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Estimation of the uncertainty of the LEM CV 3-500 transducers conversion function

***Abstract**. The paper presents the methodology for estimating uncertainty of a voltage transducer conversion function based on the example of LEM CV 3-500 transducer. The analysis was performed using the method based on the GUM Guide propagation of uncertainty law and the Monte Carlo numerical method. The article also presents a comparative analysis of the results of a voltage measurement evaluation done using the Monte Carlo simulation method, and results obtained with the use of the propagation of uncertainty law method, for a few selected voltage and frequency values.*

***Streszczenie.** W referacie zaprezentowano metodologię szacowania niepewności funkcji przetwarzania przetwornika napięciowego, na przykładzie przetwornika LEM CV 3-500. Analizę przeprowadzono przy wykorzystaniu metody opartej na Przewodniku GUM z zastosowaniem prawa propagacji niepewności oraz metody numerycznej Monte Carlo. W artykule przeprowadzono także analizę porównawczą wyników oceny pomiaru napięcia metodą symulacji Monte Carlo oraz wyników uzyskanych metodą związaną z prawem propagacji niepewności, dla kilku wybranych wartości napięcia oraz częstotliwości. (**Szacowanie niepewności funkcji przetwarzania przetwornika napięciowego na przykładzie przetwornika LEM CV 3-500**).*

Keywords: measurement uncertainty, Monte Carlo method, transducers.
Słowa kluczowe: niepewność pomiaru, metoda Monte Carlo, przetworniki pomiarowe.

Introduction

In accordance with the requirements of modern metrology [1], it is necessary to present measurement results with their estimated standard or expanded uncertainty [1, 2, 3, 4, 5]. Hence, in all fields of science and technology, such as bearing diagnosis [6], measurement of electrical quantities [7] and biomedical measurements [8]; it is necessary to estimate the uncertainty of the measurements.

Due to the extensive use of voltage converters in solving various problems of measurement, authors decided to develop a methodology for estimating the uncertainty of these transducers.

The paper presents the methodology for calculating the uncertainty of LEM CV 3-500 voltage transducers.

Two methods for determining the uncertainty of measurement are presented: traditional - the analytical method and Monte Carlo simulation - numerical method.

The analytical method is based on a convolution of the input distribution values, using a mathematical model. In this case, the designated measure of uncertainty is the expanded uncertainty, calculated as the product of the expansion coefficient k and the standard uncertainty value.

The numerical method is based on determining the uncertainty of measurement based on the extension range, which is determined by probability distribution of the measured value quantiles. The parameters of this distribution are: the expected value - as the estimate of the output value, the standard deviation and the confidence interval for a given probability level.

The study

The study was aimed at estimating the LEM CV 3-500 voltage transducers measurement uncertainty.

To estimate the uncertainty of the voltage transducer a measurement system consisting of a Fluke 5500A voltage calibrator and a Keithley 2002 multimeter was designed. A block diagram of the measurement system for testing the accuracy of the CV 3-500 transducer is shown in Fig. 1.

The tests were conducted for 6 frequency values: 50 Hz, 75 Hz, 120 Hz, 250 Hz, 500 Hz, 1000 Hz; at 6 different values of voltage: 20 V, 50 V, 100 V, 150 V, 200 V, 230 V for each of them. The measurements were repeated 50 times.

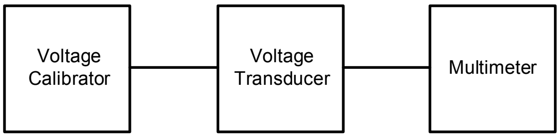

Fig. 1. A block diagram of the measurement system for testing the accuracy of the CV 3-500 transducer

The results of these studies allow to specify the uncertainty that can be expected when measuring voltage using a LEM CV 3-500 transducer.

The article presents the results of studies of 2 voltage levels: 50 V and 230 V; at 2 different frequencies: 50 Hz and 500 Hz.

Equations

The transducer output voltage u_{out} is dependent on input voltage u and the transducer conversion ratio k_u:

$$u_{out} = k_u \cdot u \quad (1)$$

Voltage measurement variance, assuming there is no correlation between the uncertainties of the measured values, in defined by the following relationship [1]:

$$u^2(u) = \left(\frac{\partial u}{\partial k_u}\right)^2 u^2(k_u) + \left(\frac{\partial u}{\partial u_{out}}\right)^2 u^2(u_{out}) \quad (2)$$

where: $u^2(u)$ - voltage measurement variance, $u^2(u_{out})$ – multimeter voltage measurement variance, $u^2(k_u)$ – transducer conversion ratio estimation variance.

Thus, in order to determine the variation of the voltage measurement $u^2(u)$ the variance of the voltage measurement $u^2(u_{out})$, and the transducer conversion ratio estimation variance $u^2(k_u)$ have to be assessed.

In order to estimate the variance $u^2(k_u)$ the following should be defined:

- calibrator output voltage variance u^2_{ku},
- multimeter voltage measurement variance, resulting from the scatter of results measured u^2_{ru},
- multimeter voltage measurement variance, resulting from the limiting error of the multimeter u^2_{mu}.

Calibrator output voltage variance u^2_k and the voltage measurement error resulting from the multimeter limiting error u^2_m was determined as type B variance, based on data provided by the manufacturer in the specification of the calibrator and multimeter, assuming rectangular probability distribution. Multimeter voltage measurement error resulting from the scatter $u^2(u_r)$ was determined as a type A variance, assuming normal probability distribution [1, 4].

The transducer conversion ratio k_u, according to formula (1), is determined by transducer input terminals voltage to transducer output terminals voltage ratio [4]. Thus, the transducer conversion ratio estimation is defined:

$$u^2(k_u) = c^2_{u_{out}} u^2(\bar{u}_{out}) + c^2_u u^2(\bar{u}) \tag{3}$$

where: $u(\bar{u}_{out})$ - the variance of the average voltage, measured by the Keithely 2002 multimeter, $u(\bar{u})$ – the variance of the average voltage at the transducer input terminals.

The sensitivity coefficients were defined:

$$c_{u_{wy}} = \frac{\partial k_u}{\partial u_{out}} = \frac{1}{u} \tag{4}$$

$$c_u = \frac{\partial k_u}{\partial u} = -\frac{u_{out}}{u^2} = -\frac{k_u}{u} \tag{5}$$

Transducer output terminals voltage variance is associated with the Keithley 2002 multimeter measurement uncertainty:

$$u^2(\bar{u}_{out}) = u_r^2 + u_m^2 \tag{6}$$

where: u_r - Keithley 2002 measurement uncertainty resulting from the measurement values spread, u_m - Keithley 2002 measurement uncertainty resulting from the limiting error, provided by the manufacturer.

For further calculations it is assumed, that:

$$u^2(u_{out}) = u^2(\bar{u}_{out}) \tag{7}$$

The Fluke 5500A calibrator was generating the input signal for the transducer. Average transducer input voltage variance $u^2(\bar{u})$ is defined by:

$$u^2(\bar{u}) = u_k^2 \tag{8}$$

where: u_k – uncertainty of the calibrator output voltage, provided by the manufacturer.

The tests were conducted for 6 frequency values for each of them at 6 different voltage values. The measurements were repeated 50 times. For each option, quantities were calculated in accordance with the relationships (3), (6), (7) and (8).

Then, based on equation (2), the complex voltage estimation variance $u^2(u)$ was calculated.

In order to estimate the uncertainty, a Monte Carlo simulation was then carried out in Microsoft Excel for $M = 10^4$ samples. The probability density function of the output voltage u_{out}, its expected value and the confidence interval was determined for a confidence level of p = 95%.

Example results of the calculations of voltage measurement complex variance are shown for 2 values of frequency: 50 Hz and 500 Hz at 2 voltage values: 50 V and 230 V.

Assuming a 95% confidence level and coverage factor k = 2, expanded uncertainty of the voltage measurement U_u was estimated:

$$U_u = k \cdot u(u) \tag{9}$$

Table 1 shows the uncertainty budget of current estimation for the frequency 50 Hz and voltage value 50 V.

Table 1. Uncertainty budget of current estimate for the frequency of 50 Hz and voltage value 50 V

Quantity X_n	Quantity estimation x_n	Standard variance $u^2(x_n)$	Probability distribution	Sensitivity coefficient c_n	Share in complex variance $u^2_n(y)$
u_{out}	1.00 V	3.74E-7 V^2	normal	50 V/V	9.35E-4 V^2
k_u	0.02 V/V	1.90E-10 V^2/V^2	normal	11500 V^2/V	2.51E-2 V^2
			Standard uncertainty $u(u)$		0.16 V
			Expanded uncertainty $U(u)$		0.32 V

Based on the calculations, the result of voltage measurement of 50 V for 50 Hz, at a given level of confidence and expansion coefficient, can be written as: U = (50.00 ± 0.32) V.

Based on the results of Monte Carlo [2] simulation numerical distribution functions were plotted. Fig. 2. shows an example numerical distribution function chart for u = 50.0 V, a frequency of 50 Hz and k_u = 0.02 V/V.

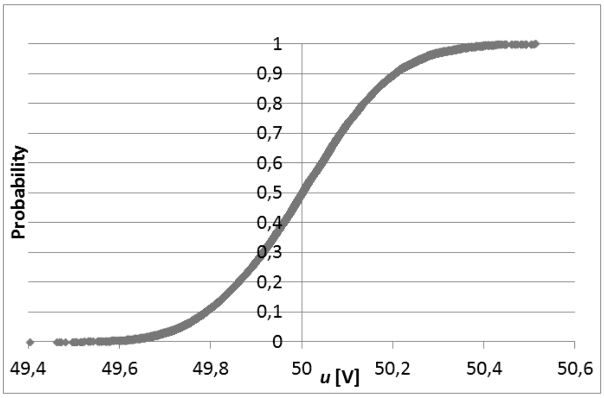

Fig. 2. Numeric distribution function for: u = 50 V, k_u = 0.02 V/V, f = 50 Hz

Based on the calculation results histograms were plotted as well. An example histogram for u = 50.0 V, k_u = 0.02 V/V and a frequency of 50 Hz is presented on Fig. 3.

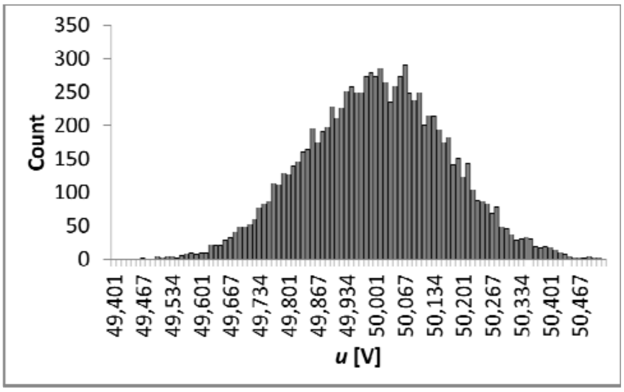

Fig. 3. Histogram for: u = 50.0 V, k_u = 0.02 V/V, f = 50 Hz

Uncertainty budget estimation was performed for a voltage value of 230 V at a frequency of 50 Hz. The results of uncertainty component calculations are shown in Table 2.

Table 2. Uncertainty budget of current estimate for the frequency of 50 Hz and voltage value 230 V

Quantity X_n	Quantity estimation x_n	Variance $u^2(x_n)$	Probability distribution	Sensitivity coefficient c_n	Share in complex variation $u^2_n(y)$
u_{out}	4.60 V	9.79E-6 V^2	normal	50 V/V	2.45E-2 V^2
k_u	0.02 V/V	2.13E-10 V^2/V^2	normal	11500 V^2/V	2.82E-2 V^2
			Standard uncertainty $u(u)$		0.23 V
			Expanded uncertainty $U(u)$		0.46 V

Measurement results for voltage of 230 V at a frequency of 50 Hz, assuming confidence level p = 95% and expansion coefficient k = 2, can be written as: U = (230.00 ± 0.46) V.

An example histogram for u_{out} equal to 230.0 V and k_u = 0.02 V/V and a frequency of 50 Hz is presented in Figure 4.

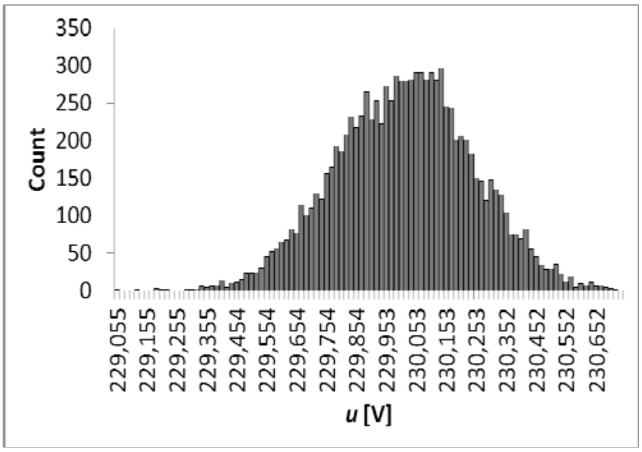

Fig. 4. Histogram for: u = 230.0 V, k_u = 0.02 V/V, f = 50 Hz

Uncertainty budget estimation for voltage value of 50 V at a frequency of 500 Hz is shown in Table 3.

Table 3. Uncertainty budget of current estimate for the frequency of 500 Hz and voltage value 50 V

Quantity X_n	Quantity estimation x_n	Variance $u^2(x_n)$	Probability distribution	Sensitivity coefficient c_n	Share in complex variation $u^2_n(y)$
u_{out}	1.00 V	4.24E-8 V^2	normal	50 V/V	1.06E-4 V^2
k_u	0.02 V/V	5.70E-11 V^2/V^2	normal	11500 V^2/V	7.54E-3 V^2
			Standard uncertainty $u(u)$		0.087 V
			Expanded uncertainty $U(u)$		0.17 V

Measurement results for voltage of 50 V at a frequency of 500 Hz, assuming confidence level p = 95% and expansion coefficient k = 2, can be written as: U = (50.00 ± 0.17) V.

An example histogram for u equal to 50.0 V and k_u = 0.02 V/V and a frequency of 500 Hz is presented in Figure 5.

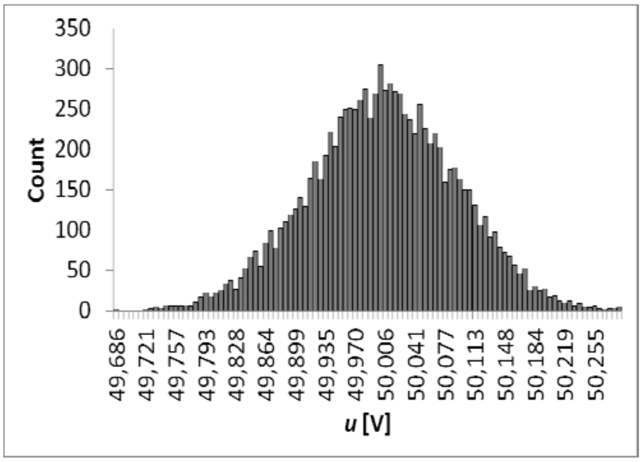

Fig. 5. Histogram for: u = 50.0 V, k_u = 0.02 V/V, f = 500 Hz

Uncertainty budget estimation was also performed for a voltage value of 230 V at a frequency of 500 Hz. The results of uncertainty component calculations are shown in Table 4.

Table 4. Uncertainty budget of current estimate for the frequency of 500 Hz and voltage value 230 V

Quantity X_n	Quantity estimation x_n	Variance $u^2(x_n)$	Probability distribution	Sensitivity coefficient c_n	Share in complex variation $u^2_n(y)$
u_{out}	4.60 V	4.82E-6 V^2	normal	50 V/V	1.21E-2 V^2
k_u	0.02 V/V	1.19E-10 V^2/V^2	normal	11500 V^2/V	1.57E-2 V^2
			Standard uncertainty $u(u)$		0.17 V
			Expanded uncertainty $U(u)$		0.33 V

Measurement results for voltage of 230 V at a frequency of 500 Hz, assuming confidence level p = 95% and expansion coefficient k = 2, can be written as: U = (230.00 ± 0.33) V.

An example histogram for u_{out} equal to 230.0 V and k_u = 0.02 V/V and a frequency of 500 Hz is presented in Figure 6.

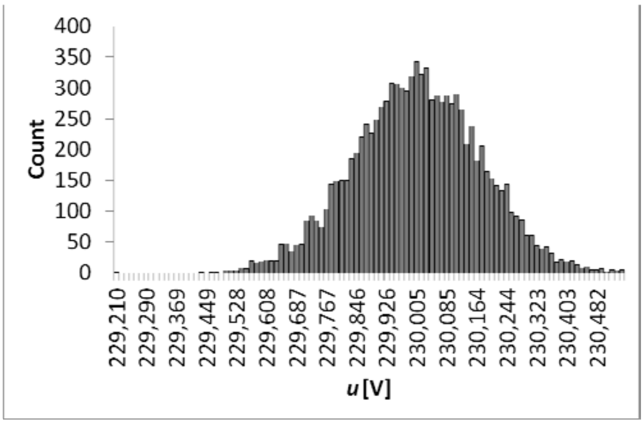

Fig. 6. Histogram for: u = 230.0 V, k_u = 0.02 V/V, f = 500 Hz

Table 5 summarizes the results of the extended uncertainty estimation using the traditional GUM method and the Monte Carlo simulation.

Table 5. Voltage estimate for the frequency of 50 Hz and 500 Hz, and voltage values: 50.0 V and 230.0 V

Voltage estimation u_{out}			
u_{out} [V]	f [Hz]	Traditional method [V]	Monte Carlo [V]
50.00	50	(50.00±0.32)	(50.00±0.31)
	500	(50.00±0.17)	(50.00±0.17)
230.00	50	(230.00±0.46)	(230.00±0.44)
	500	(230.00±0.33)	(230.00±0.33)

It can be observed, that based on the traditional method results, LEM CV 3-500 voltage transducer measurement uncertainty for 230 V at 50 Hz is equal to 0.20% of the measured voltage value, for 230 V at 500 Hz its 0.14%, for 50 V at 50 Hz its 0.64%, and for 50 V at 500 Hz its 0.34%. Lower uncertainty values can be observed for higher voltage value and higher frequency, that is 230 V and 500 Hz.

It can also be observed, that the uncertainty does not exceed 1.00% of the measured voltage value.

Summary

The article presents the problem of uncertainty estimation for voltage transducers LEM CV 3-500 processing functions using two methods: the method based on the GUM Guide using the law of propagation of uncertainty and the Monte Carlo numerical method.

Considering the results of both methods, it can be observed, that they coincide with each other.

The data obtained from this analysis allows for the estimation of measurement uncertainty for voltage transducers of this type, as well as provide a verification of the estimated uncertainty obtained by the traditional method, using the Monte Carlo method.

It can be concluded, that the estimated uncertainty of the processing function of the analyzed voltage transducer, allows for the validation of its use in measurement of voltage, with a maximal relative expanded uncertainty of 0.64%.

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