DOPPLER ESTIMATION METHOD FOR MOVING TARGET LOCATION

ROMAN SALAMON, JACEK MARSZAL

Gdansk University of Technology
Gabriela Narutowicza 11/12, 80-233 Gdansk, Poland
roman.salamon@eti.pg.gda.pl

The article presents an algorithm for target location based solely on Doppler shift of echo acoustic signals of a moving underwater object. The algorithm is designed for use in multi-static systems for protecting bodies of water from terrorist attacks by divers or submerged vehicles. The original source of signals is a transmitter emitting continuous acoustic waves. The echo signals from moving object are received by hydrophones located at fixed and known points in the water. Doppler shifts are determined from discrete Fourier transforms of hydrophone signals. The proposed iterative algorithm for target location calculates the approximate values of target speed vector and target coordinates which correspond to speed vectors as calculated previously.

INTRODUCTION

Hydro power plants, dams, retention reservoirs and onshore facilities including quays and canals are the possible targets of terrorist attacks by divers or submerged vehicles. They can be detected and located using active pulse sonars distributed in the body of water. Echo signals off the targets are masked by signals coming from all other underwater objects and from surface and bottom reverberations. In order to improve the usable signal reflected off a moving target, sonars can eliminate stationary echoes, a technique commonly used in radars (MTI Radar) [1]. Non-stationary signals such as reverberations are not eliminated effectively. Alternatively, a hydroacoustic system can be used which detects moving targets solely on the basis of Doppler shift of the echo signal frequency.

The system discussed in the article has an acoustic part consisting of a omni-directional source of a sinusoidal acoustic signal located at a fixed and known point and at least four hydrophones at known locations on the edges of the body of water. Echo signals from the hydrophones are sent to a multi-channel receiver where they are amplified and pre-filtrated before they undergo analogue to digital conversion. Once digitized, the echo signals are the basis for calculating discrete spectra in successive intervals. Next after they are moved to the primary bandwidth Doppler shifts are calculated. With known frequency shifts, we can numerically estimate the vector of target speed and its location.

The advantage of the system is that it eliminates all stationary echoes coming from stationary distracting objects especially bottom reverberations. As a result, man operated pulse sonars can be replaced by automatic Doppler system, with detection typically used in alarm systems.

The article presents algorithms used for calculating the target speed vector and coordinates and the results of a computer simulation. A simplified model of the system is presented with the minimal number of four hydrophones placed at the vertexes of a square. An additional assumption is made which is that Doppler deviated signals come from a single object. With these assumptions, the proposed method of target motion estimation can be presented in a simple form. The simplification has no effect on the essence of the method but makes the mathematical formulas simpler.

Another location method based also on Doppler shift is proposed for mobile cellular system [2], [3].

1. MODEL OF THE SYSTEM

Fig.1 shows hydrophone coordinates marked 1 to 4 and transmitter coordinates marked "o". We assume that only one moving target is under observation with momentary coordinates *x*,*y*. We will leave out the effects of submersion of the transmitter, hydrophones and target which is justified given the size of the body of water and its assumed low depth.

The distribution of pressure *p* of the sinusoidal acoustic wave with frequency f_0, emitted by the transmitter can be written as:

$$p(r,t) = \frac{A}{r}\sin(2\pi f_0 t - k_0 r)\,, \tag{1}$$

where $k_0 = 2\pi f_0 / c$ is the wave number, *c* – velocity of acoustic wave in water, *r* – distance between the object and source of sound, and *A* - a constant dependent on the efficiency of the sound source.

When the target moves, distance *r* changes in time. Hence the frequency of the wave incident on the object is equal to:

$$f = \frac{1}{2\pi}\frac{d}{dt}(2\pi f_0 t - k_0 r) = fo - \frac{f_0}{c}\frac{dr}{dt}\,. \tag{2}$$

Let us assume, that when the observation begins the target is at a point with coordinates x_0,y_0 and moves at constant speed *v* whose vector is shown in Fig.1. After time *t* the distance between the target and source of sound is:

$$r = \left|\vec{r}_0 + \vec{v}t\right| = \sqrt{(x_0 - X_0 + v_x t)^2 + (y_0 - Y_0 + v_y t)^2} \tag{3}$$

When we have calculated the derivative of this expression and substituted it to formula (2) we obtain:

$$f = f_0 - \frac{f_0}{c}\frac{v_x(x_0 - X_0 + v_x t) + v_y(y_0 - Y_0 + v_y t)}{\sqrt{(x_0 - X_0 + v_x t)^2 + (y_0 - Y_0 + v_y t)^2}}\,. \tag{4}$$

where v_x and v_y are the components of the target's velocity vector.

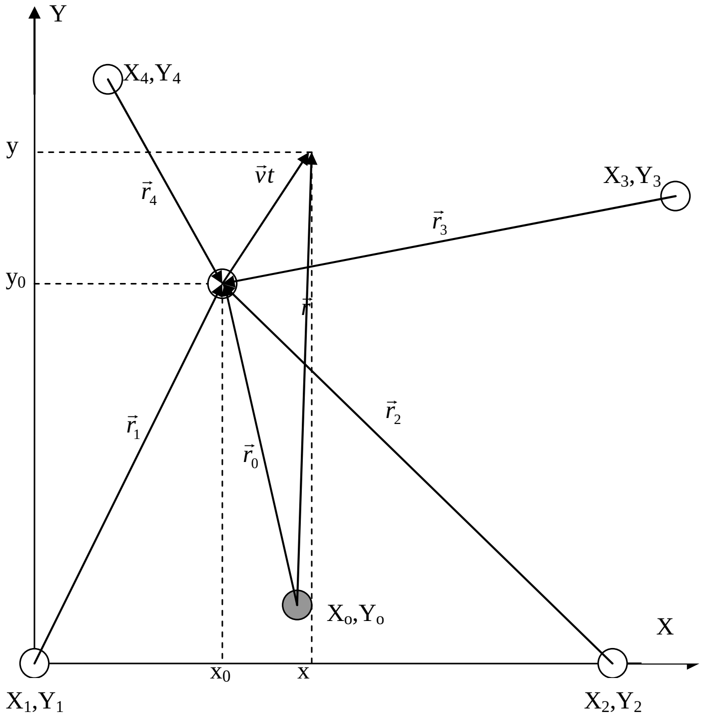

Fig.1. Position of the transmitter (gray), hydrophones and the object under observation in water

If the target moves slowly and the observation time is short, the above formula can be simplified to:

$$f \cong f_0 - \frac{f_0}{c}\frac{v_x(x_0 - X_0) + v_y(y_0 - Y_0)}{\sqrt{(x_0 - X_0)^2 + (y_0 - Y_0)^2}} \tag{5}$$

Let us note a fact which we will need further in the analysis, which is that the expression in the formula's numerator is the scalar product of the vectors $\vec{v}$ and $\vec{r}_0$, and the expression and the denominator describes length r_0 of vector $\vec{r}_0$. Hence formula (5) can be written as:

$$f \cong f_0 - \frac{f_0}{c}\frac{\vec{v}\vec{r}_0}{r_o} = f_0 - \frac{f_0 v}{c}\cos\gamma\,, \tag{6}$$

where γ is the angle between vectors $\vec{v}$ and $\vec{r}_0$.

The incident wave and the reflected wave on the surface of the moving target has frequency f which is different from the frequency of the wave emitted by the transmitter by Doppler shift. The surface of the target becomes the source of spherical wave with frequency f if the target is not moving. When it moves at speed $\vec{v}$ the frequency of the wave received by hydrophones is changing. The physical cause is a change in wave length in time $T=1$/f which for n-th hydrophone can be written as:

$$\lambda_n = \lambda + vT\cos\gamma_n\,, \tag{7}$$

where γ_n is the angle between vector $\vec{v}$ and $\vec{r}_0$.

By expressing the length of the wave through its period we obtain:

$$cT_n = cT + vT\cos\gamma_1\,, \tag{8}$$

hence we get:

$$f_n = \frac{f}{1+(v/c)\cos\alpha_n} \tag{9}$$

For a low speed v compared to the velocity c of sound in water, the above formula can be simplified to this form:

$$f_n \cong f(1-\frac{v}{c}\cos\gamma_n) \cong f_0(1-\frac{v}{c}\cos\gamma)(1-\frac{v}{c}\cos\gamma_n) \cong f_0[1-\frac{v}{c}(\cos\gamma+\cos\gamma_n)], \tag{10}$$

leaving out the component proportionate to $(v/c)^2$.

Using the relationships between formulas (5) and (6), the frequency of the wave received by the n-th hydrophone can be finally written as:

$$f_n \cong f_0[1-\frac{1}{c}\frac{v_x(x_0-X_0)+v_y(y_0-Y_0)}{\sqrt{(x_0-X_0)^2+(y_0-Y_0)^2}}-\frac{1}{c}\frac{v_x(x_0-X_n)+v_y(y_0-Y_n)}{\sqrt{(x_0-X_n)^2+(y_0-Y_n)^2}}] \tag{11}$$

where n=1,2,3,4.

With four hydrophones the result is a set of four equations with four unknowns, x_0,y_0,v_x and v_y. Frequency f_0, velocity of sound c and the coordinates of the transmitter and hydrophones are known and frequencies $f_1,\ldots,f_4$ are determined from the spectrum of the signals received. Hence it is possible to estimate both the speed of the target and its original position. It must be stressed, however, that it does not depend on any special location of the hydrophones or the transmitter, except when two or more hydrophones are at the same point.

Because there is no analytical solution to the set of equations (11), a numerical solution must be sought. Below is a presentation of one of the possible methods for approximate solving this set of equations. It uses the special geometric distribution of the hydrophones with the transmitter put in place of one of the hydrophones. As we are going to demonstrate, this helps to simplify the formulas significantly and what is more important points to effective numerical calculations.

Fig.2 shows the system's geometric configuration which involves the algorithm described in this section.

By placing the source of sound at $X_0=X_1,Y_0=Y_1$ we can simplify formula (11) for n=1 to the following form:

$$f_1 \cong f_0[1-\frac{2}{c}\frac{v_x(x_0-X_1)+v_y(y_0-Y_1)}{\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}}] \tag{12}$$

If we mark the single direction Doppler shift as:

$$F_1 \cong -\frac{f_0}{c}\frac{v_x(x_0-X_1)+v_y(y_0-Y_1)}{\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}} = -\frac{f_0}{cr_1}[v_x(x_0-X_1)+v_y(y_0-Y_1)] \tag{13}$$

and deduct it from equations (11) for n=2, 3, and 4, we obtain:

$$F_n \cong -\frac{f_0}{cr_n}[v_x(x_0-X_n)+v_y(y_0-Y_n)] \tag{14}$$

where r_n is the distance between the n-th hydrophone and the target.

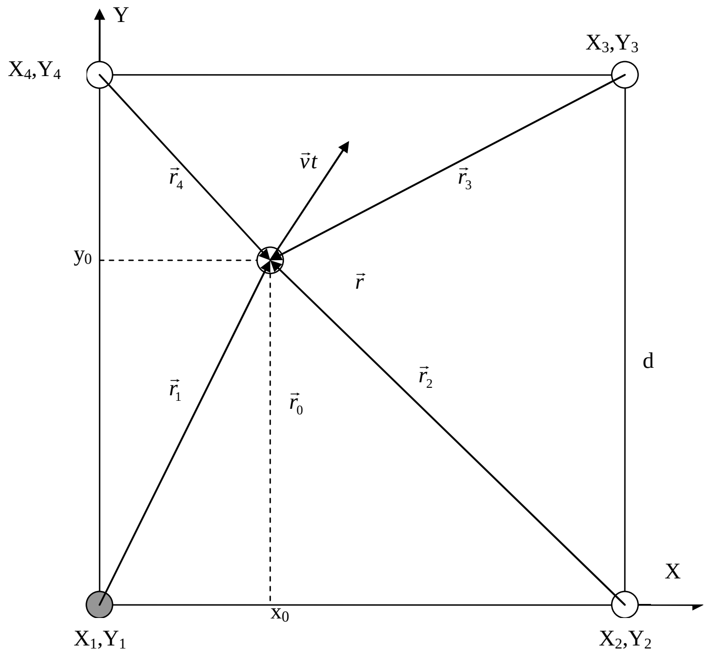

Fig.2. The transmitter and hydrophones are placed at the vertexes of the square

In the coordinates shown in Fig. 2 the formulas above take the following form:

$$
\begin{aligned}
F_1 &\cong \frac{f_0}{cr_1}(-v_x x_0 - v_y y_0) \\
F_2 &\cong \frac{f_0}{cr_2}[v_x(d - x_0) - v_y y_0] \\
F_3 &\cong \frac{f_0}{cr_3}[v_x(d - x_0) + v_y(d - y_0)] \\
F_4 &\cong \frac{f_0}{cr_4}[-v_x x_0 + v_y(d - y_0)]
\end{aligned}
\tag{15}
$$

The above set of equations is the system's mathematical model because it ties measurement data – Doppler shifts $F_1,\ldots,F_4$ – with the values that describe the vector of target velocity (v_x,v_y) and location $(r_1,\ldots,r_4)$.

2. ALGORITHM FOR ESTIMATION TARGET SPEED AND LOCATION

Let us assume first that target location is known. Under this assumption we can easily estimate the components of the velocity vector from any pair of equations (15). This is what we get:

$$v_x = \frac{c}{f_0 d}(F_2 r_2 - F_1 r_1) \quad \text{or} \quad v_x = \frac{c}{f_0 d}(F_3 r_3 - F_4 r_4) \tag{16}$$

$$v_y = \frac{c}{f_0 d}(F_3 r_3 - F_2 r_2) \quad \text{or} \quad v_y = \frac{c}{f_0 d}(F_4 r_4 - F_1 r_1) \tag{17}$$

For the purpose of further calculations, let us write the above formulas as:

$$v_x = \frac{c}{2 f_0 d}(F_2 r_2 - F_1 r_1 + F_3 r_3 - F_4 r_4)$$
$$v_y = \frac{c}{2 f_0 d}(F_3 r_3 - F_2 r_2 + F_4 r_4 - F_1 r_1) \tag{18}$$

The objective now is to estimate approximate values of target speed components based solely on our knowledge of Doppler shifts. To that end we will assume that the target is in the middle of the square, hence $r_1=r_2=r_3=r_4=d/\sqrt{2}$ and equations (18) are simplified to this form:

$$v_x' = \frac{c}{2\sqrt{2} f_0}(F_2 + F_3 - F_1 - F_4)$$
$$v_y' = \frac{c}{2\sqrt{2} f_0}(F_3 + F_4 - F_1 - F_2) \tag{19}$$

As further presented, the iteration procedure for estimating target location and speed uses coordinates v_x' and v_y' as the first approximation of the real coordinates of the velocity vector. The error of this estimation increases as the target moves further away from the center of the square. It also depends on the direction of the velocity vector. This is illustrated in Fig. 3 showing the relative errors of the velocity module and the velocity vector angle versus the velocity vector's direction. As you can see even for significant distances between the target and the center of the square, both errors are relatively small.

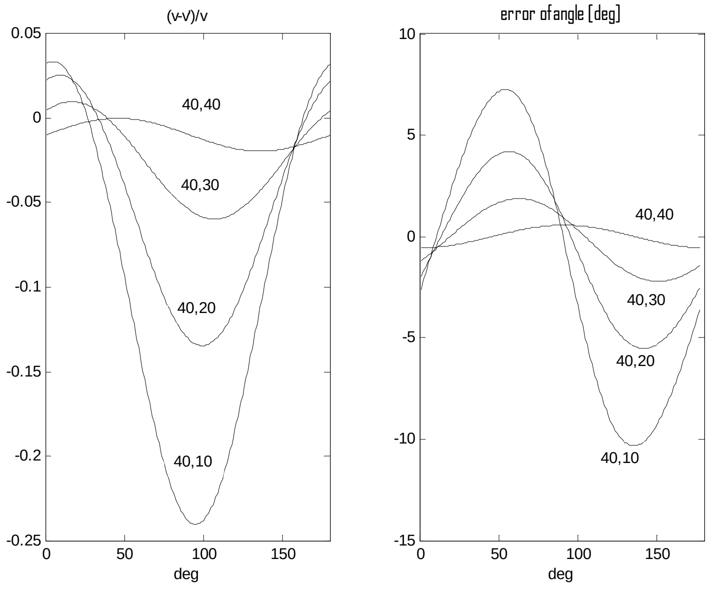

Fig.3. Errors in estimating the velocity vector (the parameters of the curves estimate the coordinate *x* and *y* of target location inside the square whose side is *d*=100 m)

With a known velocity vector we can calculate the target's approximate coordinates. In the procedure equations of straight lines are determined. The lines are the geometric place of points at which the target moving at velocity $\vec{v}'(v_x', v_y')$ reflects acoustic waves with frequency shifts $F_1,\ldots,F_4$. To that end we can use relationship (6) to estimate the differences in the angles of inclination of the straight lines and the estimated angle of the velocity vector.

The second method is to solve those equations (15) whose unknowns are the inclination factors of straight lines *m*. For this variable all equations have the same form which is:

$$a_n\sqrt{1+m_n^2} = v_x^{'} + m_n v_y^{'} \tag{20}$$

where $a_n = cF_n / f_0$.

The inclination factors of the straight lines are the roots of the above square equation and are described with the following formula:

$$m_n = \frac{-v_x^{'} v_y^{'} \pm a_n\sqrt{v^{'2} - a_n^2}}{v_y^{'2} - a_n^2} \tag{21}$$

Of the eight inclinations we dismiss:

- inclinations that have complex values and are the result of bad estimation of the velocity vector module
- inclinations which are matched by Doppler shifts different from those measured
- inclinations which estimate straight lines which do not cross the observation area.

The other inclinations are determined by straight lines in the equations:

$$\begin{aligned} y &= m_1 x \\ y &= m_2(x-d) \\ y &= m_3(x-d)+d \\ y &= m_4 x + d \end{aligned} \tag{22}$$

It should be noted that despite the selection, two straight lines can be derived from the vertexes of the square as shown in Fig. 4. This ambiguity is the direct result of evenness of the cosine function in formula (6). For particular directions of the velocity vector the target can be located on two straight lines with different inclinations and generate the same Doppler shift. This ambiguity is eliminated in the next procedure of the algorithm.

The next step in the calculation is to determine the point of intersection of straight lines which give an approximate location of the target. All of the possible pairs of equations are used to form equations with two unknowns. The solutions are the coordinates of the points we want to identify. As an example, from the second and third equation (22) we have:

$$\begin{bmatrix} 1 & -m_2 \\ 1 & -m_3 \end{bmatrix} \begin{bmatrix} y_{023} \\ x_{023} \end{bmatrix} = \begin{bmatrix} -m_2 d \\ -m_3 d + d \end{bmatrix} \tag{23}$$

The points of intersection are marked with circles in Fig. 4. The significant spread of the points is the result of an error in target location. However, the mean values of the intersection point coordinates give a relatively good approximation of the correct target location as shown in Fig.4.

The approximate values of target location are used to eliminate redundant straight lines and as a consequence the intersection points, the result of ambiguous inclinations. For this purpose, the Doppler shifts are determined at all points and the points at which errors are greatest are eliminated. Fig. 5 shows the result of the selection for the situation in Fig.4.

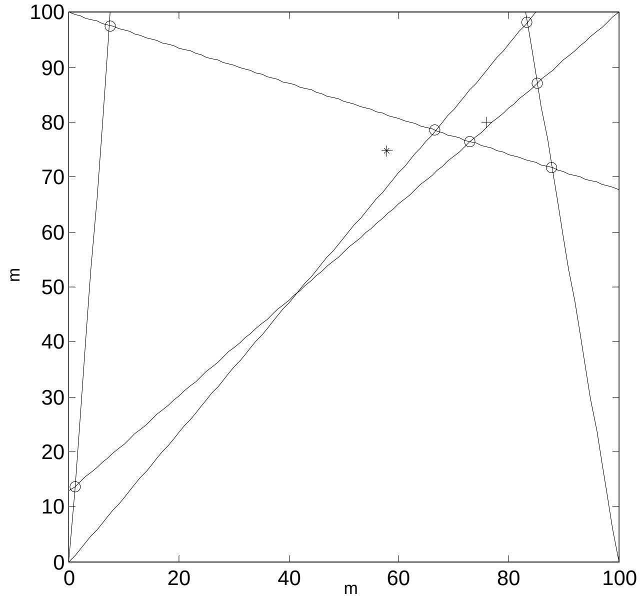

Fig.4. Straight lines described with equations (22) (+ real target location, * estimated target location)

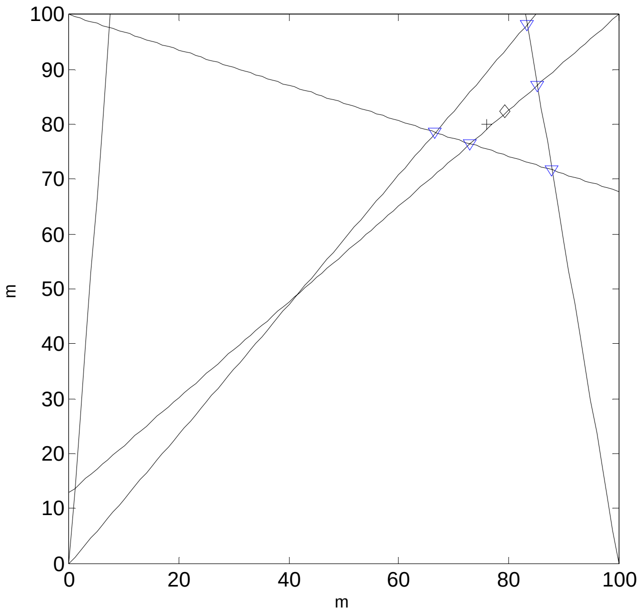

Fig.5. Result of eliminating redundant straight lines (+ real target location, diamond – estimated target location)

Following the elimination of redundant straight lines, the spread of intersection points is reduced significantly and the mean values of their coordinates give a much more accurate approximated target position. The calculations show that for targets in a relatively large central part of the square, the accuracy of determining the target location can be sufficient in some practical applications of the system.

In order to improve the accuracy of determining target speed and location, the components of the velocity vector are calculated. This is under the assumption that the target's coordinates are x_0', y_0' and have been calculated using the procedure above. Following a transformation of the equations (15) we obtain:

$$
\begin{aligned}
F_1 \frac{cr_1}{f_0} &\cong -v_x'' x_0' - v_y'' y_0' \\
F_2 \frac{cr_2}{f_0} &\cong v_x''(d - x_0') - v_y'' y_0' \\
F_3 \frac{cr_3}{f_0} &\cong v_x''(d - x_0') + v_y''(d - y_0') \\
F_4 \frac{cr_4}{f_0} &\cong -v_x'' x_0' + v_y''(d - y_0')
\end{aligned}
\tag{24}
$$

Each pair of the above equations is a set of two equations with two unknowns v_x'' and v_y''. As an example, from the second and third equation we have:

$$
\begin{bmatrix} F_2 \dfrac{cr_2}{f_0} \\ F_2 \dfrac{cr_3}{f_0} \end{bmatrix} = \begin{bmatrix} d - x_0' & -y_0' \\ d - x_0' & d - y_0' \end{bmatrix} \cdot \begin{bmatrix} v_x'' \\ v_y'' \end{bmatrix}
\tag{25}
$$

By solving six analogous sets of equations we obtain six components of the velocity vector. The mean values of the components produce more and increasingly better approximations of the real components of the target's velocity vector. The components can be used to estimate target coordinates again using the procedure above. Fig. 6 shows the result of such calculations for the situation in Fig. 4. The previous error in estimating the location of 9 m was reduced to 0.6 m. The accuracy of estimating speed has improved from 7% to 0.2% for the module and error of the angle from 3.3^0 to 0.7^0.

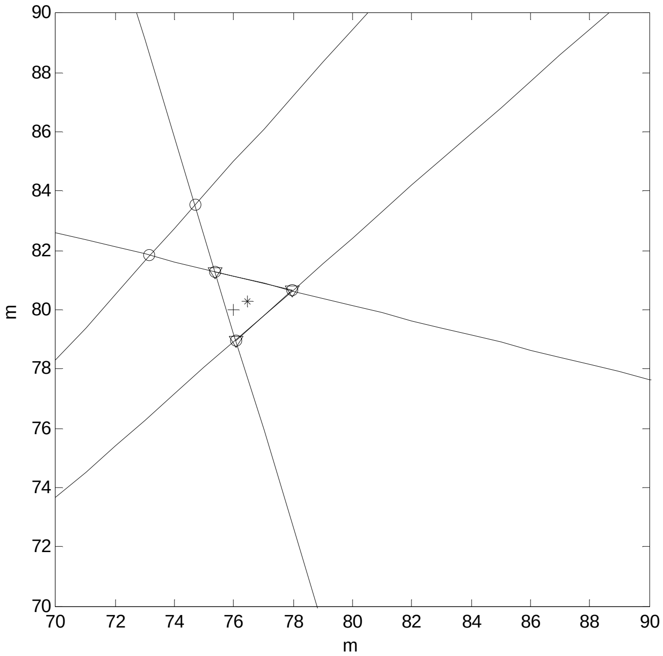

Fig.6. Target location estimated using the extended algorithm
(+ real target location, * estimated target location)

3. CONCLUSIONS

The simplified algorithm for estimation of target speed and location does not always generate results that match the real results. To avoid this the extended version of the algorithm uses additional procedures to reduce errors. Their function is mainly to eliminate straight lines whose angles of intersections are very small.

The proposed algorithm does not need a powerful computer which helps with real time system operation. It can be extended to a practical version of the algorithm with hydrophones placed in a space other than a square. A bigger number of moving targets would require more hydrophones. With more hydrophones it is easier to reduce the error when only one target is being observed.

REFERENCES

[1] M. Skolnik, Radar Handbook, Second Edition, McGraw-Hill, 1990.

[2] W. Pan, J. Wu, Z. Jiang, Y. Wang, X. You, Doppler Location Based on Two-Dimension Searching in Orthogonal Complement Space in DAS, Proc. 66-th IEEE Vehicular Technology Conference, VTC2007-Fall, Baltimore 2007, 1962 -1963.

[3] W. Pan, J. Wu, Z. Jiang, Y. Wang, X. You, Mobile Position Tracking by TDOA-Doppler hybrid Estimation in Mobile Cellular System, IEEE ICC 2007, pp. 4670 –4673.