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ENERGY ANALYSIS OF THE PROPULSION SHAFT FATIGUE PROCESS IN A ROTATING MECHANICAL SYSTEM PART III DIMENSIONAL ANALYSIS

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ABSTRACT

This article presents the third and last part of the problem of diagnosing the fatigue of marine propulsion shafts in terms of energy with the use of the action function, undertaken by the authors. Even the most perfect physical models of real objects, observed under laboratory conditions and developed based on the results of their research, cannot be useful in diagnostics without properly transferring the obtained results to the scale of the real object. This paper presents the method of using dimensional analyses and the Buckingham theorem (the so-called π theorem) to determine the dimensionless numbers of the dynamic similarity of the physical model of the propulsion shaft and its real ship counterpart, which enable the transfer of the results of the research on the energy processes accompanying the ship propulsion shaft fatigue from the physical model to the real object.

Keywords: ship propulsion shaft, dimensional analysis, Buckingham theorems, similarity criteria.

INTRODUCTION

Within the previous parts of the article, the results of model and experimental studies of the fatigue process of the propulsion shaft were presented [6,7]. The active experiments were carried out on the especially designed and built physical model of the rotary propulsion system which was mapping an operation of full-size real object. There has been proved that changes in the deflection of the rotary propulsion line are reflected in the amount of dissipated kinetic energy of masses in rotational motion and the accumulated internal energy of construction materials. After exceeding the critical values of these energies, a fatigue damage occurs, the course of which is characterized by energy residual processes: vibroacoustic and thermal. They cause, inter alia, observable diagnostic symptoms of changes in the fatigue state of the structural material from which the propulsion shaft is made. There has been proposed to adopt the high-cycle fatigue syndrome consisting of diagnostic symptoms determined from the function of the propulsion shaft action related to the transformation of mechanical energy into the way of work and heat, and the generation of mechanical vibrations and elastic waves of acoustic emission. In order to assess the diagnostic information contributed by the defined characteristics of the fatigue condition of the propulsion shaft, a program of experimental tests was developed and performed, in which two statistical hypotheses were verified: the significance of the influence of the quantities forcing the fatigue process [6] and the adequacy of the regression equation describing the fatigue life of the propulsion shaft in term of energy [7].

Experimental studies carried out on physical models that reflect the work of a real full-size object have one fundamental weakness; namely they may not consider certain phenomena and processes that have been unconsciously eliminated due to their transition to a small scale [1]. To be able to fully transfer the measurement results characterising the work of the physical model to real objects, it is not enough to simply multiply them by the dimensional scale of the model. Therefore, appropriate criteria (dimensionless numbers) for their geometric, kinematic, and dynamic similarities should be developed. An effective tool in the way of analytical solution of this problem for the considered process of fatigue of marine propulsion shafts can be its dimensional analysis, which has been widely included in publications providing theoretical guidance [2,3] and successfully used, for example, in modelling the dynamics of vehicle movement [4,5]. It allows the form of the function describing the examined process to be determined in a situation where only its arguments are known. It is then assumed that there must be full compliance of the dimension of this function (physical quantity) with the dimension of the power product created from the considered arguments, significantly affecting the course of the fatigue process which has already been partially described by the authors [6-8]. Nevertheless, there is still a noticeable lack of bibliographic items describing the problems of fatigue durability of marine propulsion systems based on the results of experimental tests carried out on a small scale.

DIMENSIONAL ANALYSIS OF THE FATIGUE DURABILITY OF THE PROPULSION SHAFT

For a rotating propulsion shaft subjected to set bending-torsional loads, one can write the relationship describing its fatigue durability τ_W as a function of the recorded physical parameters1, each of which has its own dimension, according to the International System of Units:

$$\tau_W = f(m, I, M, F, y, \omega) \tag{5.1}$$

Both the dimension of the function being searched for and the dimensions of its arguments (parameters) can be represented by the power products of three basic dimensions: length [L], mass [M], and time [T][2]:

- τ_W – fatigue durability (operating time to the development of a detectable crack on theshaft's surface), s → $[L^0 \cdot M^0 \cdot T^1]$,
- m – propulsion shaft mass (without the mass of the propeller), kg → $[L^0 \cdot M^1 \cdot T^0]$,
- I – moment of inertia (polar around the shaft axis), kg·m² → $[L^2 \cdot M^1 \cdot T^0]$,
- M – transmitted torque (by the propulsion shaft), kg·m²/s² → $[L^2 \cdot M^1 \cdot T^{-2}]$,
- F – the bending force acting on the propulsion shaft (resulting from reactions in bearingnodes), kg·m/s² → $[L^1 \cdot M^1 \cdot T^{-2}]$,
- y – shaft deflection (assumed for the middle of the shaft length, as in Fig. 1), m → $[L^1 \cdot M^0 \cdot T^0]$,
- ω – angular speed of the propulsion shaft, 1/s → $[L^0 \cdot M^0 \cdot T^{-1}]$.

The relationship (5.1) connects $m = 7$ physical dimensional quantities characterising the tested fatigue process of the propulsion shaft in the dynamic aspect, the dimensions of which include $n = 3$ basic dimensions. Thus, according to the method developed by Edgar Buckingham [9, 10] in 1914 (the so-called π theorem)[3], the fatigue life of the shaft can be described by $k = m-n$, i.e., four dimensionless similarity numbers π_k, starting from the dimensional formula in the form of the product of the powers of the important basic physical quantities in the studied process [11]:

$$\tau_W = C \cdot m^A \cdot I^B \cdot M^D \cdot F^E \cdot y^F \cdot \omega^G \tag{5.2}$$

where: $A \dots G$ – constant.

According to the Fourier dimensional consistency principle, to maintain the size equation, the dimensions of the physical quantities on the left and right side of the equation (5.2) must be the same. After replacing all the physical quantities with their basic dimensions in it, we get:

$$T = C \cdot M^A \cdot L^{2B} \cdot M^B \cdot L^{2D} \cdot M^D \cdot T^{-2D} \cdot L^E \cdot M^E \cdot T^{-2E} \cdot L^F \cdot T^{-G} \tag{5.3}$$

and after transformation:

$$T = C \cdot M^{A+B+D+E} \cdot L^{2B+2D+E+F} \cdot T^{-2D-2E-G} \tag{5.4}$$

To obtain the dimensionless form of the equation for the fatigue life of the propulsion shaft, the sum of the exponents of each physical quantity must be zero:

$$\text{for } T \rightarrow 1 + 2D + 2E + G = 0$$
$$\text{for } M \rightarrow A + B + D + E = 0$$
$$\text{for } L \rightarrow 2B + 2D + E + F = 0$$

Since there are six unknowns (exponents) and only three independent equations, it is impossible to solve the given problem without eliminating some of the arguments[4] or assuming at least three values of the sought exponents[5] as the parametric solution to the system of linear equations. On the other hand, three basic units were used for the dimensional analysis. This means that three out of the six analysed arguments of the searched persistence function may be dimensionally independent[6]. The mass m, the shaft deflection y, and the angular velocity ω were selected for further analysis, for which the dimensional independence was checked. For this purpose, the value of the determinant of the matrix composed of exponents was calculated with the dimensions of these arguments. Since the value of the determinant is different from zero, it can be assumed that these arguments are dimensionally independent:

1 Assuming that the fatigue durability depends only on the specified physical parameters, and the function describing this durability is dimensionally invariant and homogeneous.
2 This is the standard assumption of dimensional analyses in mechanics.

3 Buckingham's Theorem, also known as the π Theorem, is a key law used in dimensional analyses. It states that "if one has an equation described by a certain number of independent physical parameters, then this equation can be expressed by dimensionless modules π, the number of which is equal to the number of these physical parameters minus the number of fundamental dimensions".
4 This is not a rational procedure as it reduces the level of detail in the dimensional analysis.
5 For example, the exponent $F = -1$, which means that the fatigue durability of the shaft is inversely proportional to its deflection
6 None of their units can be expressed in combination with the others.

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 1 \neq 0 \qquad (5.5)$$

In the next step of the calculations, successive dimensionless variables $\pi_1 \dots \pi_4$ are determined using the remaining arguments and the selected arguments that are independent of the dimensions:

1. Dimensionless variable π_1 due to fatigue durability τ_W:

$$\pi_1 = \tau_W \cdot m^A \cdot y^B \cdot \omega^C \qquad (5.6)$$

Inputting basic units on both sides of equation (5.6) we get:

$$L^0 \cdot M^0 \cdot T^0 = L^0 \cdot M^0 \cdot T^1 \cdot (L^0 \cdot M^1 \cdot T^0)^A \cdot (L^1 \cdot M^0 \cdot T^0)^B \cdot (L^0 \cdot M^0 \cdot T^{-1})^C \qquad (5.7)$$

By comparing exponents with appropriate dimensions:

for $T \rightarrow 0 = 1\text{-}C \Rightarrow C = 1$
for $M \rightarrow 0 = A \Rightarrow A = 0$
for $L \rightarrow 0 = B \Rightarrow B = 0$

the following is obtained:

$$\pi_1 = \tau_W \cdot m^0 \cdot y^0 \cdot \omega^1 \qquad (5.8)$$

and finally:

$$\pi_1 = \tau_W \cdot \omega \qquad (5.9)$$

2. Dimensionless variable π_2 due to the moment of inertia *I*:

$$\pi_2 = I \cdot m^A \cdot y^B \cdot \omega^C \qquad (5.10)$$

By inserting the base units on both sides of equation (5.10) we get:

$$L^0 \cdot M^0 \cdot T^0 = L^0 \cdot M^1 \cdot T^0 \cdot (L^0 \cdot M^1 \cdot T^0)^A \cdot (L^1 \cdot M^0 \cdot T^0)^B \cdot (L^0 \cdot M^0 \cdot T^{-1})^C \qquad (5.11)$$

By comparing the exponents with appropriate dimensions:

for $T \rightarrow 0 = \text{-}C \Rightarrow C = 0$
for $M \rightarrow 0 = 1 + A \Rightarrow A = \text{-}1$
for $L \rightarrow 0 = 2 + B \Rightarrow B = \text{-}2$

the following is obtained:

$$\pi_2 = I \cdot m^{-1} \cdot y^{-2} \qquad (5.12)$$

and finally:

$$\pi_2 = \frac{I}{m \cdot y^2} \qquad (5.13)$$

3. Dimensionless variable π_3 due to the transmitted torque *M*:

$$\pi_3 = M \cdot m^A \cdot y^B \cdot \omega^C \qquad (5.14)$$

By inserting the base units on both sides of equation (5.14) we get:

$$L^0 \cdot M^0 \cdot T^0 = L^2 \cdot M^1 \cdot T^{-2} \cdot (L^0 \cdot M^1 \cdot T^0)^A \cdot (L^1 \cdot M^0 \cdot T^0)^B \cdot (L^0 \cdot M^0 \cdot T^{-1})^C \qquad (5.15)$$

By comparing the exponents with appropriate dimensions:

for $T \rightarrow 0 = \text{-}2\text{-}C \Rightarrow C = \text{-}2$
for $M \rightarrow 0 = 1 + A \Rightarrow A = \text{-}1$
for $L \rightarrow 0 = 2 + B \Rightarrow B = \text{-}2$

the following is obtained:

$$\pi_3 = M \cdot m^{-1} \cdot y^{-2} \cdot \omega^{-2} \qquad (5.16)$$

and finally:

$$\pi_3 = \frac{M}{m \cdot y^2 \cdot \omega^2} \qquad (5.17)$$

4. Dimensionless variable π_1 due to the bending force *F*:

$$\pi_4 = F \cdot m^A \cdot y^B \cdot \omega^C \qquad (5.18)$$

Putting the base units on both sides of equation (5.18) we get:

$$L^0 \cdot M^0 \cdot T^0 = L^1 \cdot M^1 \cdot T^{-2} \cdot (L^0 \cdot M^1 \cdot T^0)^A \cdot (L^1 \cdot M^0 \cdot T^0)^B \cdot (L^0 \cdot M^0 \cdot T^{-1})^C \qquad (5.19)$$

By comparing the exponent of powers with appropriate dimensions:

for $T \rightarrow 0 = \text{-}2\text{-}C \Rightarrow C = \text{-}2$
for $M \rightarrow 0 = 1 + A \Rightarrow A = \text{-}1$
for $L \rightarrow 0 = 1 + B \Rightarrow B = \text{-}1$

the following is obtained:

$$\pi_4 = F \cdot m^{-1} \cdot y^{-1} \cdot \omega^{-2} \qquad (5.20)$$

and finally:

$$\pi_4 = \frac{F}{m \cdot y \cdot \omega^2} \qquad (5.21)$$

Thus, a functional relationship that binds all dimensionless variables can be determined as:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4) \qquad (5.22)$$

which, when expanded, takes the following dimensionless form:

$$\tau_W \cdot \omega = f\left(\frac{I}{m \cdot y^2}, \frac{M}{m \cdot y^2 \cdot \omega^2}, \frac{F}{m \cdot y \cdot \omega^2}\right) \qquad (5.23)$$

and dimensional form:

$$\tau_W = f\left(\frac{I}{m \cdot y^2}, \frac{M}{m \cdot y^2 \cdot \omega^2}, \frac{F}{m \cdot y \cdot \omega^2}\right) \cdot \frac{1}{\omega} \qquad (5.24)$$

In the next step, by introducing the replacement dimensionless variable $\pi_{2\text{-}3}$, which is determined from the ratio of the variables π_3 and π_2 i.e. π_3/π_2, one obtains:

$$\tau_W = f\left(\frac{M}{I \cdot \omega^2}, \frac{F}{m \cdot y \cdot \omega^2}\right) \cdot \frac{1}{\omega} \qquad (5.25)$$

Analysing the expression (5.25) for the fatigue durability of the propulsion shaft, it can be seen that the dimensionless modulus $\pi_{2\text{-}3}$ has the form of a Newton number for its rotational motion $Ne_{(ROT)}$, while the dimensionless modulus π_4 has the form of a Newton number for the transverse movement of the shaft $Ne_{(TRANS)}$[7]. The physical sense of the first one is determined by the ratio of the transmitted torque to the accumulated kinetic energy, and the second by the ratio of the forces acting on the shaft: bending to centrifugal forces. In such a situation, formula (5.25) takes the final form:

$$\tau_W = f\,(Ne_{(ROT)}\,,\, Ne_{(TRANS)}) \cdot \frac{1}{\omega} \tag{5.26}$$

CRITERIA OF SIMILARITY

Considering the problem of the geometric, kinematic, and dynamic similarities of the fatigue process of the physical model (*M*) of the propulsion shaft and its real ship counterpart (*R*), which is made of the same material and which is subjected to analogous forces and moments, appropriate similarity scales can be defined, according to which the results obtained from the research of the physical model can be transferred to the real object:

- length scale (diameter) $S_{L/D}$:

$$\frac{L_R}{L_M} = \frac{D_R}{D_M} \rightarrow L_R = L_M \cdot \frac{D_R}{D_M} \rightarrow L_R = 8 \cdot D_R \tag{5.27}$$

where:

L_R – length of the real object propulsion shaft,
L_M – length of the propulsion shaft of the physical model (64 mm),
D_R – real object propulsion shaft diameter,
D_M – diameter of the propulsion shaft of the physical model (8 mm).

- scale of masses (moments of inertia) $S_{m/I}$:

$$\frac{m_R}{m_M} = \frac{I_R}{I_M} \rightarrow m_R = m_M \cdot \frac{I_R}{I_M} \rightarrow m_R = 0.115\text{mm}^{-2} \cdot I_R \tag{5.28}$$

where:

m_R – propulsion shaft mass of the real object,
m_M – mass of the propulsion shaft of the physical model (0.028 kg),
I_R – moment of inertia of the real object's propulsion shaft,
I_M – moment of inertia of the propulsion shaft of the physical model (0.244 kg·mm2).

- speed scale $S_{\omega/u}$:

$$\frac{\omega_R}{\omega_M} = \frac{u_R}{u_M} \rightarrow \omega_R = \omega_M \cdot \frac{u_R}{u_M} \rightarrow \omega_R = 250\text{m}^{-1} \cdot u_R \tag{5.29}$$

where:

ω_R – angular speed of the real object's propulsion shaft,
ω_M – angular velocity of the propulsion shaft of the physical model (157 s^{-1}),
u_R – peripheral speed of the propulsion shaft of the real object,
u_M – peripheral speed of the propulsion shaft of the physical model (0.628 m/s).

The comparison of the proposed scales of similarity is presented in Figure 1, with the example of a direct propulsion system, typical for cargo sea vessels.

7 Newton's Power Number, which, for mechanical systems is the ratio of the force (moment) of the resistance to motion to the force (moment) of inertia, can be determined from Newton's Second Law of Dynamics for the translational (rotational) motion of a rigid body.

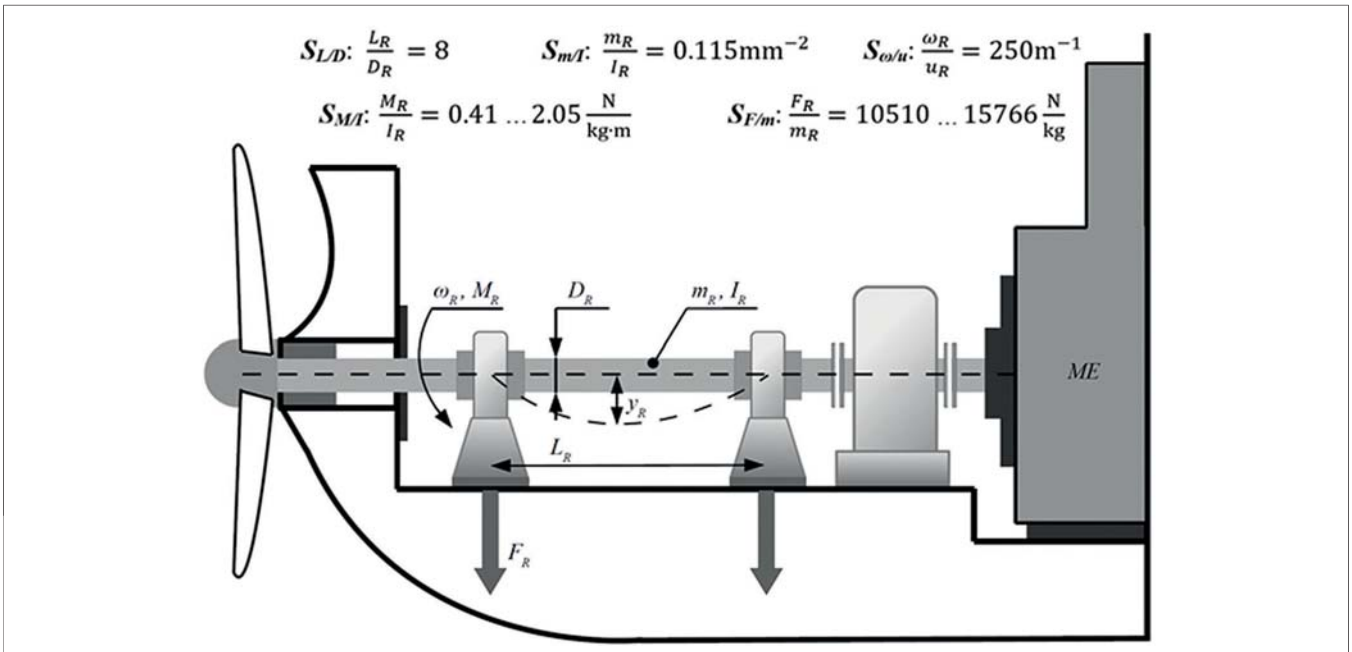

Fig. 1. Graphical interpretation of the applied scales of similarity of the physical model of the rotary propulsion shaft to the full-size ship shaft: D_R – propulsion shaft diameter, F_R – bending force acting on the propulsion shaft, I_R – moment of inertia of the propulsion shaft, L_R – propulsion shaft length, m_R – propulsion shaft mass, ME – main propulsion internal combustion engine, M_R – torque transmitted by the propulsion shaft, ω_R – angular speed of the propulsion shaft, y_R – deflection of the propulsion shaft

On the other hand, the basic invariant of the dynamic similarities of the physical model of the propulsion shaft and its real ship counterpart will be the Newton number considered in the scope of shaft rotation:

• Newton's number for a real object in rotation:

$$Ne_{R(ROT)} = \frac{M_R}{I_R \cdot \omega_R^2} \tag{5.30}$$

where:

- M_R – torque transmitted by the propulsion shaft of the real object,
- I_R – moment of inertia of the real object's propulsion shaft,
- ω_R – angular speed of the real object's propulsion shaft.

• Newton's number for a physical model in rotational motion:

$$Ne_{M(ROT)} = \frac{M_M}{I_M \cdot \omega_M^2} \tag{5.31}$$

where:

- M_M – torque transmitted by the propulsion shaft of the physical model,
- I_M – moment of inertia of the propulsion shaft of the physical model,
- ω_M – angular velocity of the propulsion shaft of the physical model.

The following condition follows from the identity of these numbers: $Ne_{R(ROT)} = Ne_{M(ROT)}$. Assuming that the values of the angular velocity (ω_R, ω_M) for the real object and its physical model, and the range of variability of the torque transmitted by the propulsion shaft of the physical model are invariant $M_M = 0.1 \ldots 0.5$ N·m, the scale of the $\boldsymbol{S_{M/I}}$ torques reflects the following relationship:

$$M_R = I_R \cdot \frac{M_M}{I_M} \rightarrow M_R = 0.41 \ldots 2.05 \frac{\mathrm{N}}{\mathrm{kg \cdot m}} \cdot I_R \tag{5.32}$$

Another analysed invariant of dynamic similarities of the physical model of the prop shaft and its real counterpart is the Newton number considered for the transverse movement of the shaft:

• Newton's number for a real object in lateral motion:

$$Ne_{R(TRANS)} = \frac{F_R}{m_R \cdot y_R \cdot \omega_R^2} \tag{5.33}$$

where:

- F_R – bending force acting on the propulsion shaft of the real object,
- m_R – mass of the real object's propulsion shaft,
- y_R – deflection of the real object's propulsion shaft.

• Newton's number for a physical model in lateral motion:

$$Ne_{M(TRANS)} = \frac{F_M}{m_M \cdot y_M \cdot \omega_M^2} \tag{5.34}$$

where:

- F_M – bending force acting on the propulsion shaft of the physical model,
- m_M – mass of the propulsion shaft of the physical model,
- y_M – deflection of the propulsion shaft of the physical model.

The identification of these numbers requires the following condition: $Ne_{R(TRANS)} = Ne_{M(TRANS)}$. By making analogous transformations and assuming that the deflection (y_M, y_R) and the angular velocity (ω_R, ω_M) of the real object shaft and its physical model are invariant, as well as assuming that the range of variability of the bending force acting on the propulsion shaft of the physical model $F_M = 294.3 \ldots 441.45$ N, the relationship representing the scale of bending forces $\boldsymbol{S_{F/m}}$ is obtained in the following form:

$$F_R = m_R \cdot \frac{F_M}{m_M} \rightarrow F_R = 10510 \ldots 15766 \frac{\mathrm{N}}{\mathrm{kg}} \cdot m_R \tag{5.35}$$

FINAL REMARKS AND CONCLUSIONS

The method of mathematical description of the fatigue life of the rotary propulsion shaft presented in this article by means of designated Newton dynamic similarity numbers enables the transfer of the diagnostic test results of its physical model to full-size ship shafts, while considering appropriate geometric and kinematic similarity criteria. However, it should be noted that the dimensional method used for identification purposes has significant limitations, as it does not penetrate into the essence of mechanical fatigue as a physical phenomenon, but only determines arbitrarily (often intuitively) the physical quantities that affect its course [12]. Hence, the dimensional analysis of the physical quantities of marginal or key significance can be accidentally included or excluded, respectively. Only their dimensions are considered, which makes it impossible to collect numerical data on the determined invariants of physical similarities. The results obtained in this way should always be confirmed experimentally or theoretically (from the analysis of mathematical equations resulting from the laws of physics).

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