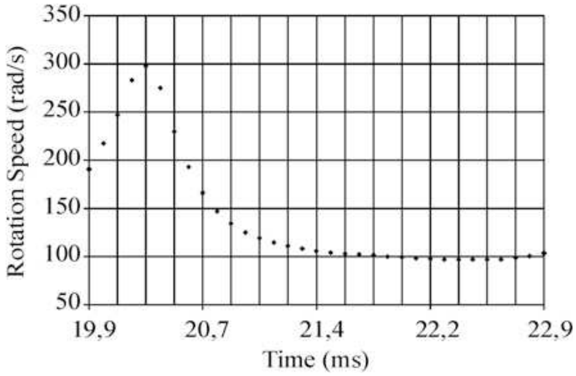


Fig. 14 Rotation speed unevenly sampled converted from a pulse scale into a timescale (pulse from 234 to 268 in Fig. 13)

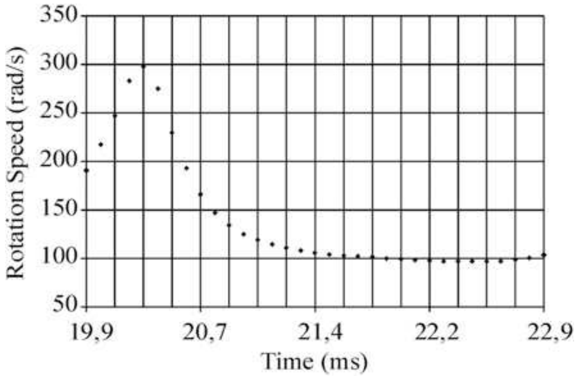


Fig. 15 Rotation speed evenly sampled converted from Fig. 14 using offline *mode* method (12)

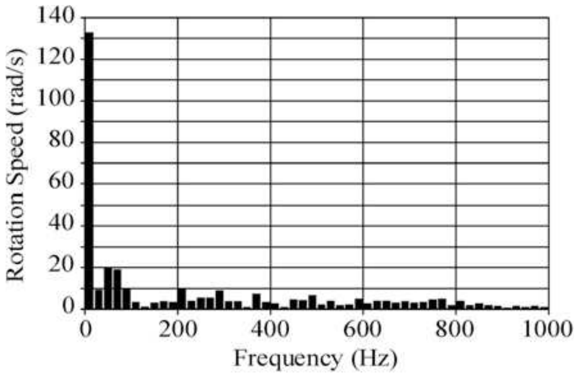


Fig. 16 Frequency spectrum speed calculated for one rotation of the motor shaft after the application of a uniform sampling FPS

(Fig. 15), and then set a frequency spectrum signal speed of the motor shaft (Fig. 16) with a visible component of a constant value of 133 rad/s and a 50 Hz component of the value of 20 rad/s.

6 Conclusions

The presented review of the methods of acquisition of the pulse signal at specific sampling instants shows that the choice of method depends on the measurement mode (online or offline) and on the nature of the change of the measured value. The simplest solution is to determine the value measured on the basis of a single inter-pulse interval, more complex methods using two or more time intervals. The presented methods allow to obtain information on the measured value at the same time for the voltage and frequency of the channel, and the samples obtained from the frequency channels can be processed by methods requiring a constant sampling frequency (e.g. The FFT, Hilbert transform). An additional advantage is the possibility to determine the resolution in the same way for both types of channels, using the effective number of bits.

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