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Direct spectrum detection based on Bayesian approach

Abstract. *The paper investigates the Bayesian framework's performance for a direct detection of spectrum parameters from the compressive measurements. The reconstruction signal stage is eliminated in by the Bayesian Compressive Sensing algorithm, which causes that the computational complexity and processing time are extremely reduced. The computational efficiency of the presented procedure is significantly better than a typical compressive sensing approach. The numerical simulations confirm the possibility of using this method to direct harmonics detection.*

Streszczenie. *W artykule zbadano wydajność podejścia bayesowskiego do bezpośredniej estymacji parametrów widma na podstawie pomiarów oszczędnych. W algorytmie bayesowskiego oszczędnego próbkowania etap rekonstrukcji sygnału został wyeliminowany, a tym samym znacznie zmniejszyła się złożoność obliczeniowa i czas przetwarzania w stosunku do typowych algorytmów rekonstrukcji. Symulacje numeryczne potwierdzają możliwość zastosowanie tej metody do estymacji składowych widma bezpośrednio na podstawie próbek sygnałów. (**Pomiar harmonicznych widma sygnału oparty na podejściu bayesowskim**).*

Keywords: reconstruction-free Compressive Sensing, Bayesian Compressive Sensing, direct spectrum sensing.
Słowa kluczowe: oszczędne próbkowanie bez rekonstrukcji, bayesowskie oszczędne próbkowanie, bezpośrednie próbkowanie widma.

Introduction

The theory of Compressive Sensing (CS) was first introduced by Candes et al. [1] and Donoho [2] as a new framework for simultaneous signal sensing and compression. The CS-based signal acquisition can be performed with a continuous sensing operator that randomly sub-samples the input data and provides compressed measurements consisting of very few linear projections of the original signal. The CS acquisition strategy is implemented by so-called Analog-to-Information Converter (AIC). It is based on different types of configurable architecture (i.e. Random Demodulation (RD), Random Modulation Pre-Integrator (RMPI), Compressive Multiplexer (CMUX), or Non-Uniform Sampler (NUS)) [3-5]. The original signal can be faithfully recovered from the fewer random measurements by using CS reconstruction algorithms. There is a wealth of literature on all these aspects of CS. The reconstruction algorithms are computationally complex, consume a lot of energy, and their hardware implementation is exceptionally labor-intensive. So the performance optimization of these algorithms is an active field of research in many applications [6-12].

In numerous cases, reconstruction algorithms perform moderately at low measurement rates and are computationally expensive. In practice, the purpose of measurement is not always to correctly reconstruct the input signal, but to estimate some of its parameters. Recent advances in CS theory have shown that effective inference is possible directly from the compressive measurements, without a reconstruction stage [13-20]. Especially in signal detection, the Bayesian-based approach was proposed to directly process the signals in the compressive domain [21]. A promising solution consists in implementation a Bayesian Compressing Sensing (BCS) for spectrum sensing in Cognitive Radio Networks [10, 22-24]. The reconstruction-free BCS approach is that by extracting spectrum components directly from the compressed measurements. The reconstruction stage in the receiving node can be completely bypassed, what significantly reduces the computational complexity of the system.

This paper presents an investigation of the Bayesian framework's performance for a spectrum determination directly from compressive measurements. The paper's organization is as follows: in the next section, it is discussed the reconstruction-free CS technique along with the related signal model, which exhibits sparsity so that CS can be applied. In the subsequent section, the CS inversion problem from a Bayesian perspective and associated relevance vector machine (RVM) algorithm are detailed. Then, the estimation of spectrum parameters is presented. Finally, the simulation results for performance comparison with an emphasis on computation complexity, and accuracy are presented. Conclusions and future scope are discussed in the last section.

Reconstruction Free CS concept

The conventional CS signal processing scheme contains both acquisition and reconstruction stages (see Fig. 1). Suppose that the signal x of length N samples is represented by a linear combination of known basis functions ψ_i in sparsity basis ψ with $a \in R^N$ transform domain coefficients of x. When the number of nonzero coefficients in x is K that signal $x \in R^N$ can be transformed in sparse transform domain to K orthogonal vectors, where $K \ll N$. The compressive measurements $y \in R^M$ ($M < N$) are obtained using:

$$y = \varphi \cdot x = \varphi \cdot \psi \cdot a = \Theta \cdot a \tag{1}$$

where: $\varphi \in R^{MxN}$ - a measurement matrix, $\Theta \in R^{MxN}$ - a reconstruction (sensing) matrix.

Let, the examined signal is a multicomponent waveform that consists of K sinusoids, defined as:

$$x_n = \frac{1}{N}\sum_{k=1}^{K} X_k \cdot \exp(j \cdot \frac{2\pi}{N} \cdot n \cdot k) \tag{2}$$

where: X_k a vector of DFT coefficients wherein K coefficients, at most, are nonzero.

Then, the acquisition process is described by:

$$y = \Theta \cdot X_K \tag{3}$$

The relationship between the measurement lengths is K << M < N.
In reconstruction-free CS model, the reconstruction stage is omitted (see Fig.1 – the bottom path). Assumed that, measurements may be noisily, therefore:

$$y = \Theta \cdot X_K + n_G \tag{4}$$

where: n_G - zero mean uncorrelated Gaussian noise with variance σ^2.

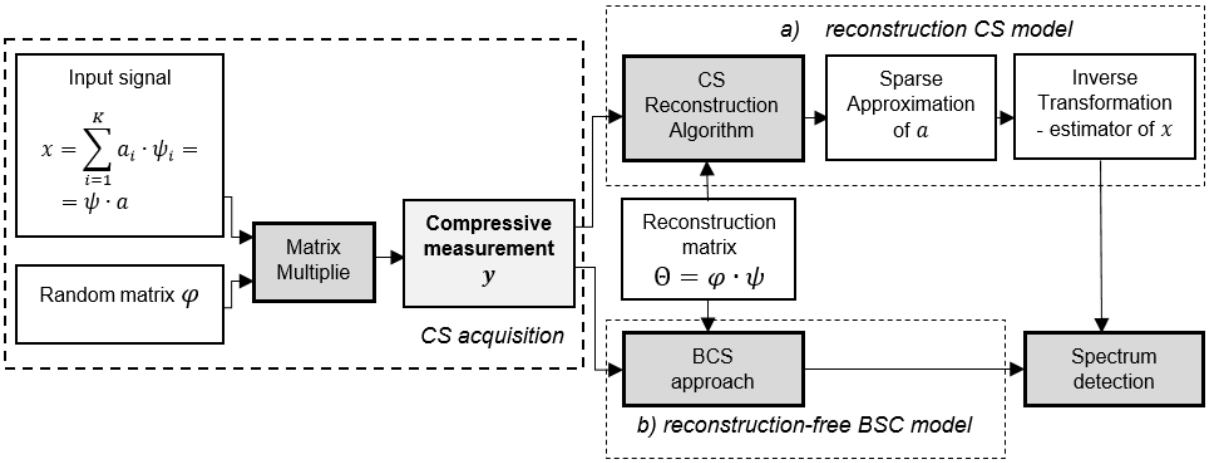

Fig. 1. CS scenarios to detect a signal spectrum

The expression (4) shows the relationship between the compressed time domain signal y, and the frequency domain sampled signal X_k.

BCS Inversion framework

In Bayesian modelling, all variables are treated as stochastic quantities defined by probability distribution functions. A vector X_k is an unknown model parameter with a priori assigned probability distribution $p(X_K|\beta_2)$, where $\frac{1}{\beta_2}$ is the variance of the Gaussian probability density function and denotes an $M \times 1$ hyperparameter vector of the Bayesian model. The probability of noise terms n_G is given as:

$$p(n_G) = \prod_{i=1}^{M} N(n_{Gi}|0,\sigma^2) \tag{5}$$

where: $\sigma^2 = \frac{1}{\beta_1}$ and the quantity β_1 is a hyperparameter scalar of the Bayesian model.

Taking account the equations (4) and (5), the compressive measurements y is a random process with conditional probability distribution inferred by Gaussian likelihood [22]:

$$p(y|X_K,\ \sigma^2) = \prod_{i=1}^{M} N(\Theta \cdot X_K, \sigma^2) = \\ = (2\pi\sigma^2)^{\frac{-M}{2}} \cdot exp\left(-\frac{1}{2\sigma^2}\|y - \Theta \cdot X_K\|_2^2\right) \tag{6}$$

where: $\|\cdot\|_p$ denotes the ℓ_p – norm and is calculated as $\|x\|_p = \left(\sum_{n=1}^{N}|x_n|^p\right)^{\frac{1}{p}}$.

Therefore, suppose that a zero-mean Gaussian prior distribution is defined on the frequency domain sampled signal X_k with a hyperparameter vector β_2 [22]:

$$p(X_K|\beta_2) = \prod_{i=1}^{M} N(X_K|0,\beta_{2i}^{-1}) = \\ = (2\pi)^{\frac{-M}{2}} \prod_{i=1}^{M} \beta_{2i}^{\frac{1}{2}} \cdot exp\left(\frac{-\beta_{2i}}{2} X_i^2\right) \tag{7}$$

$\{\beta_{2i}\}$'s are independent hyperparameters that form the $\beta_2=[\beta_{21}, ..., \beta_{2K}]^T$ vector and control the strength of the prior over associated X_i coefficients individually.

The posterior conditional distribution can be expressed via Bayes' rule as [23]:

$$p(X_K|y,\beta_1,\beta_2) = \frac{p(y|X_K,\beta_1)\cdot p(y|X_K,\beta_2)}{p(y|\beta_1,\beta_2)} \tag{8}$$

Combining a linear model within a Gaussian likelihood with a Gaussian prior, this distribution is also a Gaussian distribution $N(\mu, \Sigma)$ [22]:

$$p(X_K|y,\beta_1,\beta_2) = \frac{(2\pi)^{\frac{-M}{2}}}{|\Sigma|^{\frac{1}{2}}} \exp\left\{-\frac{(X_K-\mu)^T(X_K-\mu)}{2\Sigma}\right\} \tag{9}$$

The posterior mean μ, and covariance Σ of the above distribution are expressed respectively by [25]:

$$\mu = \beta_1 \cdot \Sigma \cdot \Theta^T \cdot y \tag{10}$$

$$\Sigma = \left(B + \beta_1 \cdot \Theta^T \cdot \Theta\right)^{-1} \tag{11}$$

where: B is a vector $B=diag\{\beta_{21}, \beta_{21,...,}\beta_{2K}\}$

In (10) and (11), for a chosen CS acquisition strategy, the sensing matrix Θ is given, while the unknown values of hyperparameters should be projected. Marginal likelihood maximization technique such as RVM can be applied to calculate these hyperparameters iteratively from the data [26]. Maximization of the marginal likelihood function with respect to logarithm of β_1 and β_2 can be expressed as [26]:

$$L(\beta_1,\beta_2) = -\frac{1}{2}\left[M\log(2\pi) + \log|C| + y^T C^{-1} y\right] \tag{12}$$

where: $C = \sigma^2 \cdot I + \Theta \cdot B \cdot \Theta^T$, and $I \in R^{MxM}$ is an identity matrix.

Differentiating equation (12) with respect to β_1 and β_2, and equating it to zero gives the following expressions that can be solved iteratively [26]:

$$\begin{cases} \beta_1^{new} = \dfrac{M - \sum_i \gamma_i}{\|y - \Theta \cdot \mu\|^2} \\ \beta_{2i}^{new} = \dfrac{\gamma_i}{\mu_i^2} \end{cases} \quad i \in \{1,2,...,K\} \tag{13}$$

where: μ_i denotes the i^{th} posterior mean weight from (10), and $\gamma_i \triangleq 1 - \beta_{2i}\Sigma_{ii}$, Σ_{ii} is the i^{th} diagonal element of the posterior signal covariance in (11) calculated with the updated β_1 and β_2 values.

Thus, μ and Σ can be obtained through an iterative algorithm, which iterates between equations (10) and (11), and (13) until a certain convergence criterion has been satisfied.

The spectrum parameters estimation

The goal of BCS is to estimate the sparse parameter vector X_K by using a sparsity endorsing prior through the estimation of β_1 and β_2. The hyperparameters are calculated iteratively. Suppose there is a specific threshold T for which the spectral coefficient is close to zero. The base vector, which is the corresponding column in the random measurement matrix, belonging to the relating spectral coefficient, can be removed in the estimation process. At each iteration, if $\mu_i > T$ the related coefficient X_i will be included in the iteration process. To complete the estimate, another stopping criterion must be defined as the convergence criterion that measures the changes between the updated values of the hyperparameter vector. The difference value δ is described by [23]:

$$\delta = \sum_{i=1}^{M} \left| \beta_{2i}^{n+1} - \beta_{2i}^{n} \right| \tag{14}$$

where: β_{2i}^{n+1} and β_{2i}^{n} are inverse variance estimation of the ith prior at the (n+1)th and nth iterations, respectively.

The ith point sparse vector X_K can be described by three parameters [22]:

$$\left\{ \left[f_i, \mu_i \right], \Sigma_{ii} \right\} \tag{15}$$

where: f_i is the spectral location, μ_i denotes the magnitude, and Σ_{ii} represents the estimation error.

Taking into account that for a kth component of spectrum:

$$\begin{gathered} \forall\ k_c \ \ s.t. \ \ \mu_k > T, \quad f_k \in \Delta f \\ \Delta f = f_{iH} - f_{iL} \end{gathered} \tag{16}$$

where: k_c is the index for point estimates contiguous in the frequency domain.

The sparse vector X_K can be described by an estimator of a central frequency, an average magnitude, and an accuracy [22]:

$$\left\{ \left[\hat{f}_k, \hat{\mu}_k \right], \hat{\Sigma}_k \right\} \tag{17}$$

where: $\hat{\Sigma}_k = \sum_{j \in \{i_L, i_H\}} \Sigma_{jj}$ denotes the accuracy of the point estimate μ_k.

If the composite feature point kth described by the pair of parameters will be multiplied by the inverse of the accuracy, we get [22]:

$$\hat{\Sigma}_k^{-1} \begin{bmatrix} \hat{f}_k \\ \hat{\mu}_k \end{bmatrix} = \sum_{j \in \{i_L, i_H\}} \Sigma_{jj}^{-1} \begin{bmatrix} f_j \\ \mu_j \end{bmatrix} \tag{18}$$

Thus, the average magnitude and central frequency of the kth spectral component is expressed by [22]:

$$\begin{bmatrix} \hat{f}_k \\ \hat{\mu}_k \end{bmatrix} = \hat{\Sigma}_k \cdot \sum_{j \in \{i_L, i_H\}} \Sigma_{jj}^{-1} \begin{bmatrix} f_j \\ \mu_j \end{bmatrix} \tag{19}$$

Performance analysis

The numerical simulations were carried out in the LabVIEW programming environment to verify the performance of directly estimating spectrum parameters. The input signal is a 512-length multi-tone waveform with fundamental harmonic 50 Hz and sparsity level K set to 7. The resolution of DFT Δf is equal to 10 Hz. In CS acquisition stage, the random Bernoulli matrix is used as the measurement matrix. The sensing matrix Θ represents a partial random inverse Fourier transform matrix obtained by omitting rows from the transformation basis ψ. The measurement value *y* is generated by Equation (4) with a zero-mean white Gaussian noise of variance $\sigma = 0.005$.

The following performance metrics are defined to evaluate the accuracy of direct spectrum detection. The estimation error (*EE*) of the average magnitudes is defined as:

$$EE = \frac{\left\| \hat{\mu} - \mu \right\|}{\left\| \mu \right\|} \tag{20}$$

The accuracy of spectral components localization in the frequency domain can be assessed by the frequency detection error (DE_K) calculated for each components X_k:

$$DE_K = \frac{\left\| \hat{f}_K - f_K \right\|}{\left\| f_K \right\|} \tag{21}$$

The estimation error value depends on the number of iterations (the compressive measurements) (see Fig. 2). The minimum number of samples M_{min} to satisfy the hyperparameter convergence criterion (14) is 160 samples, which corresponds to the compression ratio (i.e. M/N) equal to 0.31. If the difference value δ is set low, the required number of iterations will increase. Comparing of the magnitude estimation performance of direct spectrum detection and conventional reconstruction algorithms, based on CS convex optimization algorithms, and greedy algorithms [11, 23, 25] is shown in Table 1. Note that the direct spectrum detection's estimation error is less than the error sustains by Basis Pursuit (BP), discrete Radon transform CS (CS-DRT) algorithms, with similar input conditions. The number of the compressive measurements also influence the accuracy of localization of spectrum components (see Fig. 3). For M_{min}, the detection error does not exceed 3% for the highest harmonics.

A computational complexity and a computation time mainly depends on the RVM algorithm and precisely the number of rest of the basis vectors at the end of the iterations in BCS Inversion procedure. This relates to the number of nonzero coefficients K, and results as $\mathcal{O}(MK^2)$, called square complexity with linear multiplier [23]. The algorithm was run on an Intel (R) Core (T) i7-2600 CPU @ 3.4 GHz processor with 8 GB RAM. The CPU load during the executing code was assessed with software Process Lasso (https://bitsum.com/). The computer burden did not exceed 10%. The run-time of an algorithm was appointed based on 100 trials. The obtained results confirm that the reconstruction free CS approach reduces the processing time compared with conventional CS methods (Tab. 1).

Conclusion and future scope

The paper highlights the possibility of direct estimation and detection of spectrum parameters from the compressive measurements using the Bayesian approach.

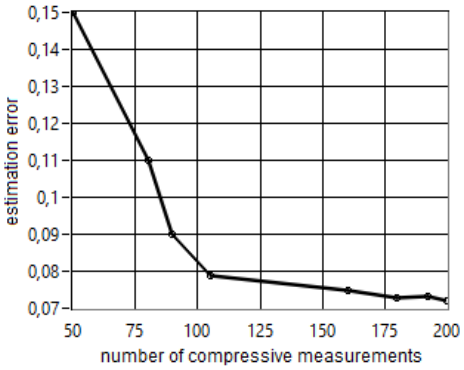

Fig. 2. The estimation error (EE) versus the number of compressive measurement. N=512, K=7, $\sigma = 0.005$, δ=0.075

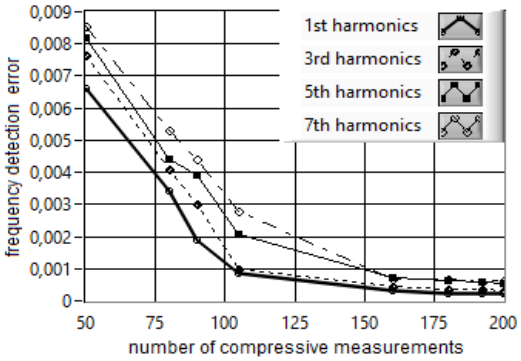

Fig. 3. The frequency detection error versus the number of compressive measurement. N=512, K=7, $\sigma = 0.005$, δ=0.075

Table 1. The summary of the performances of the direct spectrum detection (DSD), BP, and CS-DRT algorithms (N=512)

CS algorithm	*K*	M_{min}	Estimation error	Processing time [s]
BP	6	223	0.2381 (MSE*)	1.3482
CS-DRT	7	60	0.15 (MSE)	0.54
DSD	7	160	0.075	0.27

* *MSE - mean square error*

The reconstruction stage is eliminated in the algorithm, and thus the computational complexity and processing time are significantly reduced. Furthermore, the computational efficiency of the presented procedure has been found to be much better than typical reconstruction algorithms, for the cases considered. Numerical simulations were performed for signal, which can be modelled as a superposition of a small number of sinusoids and the DFT basis forms its sparse domain. Based on this study's results, the future work will be related to the implementation of BCS for direct spectrum sensing in a more dynamic spectral environment.

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