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EQUILIBRIUM PRICE – MODELLING, FORECAST AND APPLICATION

Abstract: Modern economy is characterized by growing importance of prices. The knowledge of the general rules of price mechanism and price researches allows enterprises for foreseeing the equilibrium price and making profitable decisions, especially on the market in a homogeneous oligopoly, where entrepreneurs have to take competitors' reaction into account. An econometric dynamic model of gasoline equilibrium prices, on the market of liquid fuel in Poland, enables to determine the factor elasticity of gasoline prices and seasonal effects. It also allows for constructing the accurate, conditional, short-run gasoline prices forecast.

Key words: equilibrium price, forecasting, fuel prices.

1. Introduction

One of the features of modern economy is the fact that prices importance grows more and more. The reason for it is technological development and innovations. They contribute to introducing new products which have similar qualities. Consequently, differences among market sections are fading away and demand is getting more price-elastic. Finally when there is a production surplus at many markets, the weight of prices in the sale, in the market share and in the profit control is even bigger.

The price analysis and forecasts are used more often at the oligopolistic markets where other marketing tool values are relatively low. The knowledge of the general rules of price mechanism makes enterprises foresee the equilibrium price and influence price modifications.

The knowledge of equilibrium prices and quickness of adaptation to these prices is very important, especially for oligopolistic markets where entrepreneurs have to take competitors' reaction into account. Knowing that prices are falling down, an enterprise with a smaller market share should adapt them faster. As a result, the company makes bigger sales. If marginal costs are fixed, adaptation before the leader[1] makes oligopolist's profits higher. If the prices are increasing, an oligopolist should wait for the reaction of the leader.

[1] Enterprises with higher market share or/and lower marginal costs.

The main goal of the article is to construct a market price model and show that forecasts of market prices can be used for efficient price management in an enterprise operating on the market in a homogeneous oligopoly.

2. Equilibrium prices – theoretical approach

Analyzing different types of a market structure it can be noticed that equilibrium prices react in the same way. Considering, for instance, demand growth (or supply decline) equilibrium price growth will observed. It does not matter if an enterprise operates on a perfect competition market or under conditions of pure monopoly (see Figure 1a, b). Oligopoly is a market structure existing somewhere between perfect competition and pure monopoly. That is why the response of equilibrium prices to demand or supply changes is the same.

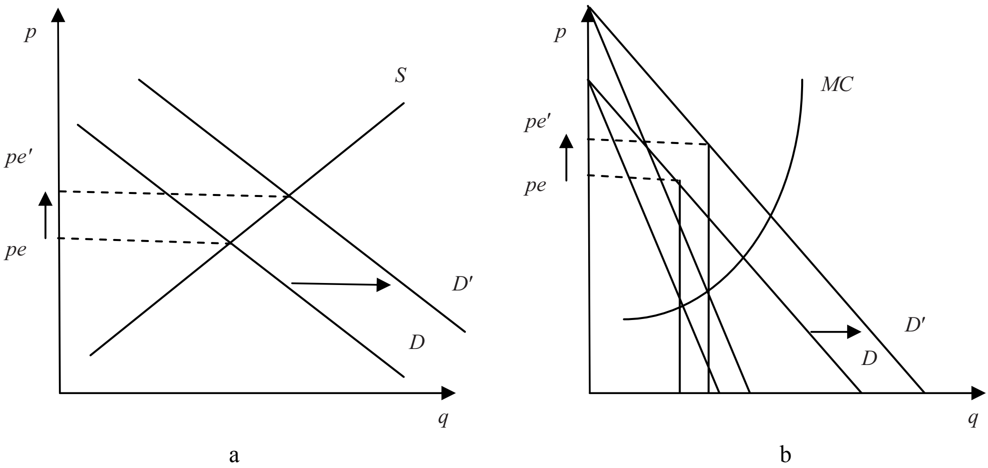

Figure 1. Reaction of equilibrium prices depending on a market structure (in the case of demand growth); a – perfect competition, b – pure monopoly

Source: own research.

If an equilibrium price model could be constructed and used for the prediction of a further progress of these prices, an enterprise will able to react much quicker. It is especially important when competitors' reactions should be considered.

The market of liquid fuel in Poland is an example of a market where price reactions are very significant. There are two main suppliers on this market and liquid fuel is impossible to ascribe to any of them. Thus, the market of fuel can be recognized as a homogeneous oligopoly. Consequently, fuel prices are the main competition tools. To construct a model of equilibrium prices, demand function and supply function should be determined first. Demand for fuel in Poland depends, most of all, on the average retail fuel prices (P) and on the average wages of consumers (W)[2]. Fuel supply depends, most of all, on retail fuel prices (P), world prices (WP), USD rates (USD) and taxes (excise – EX). Thus, the general functions of demand and supply for fuel can be presented as follows:

$$D_t = f(P_t, W_{t-i}), \tag{1}$$

$$S_t = f(P_t, WPt, USD_{t-i}, EX_t). \tag{2}$$

Equation (1): Some delay in economy activity influencing demand for fuel has been assumed. Equation (2): It has been assumed that USD variable is lagged, which is related with trade agreement.

Regular gasoline ($R95$) is assumed to be the dependent variable. The period of the analysis is 2004m1 – 2008m4. If demand is compared with supply, the following equilibrium price function can be achieved:

$$R95_t = f(W_{t-i}, RW95_t, USD_{t-i}, EX_t). \tag{3}$$

The average wages (W_{t-i}) became insignificant and were excluded from the analysis. In the analyzed period, the level of the gasoline excise changed a few times (see Figure 2). Because of its small variability, it is impossible to show its influence on the gasoline equilibrium prices. That is why variable EX_t was also excluded from the analysis[3].

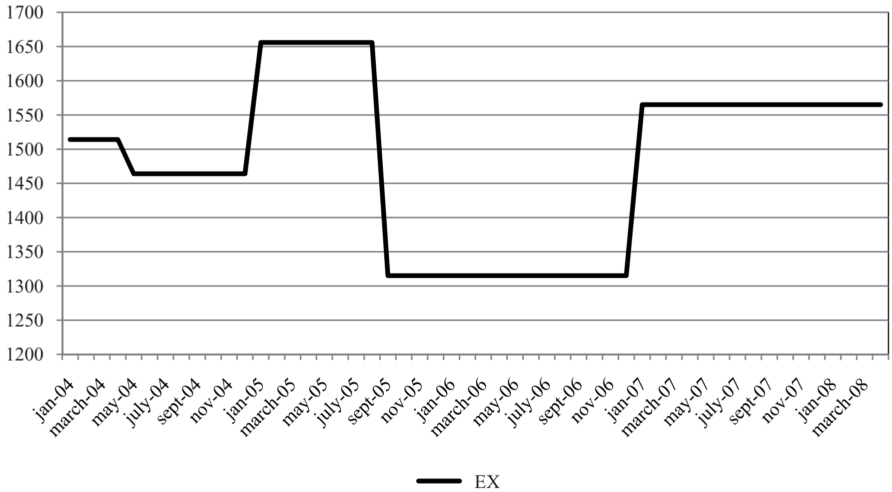

Figure 2. The gasoline excise in PLN. Data collected from Lotos Group

Source: own research.

[2] According to the previous research: [Sobiechowska-Ziegert 2002].

[3] The influence of the taxes was eliminated using the following formula: $R95_t = \frac{R95_{retail}}{1.22} - EX_t$.

The data collected, first of all, allows for investigating gasoline market price adjustment. The idea was invented by Walras [Klimczak 1993, p. 97] and developed by Nerlove [Goldberger 1972, p. 352]. It consists in market price adjusting to the equilibrium level. From the econometric point of view, the lagged dependent variable should be included among explanatory ones.

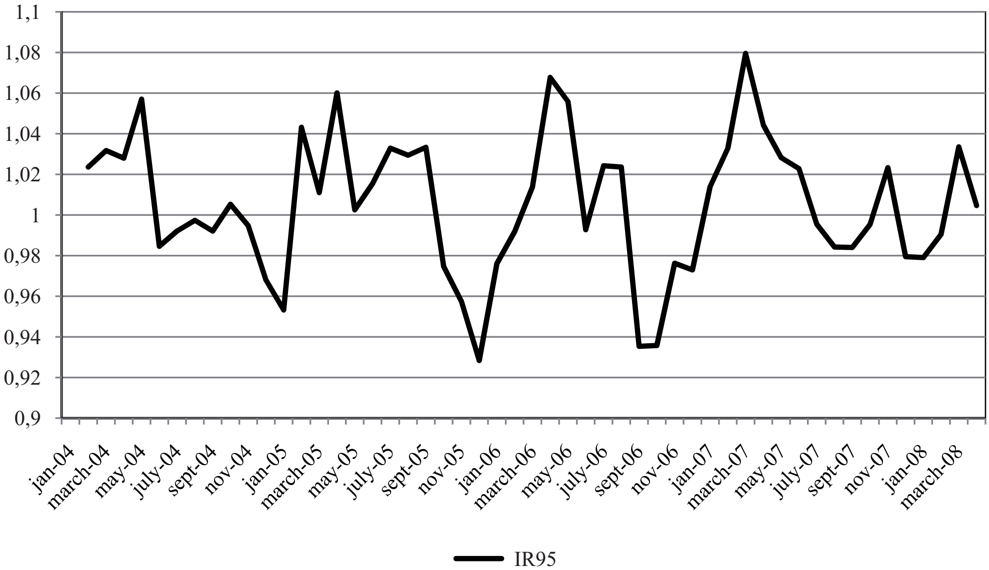

Figure 3. Monthly dynamics of fuel prices in Poland (previous month = 1.000)

Source: http://www.autocentrum.pl/tankuj-z-nami/.

Considering the monthly frequency of the statistical data and some variations during the examined period (see Figure 3), an econometric model of market fuel prices also including seasonal dummies can be constructed. It can be presented as follows:

$$y_t = b_0 + ay_{t-1} + \sum_{i=1}^{k} b_i x_{ti} + \sum_{j=1}^{11} c_j V_{tj} + u_t, \tag{4}$$

where: V_{tj} – seasonal dummies relative to the last season.

3. Econometric model of the gasoline equilibrium price

To make interpretation easier and to test statistical hypothesis, the power-exponential form of the model (4) can be used:

$$y_t = e^{b_0} \cdot y_{t-1}^{a} \cdot \prod_{i=1}^{k} x_{ti}^{b_i} \cdot e^{\sum_{j=1}^{11} c_j V_{tj}} \cdot e^{u_t}. \tag{5}$$

Apart from the appropriate factor elasticity of gasoline prices, the model presented above allows for determining seasonal effects as well. It should be remembered, however, that in the case of dynamic models, the following equation structure should be solved [Ossowski 2007]:

$$\begin{aligned} e_1 &= a \cdot e_{12} + c_1, \\ e_2 &= a \cdot e_1 + c_2, \\ e_3 &= a \cdot e_2 + c_3, \\ &\ldots\ldots\ldots\ldots\ldots, \\ e_{12} &= a \cdot e_{11} + c_{12}, \end{aligned} \quad (6)$$

where: e_j – seasonal deviation in month j.

To solve the equation structure given above, the value of e_{12} has to be determined according to the following formula:

$$e_{12} = \frac{a^{11}c_1 + a^{10}c_2 + a^9 c_3 + \ldots + ac_{11} + c_{12}}{1 - a^{12}}. \quad (7)$$

After the value of the equation (7) is determined, it is placed in the (6) structure, and the following seasonal effects are calculated. Relative seasonal deviations can be evaluated using the following formula:

$$Efs_j = \frac{p_t - Trsw(t, x_{ti})_t}{Trsw(t, x_{ti})_t} = \left(e^{e_j} - 1\right) \cdot 100\%, \quad (8)$$

where: Efs_j – seasonal effect in j-th month; $Trsw(t, x_{ti})_t$ – switching trend, determined by the factors in the model. The price levels from the following months relative to this trend are compared.

Substituting the appropriate symbols, finally the following equation can be obtained:

$$R95_t = e^{b_0} \cdot R95_{t-1}^{a} \cdot RW95_t^{b_1} \cdot USD_{t-3}^{b_2} \cdot e^{\sum_{j=1}^{11} c_j V_{tj}} \cdot e^{u_t}, \quad (9)$$

where: $R95_t$ – regular gasoline average price in Poland in PLN/l, $RW95_t$ – regular gasoline average world price in USD/t, USD_{t-3} – USD rate in PLN (this variable is assumed to be lagged, which is relating with trade agreements).

Using the OLS method, the equation (9) should be transformed into linear form. It can be done by applying natural logarithms on both sites:

$$\ln R95_t = b_0 + a \ln R95_{t-1} + b_1 \ln RW95_t + b_2 \ln USD_{t-3} + \sum_{j=1}^{11} c_j V_{tj} + u_t. \quad (10)$$

After estimation the following results were received (Table 1).

Table 1. Estimation results. Based on the OLS estimation using MFit

Parameter	Coefficient	*T*-ratio
b_0	−2.9704	−7.116
a	0.4959	8.906
b_1	0.4317	8.058
b_2	0.4493	4.750
c_1	−0.0877	−4.852
c_5	0.0448	2.420
c_9	0.0443	2.438

Source: own research.

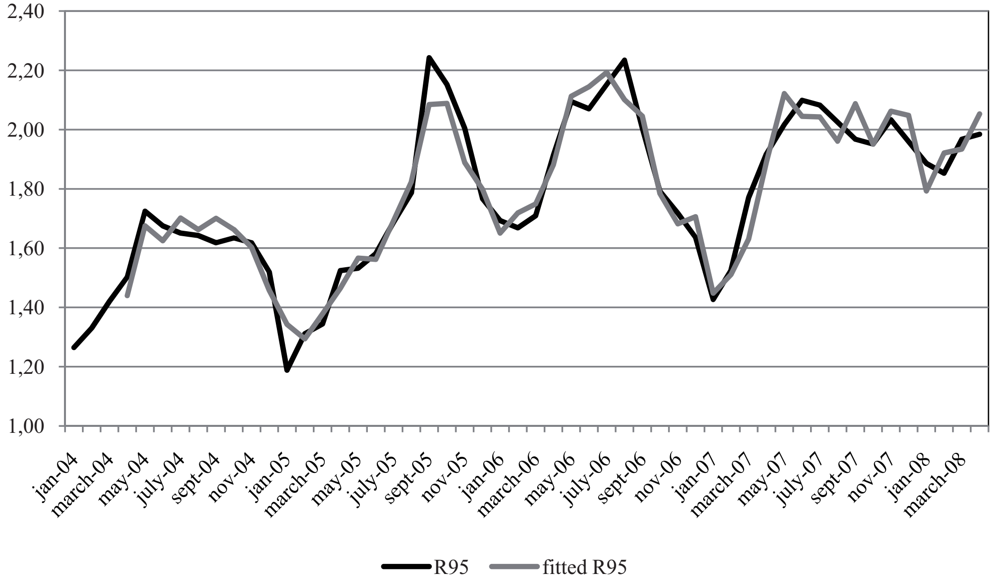

Figure 4. Empirical and theoretical values of gasoline prices

Source: own research, based on Table 1.

Looking at the *T*-ratio and assuming the standard significance level $\alpha = 0.05$, it can be noticed that the parameters are significantly different from zero (see Table 1). Analyzing goodness of fit measures and other statistics (see Table 2), it can be concluded that about 93% of the total variation of gasoline prices has been explained by the presented regression. On the basis of the standard error of regression, it can be estimated that the average ratio of actual values in fitted values is about 96-104%, which means that the above model is fit well. The correctness of the price model has also been confirmed by serial correlation test statistic, functional form statistic, normality and heteroscedasticity test statistic. Assuming standard significance level 0.05

there is no reason to reject null hypothesis about the lack of autocorrelated residuals, about functional form, normality of random disturbance term and constant variance.

Table 2. Goodness of fit measures and test statistics

Measure/test statistic	Value
R^2	0.93030
S_e	0.040802
Dh-statistic	–1.2392 [0.215]
Ramsey's χ^2	1.8891 [0.169]
Jarque's and Berr's χ^2	1.6449 [0.439]
White's χ^2	0.57804 [0.447]

Source: own research.

The above results allow for drawing the following conclusions about the influence of explanatory variables on gasoline prices:

- short run elasticity of gasoline prices $E_{R95(RW95)}$ = 0.4317 means that 1% of world gasoline prices increase causes the 0.43% prices increase in Polish market;
- long run elasticity of gasoline prices $E^L{}_{R95(RW95)} = \dfrac{0.4317}{1-0.4959} = 0,856$ means that, holding all else constant, 1% of world gasoline prices increase causes the 0.86% equilibrium price increase in Polish market;
- short run elasticity of gasoline prices $E_{R95(USDt-3)}$ = 0.4493 means that, holding all else constant, 1% of USD rate increase in time t causes the 0.45% gasoline prices increase in Polish market in the next quarter;

Table 3. Seasonal effect

Period	Seasonality relative index
January	–8.21%
February	–4.16%
March	–2.09%
April	–1.04%
May	4.04%
June	1.99%
July	0.98%
August	0.48%
September	4.78%
October	2.34%
November	1.15%
December	0.41%

Source: own research.

- long run elasticity of gasoline prices $E^{L}_{R95(USD_{t-s})} = \frac{0.4493}{1-0.4959} = 0,891$ means that, holding all else constant, 1% of USD rate increase leads to the average equilibrium price increase of about 0.89% in the next quarter.

Seasonal deviations calculated from the formula (8) are presented in Table 3.

4. Short-run gasoline price forecasting

The short-run forecast of gasoline prices will be determined from the following formula:

$$\ln R95_T = \hat{b}_0 + \hat{a}\ln R95_n + \hat{b}_1 \ln RW95_T + \hat{b}_2 \ln USD_{T-3} + \sum_{j=1}^{11} \hat{c}_j \, v_{Tj}. \quad (11)$$

To use the above model for the prediction of gasoline prices for 2008m5, the future values of explanatory variables should be calculated first. $R95_n$ variable is the last observable value of gasoline prices, USD_{T-3} variable is the monthly average USD rate from February. The only problem is to set the value of the $RW95_T$ variable. The possible solution in this case is to use the time trend model or the autoregressive model of this variable. Taking into account the fact that the time trend model of regular gasoline world prices did not describe empirical data well, the autoregressive model of this variable was used:

$$RW95_t = b_0 + b_1 RW95_{t-1} + u_t, \quad (12)$$

where: $RW95_{t-1}$ – regular gasoline world prices from the previous period.

The estimation results and the verification measures of model (12) are presented in Table 4. According to the data presented in Table 4, the model of regular gasoline world prices (12) can be good enough to predict values of the explanatory variable for the following month.

Table 4. Estimation results and verification measures. Based on model (12)

Parameter/measure/test statistic	Value [Prob] (T-ratio)
b_0	39.1560 (1.2380)
b_1	0.05323 (17.8998)
R^2	0.8674
S_e	56.1095
V	0.0955
Dh-statistic	1.4043 [0.160]
Ramsey's χ^2	0.12437 [0.724]
Jarque's and Berr's χ^2	3.7006 [0.157]
White's χ^2	1.8462 [0.174]

Source: own research.

The forecast of the world gasoline prices can be calculated from the following formula:

$$RW95_T = \hat{b}_0 + \hat{b}_1 RW95_n, \quad (13)$$

where: $RW95_n$ – the last observable value of the gasoline world prices.

The results of the forecast of the $RW95_T$ variable are presented in Table 5.

Table 5. Forecast results. Based on model (13)

Forecasting period	Forecast [USD/t]	Standard error of the forecast [USD/t]	Relative mean prediction error [%]
	(1)	(2)	(2)/(1)
2008m5	924.2644	59.6977	6.46

Source: own research.

The above results, especially the relative mean prediction error, allow for concluding that the above forecast is permissible (6.46% < 10%). That is why it can be used for the prediction of gasoline prices. The prediction results, using model (11), were presented in Table 6. Under the condition that retail world gasoline prices will achieve the level of 924.26 USD/t, gasoline prices in Poland will reach the level of 2.14 PLN/l.

Table 6. The gasoline price forecast results. Based on model (11)

Forecasting period	Forecast [PLN/l]	Standard error of the forecast PLN/l]	Relative mean prediction error [%]
	(1)	(2)	(2)/(1)
2008m5	2.14	0.10	4.72

Source: own research.

From the point of view of the permissibility, the above forecast can be qualified as a good forecast, because the relative prediction error is less than 5% [Zeliaś et al. 2003]. Using standard error of the forecast, the prediction interval of gasoline prices can be calculated (see Table 7).

Table 7. The 95% prediction interval for gasoline prices. Based on Table 6

Forecasting period	*T*-Statistic	Lower limit [PLN/l]	Upper limit [PLN/l]
2008m5	2.018082	1.94	2.34

Source: own research.

To determine the forecast quality, the ex-post errors were calculated. Analyzing the measures presented in Table 8, it can be concluded that the gasoline prices forecast is accurate. The forecast error makes only 1.42% of the real gasoline price. Moreover, the real and the forecasting prices change directions are the same. Forecasting prices are higher than the last observable value. It implies that an enterprise with smaller market share (the follower) should wait for the move of the market leader. Eventually, the real value of gasoline prices is higher than the last observable one as well, which means that the decision made by the follower would be correct.

Table 8. Forecast quality measures

Forecasting period	Real value [PLN/l]	Ex-post error [PLN/l]	Relative ex-post error [%]	Real change of prices	Forecasting change of prices
2008m5	2.11	−0.03	1.42	(+)	(+)

Source: own research.

5. Conclusions

Analysts very rarely use only one forecasting method. In this case, to achieve future gasoline prices, taking into account seasonal deviations, some alternative ways can be used. One of the above mentioned methods is a time trend model or an autoregressive model with seasonality, or Winters' smoothing. The first two methods allow for estimating the standard error of the forecast but do not describe any factors influencing gasoline prices. The third, enables to calculate the mean forecast accuracy, for instant MSE, MPE, MAPE [Hanke 2005, p. 80] and evaluate the forecast usefulness but also does not determine any price factors. The choice of the appropriate method depends, most of all, on the purpose of the forecast. For price management, when the future price level is as much important as its change direction, an econometric model seems to be more preferable. It allows for recognizing main causes influencing equilibrium prices and reacting immediately after the movements observed on the market. For the purpose of the analysis, the time trend model and the autoregressive model of gasoline prices were constructed. None of them, however, was good enough for the forecasting. In the time trend model there was observed serially correlated errors, whereas in the autoregressive model there was violated the assumption of the error term normality. Although it is possible to produce a forecast with autocorrelated errors [Schmidt 2005, p. 315] and the assumption of normality is not required for the OLS estimation [Studenmund 2006, p. 95], both models were a poor description of the data generating process. Comparing Akaike's Information Criterion and Schwarz Criterion for the econo-metric specifications used for the analysis, the best model of gasoline equilibrium prices was the cause-effect model presented in the article (equation (10)). Alternatively Winters' smoothing method with experimentally selected weights (by minimizing a measure of the forecast error) was also employed. Mean forecasting errors were, however, not satisfying enough to use it for the prediction.

Summarizing, the above presented model seems to be worth of the recommendation in the case of the short-run forecasting of gasoline equilibrium prices. The model allowed for achieving the accurate forecast and would enable the follower to make the right decision about prices. It should be remembered, however, that the application of cause-effect econometric models needs the updating of the relations among the variables. That is why some market monitoring system should be constructed. Its principal task should be collecting the appropriate data, verifying relations among variables and checking forecast accuracy permanently. Still, the main problem remains the calculating future values of the explanatory variables. To make forecasting process easier, autoregressive models of the explanatory variables or the time trend models can be used.

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CENA RÓWNOWAGI – MODELOWANIE, PROGNOZA I ZASTOSOWANIE

Streszczenie: nowoczesną gospodarkę charakteryzuje wzrastające znaczenie cen. Znajomość ogólnych zasad mechanizmu kształtującego ceny oraz analizy cen umożliwiają przedsiębiorstwom przewidzenie ceny równowagi i podjęcie przynoszącej zyski decyzji, zwłaszcza na rynku oligopolu homogenicznego, gdzie przedsiębiorcy muszą brać pod uwagę reakcję konkurencji. Dynamiczny model ekonometryczny cen równowagi benzyny, na rynku paliw płynnych w Polsce, umożliwia wyznaczenie czynnikowej elastyczności cen benzyny oraz efektów sezonowych. Pozwala również na skonstruowanie dokładnej, warunkowej, krótkoterminowej prognozy cen.