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### Experimental study on ice drift under the wind effect

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#### Abstract

This study aims at wind and free ice drift interaction, which is an important aspect in sea ice, and low flow inland waters. Ice drift is caused by dynamic balance of water drag, gravitational acceleration, resistance force and wind drag. To have a clear point of view on wind to ice interaction, the external forces for this experimental study were limited to wind effect. The experiments were conducted in the Institute of Hydro-Engineering of Polish Academy of Sciences (IHE PAS). The flume for performing the experiments was not previously dedicated to wind measurement, therefore, parts dedicated to the wind measurement had to be set up and built. The considered section of the flume was 0.6 m wide and 3.95 m long. The depth of water and wind sections were 0.595 m and 0.2 m, respectively. Initial water was considered for experiments under the influence of varying wind velocity fields with the approximate averaged values of 0.5, 1.1 and 1.5 m/s. To measure the wind velocity, the image processing method was selected, therefore, a two-dimensional Particle Image Velocimetry (PIV) technique was applied. For the seeding of the wind field, the water mist was chosen. It was considered as a novelty in the work since water mist was not used for PIV technique for this range of wind velocity, in the past literature. To measure the ice velocity, Particle Tracking Velocimetry (PTV) technique was applied. The PTV technique was performed by taking use of PTVlab software, considering different concentration of artificial ice and wind velocity. The results showed the relation between the concentration of ice, wind, and ice velocity.

**Keywords:** laboratory experiment, ice drift, wind drag, ice dynamics, PIV technique

#### 1. Introduction

Associated to river hydraulics, the interactive ice effect with a river is of importance, and in deep knowledge of river ice is needed (Ashton, 1986). Various types of ice may appear in different stages of ice process during wintertime, although, in this study merely ice run is

considered. Specifically, ice cover and ice pieces after breakup are simulated in an experimental study, under different wind velocities influences.

The surface ice run is under the control of wind and water drag forces, as well as the force of gravity. The ice run is also influenced by interaction of ice and hydraulic structures (Shen, Su and Liu, 2000). Surface ice run includes varying shapes and sizes of the pieces, which for surface ice transport and their interaction could be considered as a continuum in the natural rivers since the scale of a natural river channel to the ice pieces is larger (Shen, Shen and Tsai, 1990).

Lal and Shen (1993) featured the rate of work done by wind on ice to shear stress of the wind and the water velocity near the surface in turbulent fluctuations. Shen et al. (1995) improved ice process formulation for a channel network in the upper Niagra River, including wind effect. For the river hydraulics computation, the equations of continuity and momentum were solved. Shear tension of the air was considered for water and different ice concentrations. By considering an equilibrium state of the ice, the shear stress on top of the ice and that of between water and moving ice was assumed to be equal. Furthermore, the resistance for the momentum equation, was attributed to both underside of ice and bed resistances (Shen, Wang and Lal, 1995).

Image based velocimetry is considered as a non-invasive technique which provides continuous flow field which is in contrast with point-based results (Lin, Grundmann and Eltner, 2019). Imaged-based velocimetry technique includes various techniques; one of them is Particle Image Velocimetry (PIV) and the other is Particle Tracking Velocimetry (PTV). PIV technique is an Eulerian-based approach, while the PTV technique is based on a Lagrangian frame of reference; the former estimates velocities at image subregions and the latter calculates velocities based on the detected particle movements in a view field. In the PIV technique, the seeding material needs to reflect the flow field. In this technique, the velocity field needs to be traced and illuminated with the seeding material and a laser sheet, respectively. Each of the two pulses of the laser generated in regular intervals provides the condition to record the succeeding positions of the particles.

Both PIV and PTV are the most popular conventional image processing methods for measuring the flow fields. Choosing the proper algorithm is based on the tracer concentration in the flow field. PIV technique is suggested when a group of particles or other quantities related to the flow form a homogeneous field in successive images. In the case of low tracer concentration, it is useful to measure the displacement of individual particles. This low image density algorithm is referred to as Particle Tracking Velocimetry (Fujita, Muste and Kruger, 1998). PTV first needs to spot the possible tracer particles, and then find the trajectory of the particles. Compared to PIV, it is more suitable for changing environmental conditions (Lin, Grundmann and Eltner, 2019).

A seeding material for the PIV measurement in the wind field needs to be chosen, in a way to satisfy both wind velocity field measurements and experimental set up condition, e.g., the place for installing the required devices. There are related studies to PIV measurements; each of which is dedicated to various PIV measured fields. In the studies considering different fields to be measured with PIV technique, for each section separate type of seeding material is needed (Techet and McDonald, 2005; Zhang, Wang and Lee, 2008). It can be noticed that the



particles with higher density which are blown in the air are more probable to affect the velocity of the wind field; seeding particles in the wind field may act as such (Melling, 1997). This is a consideration that can be taken into account for identifying seeding materials. It should be noticed that seeding particle materials need to be large enough to scatter the laser light and small enough to properly track the flow velocity field. Apart from size of the seeding particle material, its choosing appeals specific concern to be chemically inactive, non-abrasive, enduring (steady and being constantly supplied), and non-poisonous. Fulfilling these characteristics, different materials can be suggested, which can be found in Melling (1997), for both liquid and gas fields. For this study different materials were considered for seeding the wind fields, with the aim of conducting the PIV technique measurement. Finally, based on the requirements of the laboratory conditions, water particles (water droplets) were used as the seeding material.

Dal Sasso et al. (2020) compared PTV and large-scale PIV techniques in terms of accuracy for different seeding conditions and case studies for calculating surface velocity. They noticed both techniques are sensitive to variation of dimension and dispersion of the seeding particles as well as seeding density for the image velocimetry. The authors mentioned that environmental noises, e.g., shadows and illumination, can negatively affect the seeding particles quantification. Furthermore, low seeding material resulted in incomplete flow field. They also suggested increasing the duration of the videotapes to reduce error and find optimal frame windows (Dal Sasso, Pizarro and Manfreda, 2020). Tauro and Salvatori (2017) analysed videos through PTV technique to estimate river surface velocities in comparison to radar data, noticing that PTV was not possible to be applied for videos related to night hours. They found a right angle for the optical sensor dedicated to captured videos for the most accurate velocity results. Also, fisheye lens led to less accurate velocity results and was intensely affected by illumination. They concluded PTV can be successful in low flow velocities (Tauro and Salvatori, 2017).

This study relates to the wind effect on ice drift. It includes experimental tests, which were performed in the wave flume located in the Institute of Hydro-Engineering of the Polish Academy of Sciences (IHE PAS). Ice sheet models of rectangular shape were prepared as propylene pallets. For the wind and water velocity measurement, PIV technique (Particle Image Velocimetry) was used. While the ice velocity was measured with PTV technique (Particle Tracking Velocimetry). Three different wind fields were considered for the experiments with adjusting a wind generator. The main objective of this study is to investigate the effect of wind on surface ice drift. In the past literature, there were not many studies related to the effect of wind on ice dynamics. Therefore, in this study, different parameters attributed to the freshwater ice dynamics under the wind influence are considered, including size and concentration of the ice as well as wind velocity.

This study concerns the effect of air movement on the ice transport, which is applicable in stationary water bodies, in which drag of wind dominates that of water. The appraised solutions and the research layout are original. The experimental research does not concern any commercial incentives and is self-funded research, thus the aim was to run a set of basic experiments, with cost considerations.

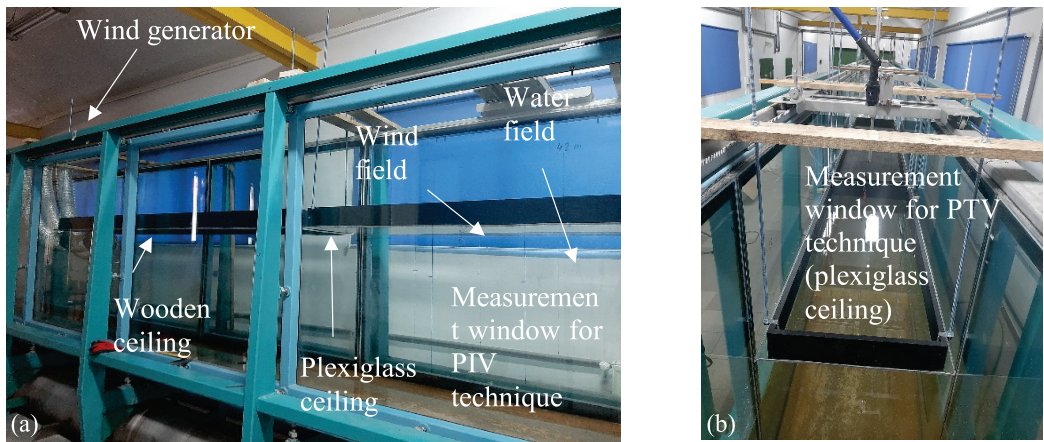
## 2. Experimental setup

Experiments were performed in the wave flume of Institute of Hydro-Engineering of Polish Academy of Sciences (IHE PAS). The flume was 64.1 m long and 0.6 m wide, as well as 1.4 m high. The aim of the experiments was to investigate the effect of wind on the ice movement that can be defined by Equation 1 (Kolerski, Shen and Kioka, 2013).

$$\vec{F}_a = [\rho_a C_a |\vec{W}| (W_x) N] \vec{i} + [\rho_a C_a |\vec{W}| (W_y) N] \vec{j} \quad [1]$$

$\vec{F}_a$  is the wind force exerted on ice.,  $\rho_a$ ,  $C_a$ , and  $\vec{W}$  are the air density, wind drag on ice coefficient, and wind speed vector at 10 m level above the water surface, respectively.

Different experiments with a variety of wind velocities were conducted to consider their impact on the tension imposed on ice and consequently its velocity. This laboratory in which the experiments were conducted is mainly equipped with a wave flume and it was not designed for experiments related to wind measurements. Therefore, extra preparation for provision a wind flume was done. This preparation of the wind flume included designing and installing a ceiling above the water flow and a wind generator with the aim of wind blowing above the water. In Figure 1, illustration of the flume is depicted, which shows both wind field and water field.



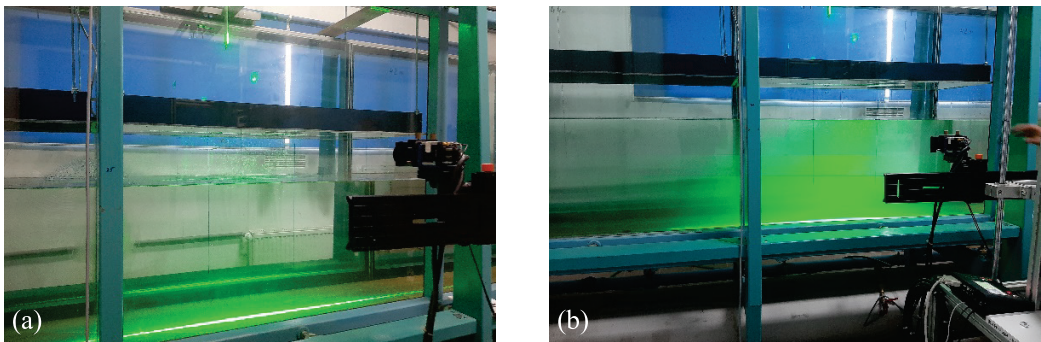
**Figure 1.** An illustration of the wind flume from the side (a), a view of plexiglass ceiling (b)

The wind generator was made from three fans and was located at the flume inlet. Three wind conditions were considered related to air flows from one fan, two, and three working fans (number of operating fans) (Figure 1). To isolate the wind, a covered area (wind flume height was 20 cm) with the help of the ceiling was considered. The measurement window for PTV technique was made of plexiglass (1.4 m long), to make ice pieces movements visible (Figure 1), and to transport the laser light (in the PIV measurement) through the glass. While the part of the ceiling at the immediate distance from the wind generator was made from wood (2.5 m long).

Zero water discharge was applied for all the experiments, although, the water velocity under the influence of wind velocity was calculated. Due to the experiments being conducted in room temperature, ice models were made from polypropylene. The ice pieces of 10×10 cm

size with the thickness of 1 cm were considered. Apart from the ice pieces, larger pieces of ice, indicating ice cover was considered. For the ice cover simulation, 4 pieces of polypropylene with lengths of edges of 50×40 cm, and the thickness of 1 cm were taken into account. To carry out the measurements for the ice pieces movements, the particle track velocimetry (PTV) was applied. This method tracks the route of individual particles which in this method are ice pieces. Based on this approach, the velocity of the pieces is to be calculated as a function of their displacement in specific timing, from the recorded videos during an experiment. The videos are captured from the top of the flume parallel to the top measurement window (plexiglass section) (Figure 1).

To carry out the measurement of the wind velocity and two-dimensional water velocity field, PIV technique was used. The method requires a computer connected to a camera to grab frames and software to analyse the series of pictures (Ettema et al., 1997). The measurements of these velocity fields were performed by a Flow Master PIV system. It included a CCD camera, with a resolution of 1280×1024 pixels, and a 50 mJ dual laser-head system (repetition of 50 Hz). The seeding materials for water and wind velocity were used to reflect the velocity field under the light of pulsed laser. The reflection was recorded by a camera as a pair of two single-exposure images. The time interval between two was 0.0018 s. Based on the time intervals between captured images and the displacement of the seeding particles, the velocity vectors were defined in so-called “interrogation windows” (selected subareas). The velocity measurements were performed in a plane parallel to the flow direction after calibration (see e.g. Biegowski, Paprota and Sulisz, 2020). In Figure 2, illuminated wind and water fields under the laser pulse are shown.



**Figure 2.** The illuminated wind field (a) and water field (b) by the laser light

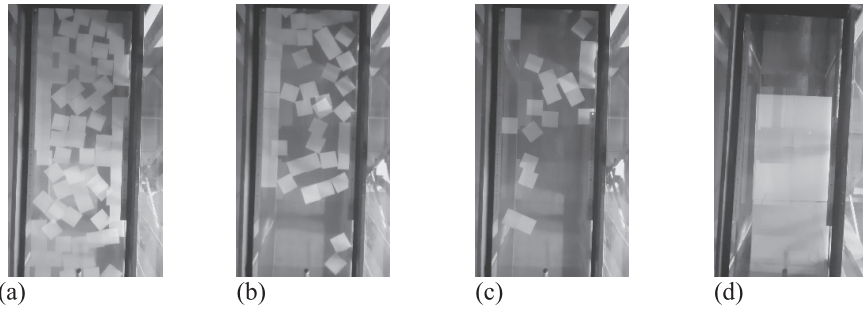
There were different choices of seeding materials for illuminating wind velocity field under laser light. At the aim of homogenous seeding of wind field, a large quantity of seeding particle material in the wind field was needed; therefore, erosion and pollution could happen, in case of using corruptive and pollutive materials as the seeding particle (Stanislas, Kompenhans and Westerweel, 2013). Materials such as, microballoon, Aluminum Oxide, olive oil, etc. were possible to be used (Melling, 1997). However, such materials were excluded from testing since the chances were high to stagnate the laboratory environment and the flume. For instance, using olive oil as the seeding particle could contaminate the flume, due to required long time to run the experiments (Stanislas, Kompenhans and Westerweel, 2013).



Based on what was mentioned above, other types of benign materials for the wind field reflection were considered. Firstly, a humidifier device was used to produce water vapour to see if the water vapor may be recognised by the camera. The results did not suffice spatial resolution (light reflection by the seeding particles), and the seeding particles were not successfully detected by the PIV camera. It means, the water vapour could not reflect the laser light properly, to make the wind field visible. Even if it did, it would rarely happen and in random areas of the wind field. It could be due to the low concentration of the water vapour in the wind field. For this experimental setup, it was not easy to achieve high concentration of water vapour. The next testing material was the smoke from the burning aromatic sticks to test if it can be detected by the camera. The material was found out to be ineffective due to being insufficiently particulate. This means that the produced particles were mainly grouped which would not cover a large part of a wind field. Lastly, water particles were tested, and seemed to be functioning. Water particles are advantageous due to not harming the laboratory environment, thereby a healthy environment would be created. On the other hand, water particles may menace the flume by corrosion and condensation (Stanislas, Kompenhans and Westerweel, 2013). Although, the mention threads were not considered as problematic for this study. A sufficient and constant seeding of the particles in a gas field (wind field in this study) must be ensured (Melling, 1997). Therefore, in the wind velocity field, a section for a set of spray nozzles connected to a water tab was added to the experimental setup.

Experiments concerning PTV calculation of the movement of the ice were conducted under the influence of different wind fields. A view field from the top of the flume was considered, for measuring the ice movements. A camera was located on top of the flume to capture ice movement. At first, the velocity of the ice pieces (with the edge lengths of  $10 \times 10 \times 1$  cm) was measured, then the ones attributed to the ice covers (with the edge lengths of  $50 \times 40 \times 1$  cm) were captured. The velocity of ice pieces and ice covers under the influences of generated winds by one, two and three fans were measured. The duration of the experiments was from 3 to 5 minutes. For the ice cover velocity measurement, constant concentration was considered. It was due to the fact that the ice covers would shield the view field, producing a concentration equal to 1. While, for the ice pieces, different ranges of concentration were considered; 0 to 0.35, 0.35 to 0.7 and 0.7 to 1. The calculation of the concentration was done by the number of pieces in 1 m length of the view field ( $0.6 \text{ m}^2$  of view field).

For each wind field, three separate repetitions of experiments for ice pieces and covers were accomplished. For the ice pieces (as opposed to the ice covers) that the concentration of ice was important, the three mentioned ranges of the ice concentration (considered for ranging of ice concentration in PTV technique) were collected from the experiments. In other words, three ranges of concentration were obtained within the duration of each experimental run. In Figure 3, examples of each of the three ranges of concentration and ice cover are provided. The ice velocity field calculation was performed by the free open source PTVlab Software (AntoineP, 2024). To calculate ice velocity fields, the maximum time duration of 60 minutes for each range of ice concentration and for each experiment test was used. This time duration was the most efficient time expansion in terms of application as an input data in PTVlab software, based on trial-and-error technique for obtaining the velocity fields. To avoid the effects of the boundaries on the flume wall, the PTV measurements were narrowed down to the central part of the measurement window.



**Figure 3.** Samples of snapshots for low (0–0.35) (a), medium (0.35–0.7) (b) and high (0.7–1) (c) ranges of concentration of ice, as well as ice cover (d)

For the ice velocity measurement, surface of the ice (which could be detected by the PTVlab Software in the duration of velocity calculation) were used for tracking their routes. Ice covers did not show any movement under the influence of the wind by one working fan. Therefore, the related experiments would finish right after starting.

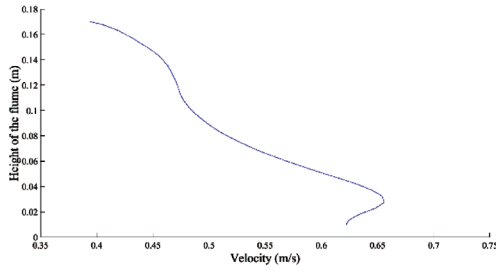
### 3. PIV measurements and results (related to the wind and water velocity)

In PIV technique, the velocity vectors of seeding particles (water particles in the wind field and hollow glass spheres in water field) were captured in time intervals 0.0018 s. For each PIV trial measurement, 900 velocity vector fields are produced. A Matlab code was needed to access each of these vector velocity fields, consecutively. The velocity vectors refer to the displacement in certain time interval in x and y directions. The velocity vectors were derived at each time instance (0.0018 s).

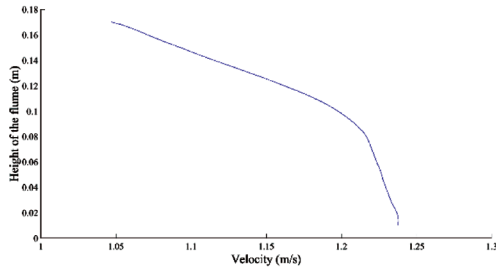
To perform data analysis, a 3D matrix was coded to hold the data. The first, second and third dimensions were velocity vectors (for each x and y coordinates of the measurement plane), time, and different trials, respectively. Consequently, the average velocity of each trial along the height of the wind flume (height level of the field represented in y coordinate), were calculated. In Figure 4, the calculated averaged velocities of all trials along the height of the flume is presented for three wind fields by blue lines (Figure 4 (a), (c) and (e)). Same profiles are provided for water velocity measurement under the wind influences (Figure 4 (b), (d) and (f)); the figure shows the water velocity under the influence of wind field versus the depth of the water field which its origin coordinate was at the water surface.

As can be seen in the pictures, the most symmetric wind velocity distribution along the height of the flume is observed for the wind field of three working fans and the least for that of one fan. The coordinate origin is located at the water surface and the wind flume ceiling was located at the distance of 0.2 m from the coordinate origin. It also can be noticed, at the adjacent of the ceiling, due to the effect of ceiling boundary, the most effects in the wind velocity profiles are observed for the wind fields related to one working fan and two working fans.

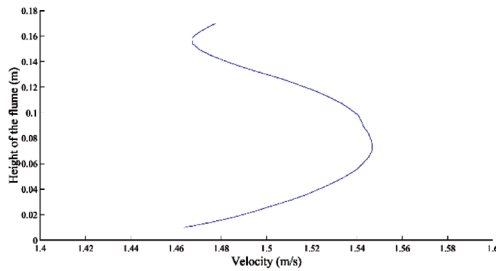




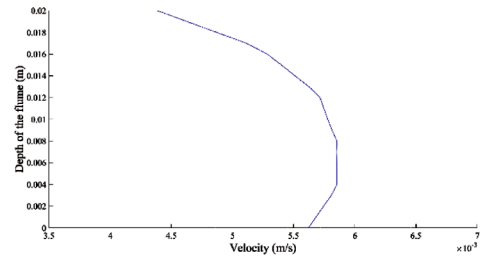
(a)



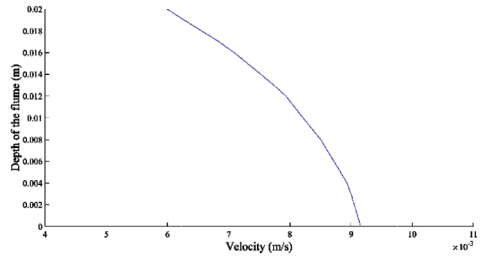
(c)



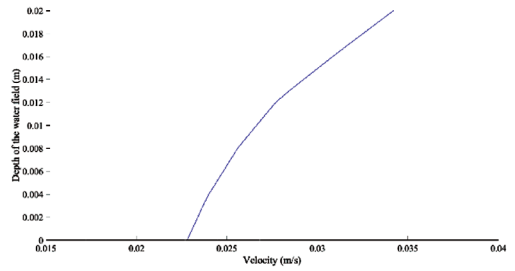
(e)



(b)



(d)



(f)

**Figure 4.** Wind velocity fields for one working fan (a), two (c), and three (e) working fans (grey hue shows changing from first to ninth experimental trials by getting lighter), and water velocity (b), (d) and (f) as such, based on averaging velocity vectors for each trial and at each height level. The averaging of the velocity values of all trials are shown along each height of the wind field with a blue line

## 4. PTV measurement and results

The averaged velocities of the pieces and covers were average of the velocity vectors (averaged area velocity) which were located at an area around the location of the PIV measurement (the area for PTV calculation of ice velocity was at the location of the PIV measurement for wind velocity). The ice pieces concentrations would be determined by counting the pieces in the observation area of one meter length ( $1 \times 0.6 \text{ m}^2$  of area). The number of ice pieces shows total area of the ice (surface area of one piece in the experiments was  $0.01 \text{ m}^2$ ), which was used in the concentration calculation indicated in Equation 2. In this study, three ranges of concentration, associated to low, medium and high were considered and are provided in Table 1. The three concentration ranges were extracted, and the velocity of ice was calculated with PTV technique.



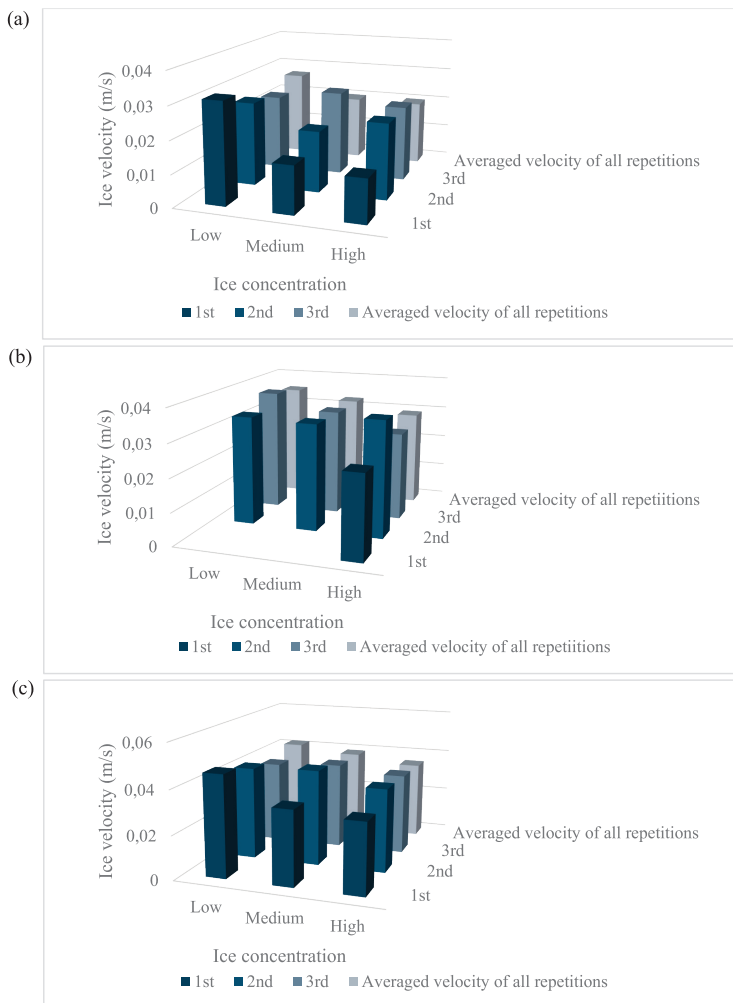


**Table 1.** Different ranges for the concentration based on the number of pieces.

Types	Concentration	Number of ice pieces
Low	0–0.35	0–20
Medium	0.35–0.7	21–41
High	0.7–1	41–59

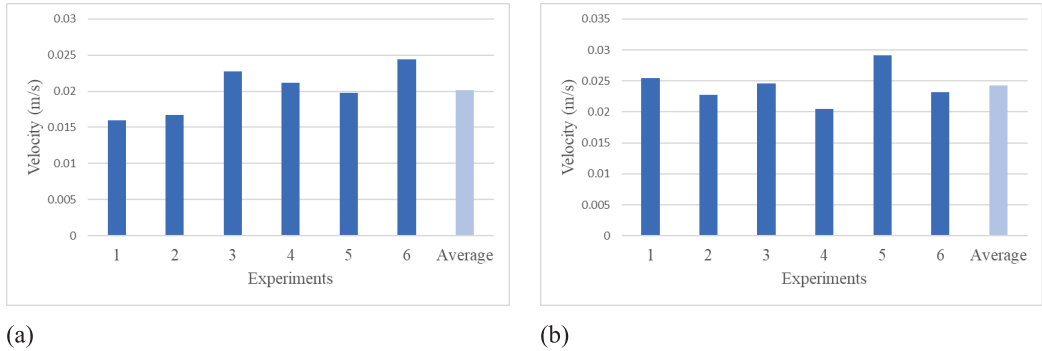
$$\text{Concentration} = \frac{\text{Total area of the ice}}{\text{Observation area of water surface}} = \frac{\text{Total area of the ice}}{1 \times 0.6 \text{ m}^2} \quad [2]$$

The averaged ice area velocities for different ice concentrations (low, medium and high) and each experiment run are shown in Figure . It can be seen in Figure that there are averaged ice area velocities of three trials. They are shown in a blue shade (except for the Figure (b), in which there are two repetitions for low and medium ice concentrations). The calculations were also repeated for different wind velocity fields and ice concentrations.



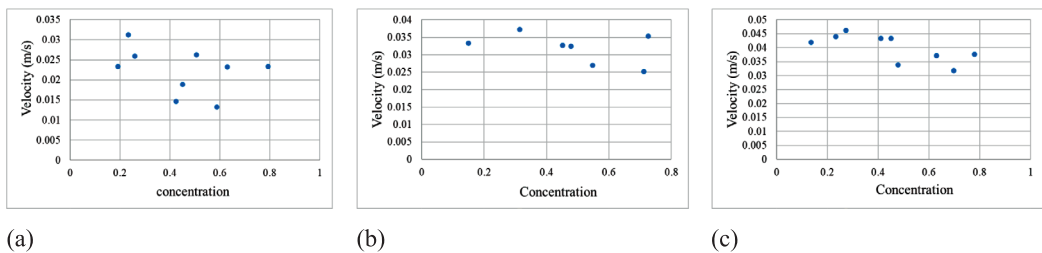
**Figure 5.** The averaged ice area velocity for ice pieces and under the wind field of one working fan (a), two (b), and three (c) working fans

In Figure 6, the averaged ice area velocities for the covers under the influence of two wind fields and their averaged values (light blue bar) are shown. As mentioned before, the ice covers exhibited movements under the wind effects of two and three working fans. As can be seen, six experiments are provided for each wind field.



**Figure 6.** The averaged ice area velocity for covers under the wind field of two (a) and three (b) working fans

In Figure 7, the ice velocities related to the Figure 5, with changing ice concentration are provided.



**Figure 7.** Ice velocity versus concentration under the influence of one (a), two (b), and three (c) working fans

## 5. Conclusions

Using water particles as seeding material for PIV measurement led to reasonable results for wind velocity field measurement. The averaged values of the velocity profiles from the PIV measurements are 0.514, 1.165 and 1.499 m/s for one, two, and three working fans, respectively. Although, marginal values for water velocity were observed; the averaged water velocity field values which were obtained under the influence of the wind fields related to the one, two and three working fans were 0.0049, 0.0074 and 0.0283 m/s, respectively.

No relation between the ice concentration and ice velocity was observed, apart from an overall inverse relationship between the ice concentration and velocity (increasing concentration led to reduced velocity). As can be seen, taking the averaged values of the experimental trials (light blue bar in Figure 5) into account, the lower concentrations would lead to higher velocities. The ranges of averaged velocities (light blue bar in Figure 5) for the wind fields of one, two, and three working fans are 0.019 to 0.026, 0.029 to 0.035 and 0.035 to 0.041 m/s, respectively. It can be noticed that less variations in the averaged ice area velocities are observed related to the



wind field of two working fans. Shen et al, (1990) related the interaction of the particles (ice) to their concentration, the interaction of the particles to the time duration of their interaction and consecutively to the ice flow regime (Shen, Shen and Tsai, 1990). They mentioned the low and high concentrations basically occur in high and low ice velocity, the former and the latter can be called as rapid and slow flow regimes, respectively (Shen, Shen and Tsai, 1990). The averaged ice cover velocities related to two and three working fans are 0.0195 and 0.0265 m/s, respectively (Figure 7. As might be expected, changing the ice concentration is not concerned for the ice covers and is only considered for the ice pieces.

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