

FPGA Implementation of the Multiplication Operation in Multiple-Precision Arithmetic

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Abstract—Although standard 32/64-bit arithmetic is sufficient to solve most of the scientific-computing problems, there are still problems that require higher numerical precision. Multiple-precision arithmetic (MPA) libraries are software tools for emulation of computations in a user-defined precision. However, availability of a reconfigurable cards based on field-programmable gate arrays (FPGAs) in computing systems allows one to implement MPA algorithms in hardware. Whereas addition and subtraction operations of two n -digit numbers require $O(n)$ operations, the basecase multiplication is equivalent to the convolution computation that requires $O(n^2)$ operations. Therefore, an efficient implementation of the multiplication operation is crucial for application of the reconfigurable hardware in MPA computations. In this contribution, our implementation of the basecase-multiplication algorithm in MPA on FPGA is presented. The method is implemented using the very high speed integrated circuit hardware description language (VHDL) and benchmarked on Xilinx Artix-7 FPGA. In the developed implementation of the MPA multiplication, the multiplication of two integer 1024-bit numbers (2048-bit numbers) takes 205 nsec (819 nsec) with the use of 40 DSP modules. It gives two-fold speedup in comparison to the reference results published in the literature. The developed digital circuit of the MPA multiplier works with integer numbers of precision varying in the range between 16 bits and 32 kbits. Such a scalability allows one to use the developed method not only in scientific computing, but also in embedded systems employing encryption based on MPA.

Keywords—FPGAs, multiple-precision arithmetic, scientific computing, parallel processing, embedded systems.

I. INTRODUCTION

Multiple-precision arithmetic (MPA) can be considered as a new approach for solving scientific problems which are difficult in terms of convergence, ill-conditioning and precision. It may be anticipated that MPA will be an increasingly important research tool in the future. The approach of a new era of computing is predicted in the literature [1], [2], in which the numerical precision required for computations is as important to the design of scientific codes as are algorithms and data structures. The review of MPA applications in scientific computing is presented in [1], [2]. Further MPA applications in computational electromagnetics are presented in [3]. Currently, MPA applications in scientific computing can be classified into the following categories:

- Generation of special mathematical functions applicable in the mathematical modeling.

- Solving ill-conditioned linear systems of equations.
- Difficult simulations of scientific and engineering problems (e.g., climate modeling, fluid dynamics, etc.).
- Derivation and verification of novel formulas and theorems in mathematics.

Elementary algebraic operations such as addition, subtraction and multiplication are crucial for any investigations requiring MPA computations. Whereas the addition and subtraction of two n -digit numbers require $O(n)$ operations, their multiplication requires $O(n^2)$ operations in the standard implementation (referred to as the basecase multiplication here). Therefore, the main effort in the implementation of the elementary MPA computations focuses on the multiplication operation. Although MPA multiplication can employ the Karatsuba, Toom-Cook and Schönhage-Strassen methods (which respectively require $O(n^{1.585})$, $O(n^{1.465})$, $O(n \log(n) \log(\log(n)))$ operations [4], [5], [6]), these algorithms are efficient for very-high-precision operands. For small precision of MPA operands, the extra shift and addition operations in those aforementioned algorithms make them slower than the basecase method. Therefore, for instance, GNU MPA library (GMP) [7] switches the multiplication method from the basecase to Karatsuba method when precision of operands is increased over 1920 bits (30 limbs, each limb equals 64 bits). Hence, efficient implementation of the basecase multiplication still remains a fundamental requirement for any MPA computations, regardless of the field of application, software tools and architecture of processing units.

There are several MPA libraries running on central processing units (CPUs), e.g., ARPREC, GMP, QD, MPIR (GMP-compatible) [3]. In the last few years, graphics processing units (GPUs) have been applied as a low-cost architecture for an acceleration of scientific codes. Therefore, MPA libraries were also implemented on GPU. There are several MPA libraries running under this parallel architecture, e.g., GARPEC, GQD, CUMP (GMP-compatible) [3]. However, the emulation of MPA computations on standard processors provides its significant slowdown when the precision of computations is increased. According to [2], computations on CPU in double-double precision typically run 5–10 times slower in comparison with 64-bit implementation. These slow down

at least 25 times for quad-double arithmetic, more than 100 times for 100-digit arithmetic, and over 1000 times for 1000-digit arithmetic. These facts demonstrate the need for new computing architectures accelerating MPA computations.

The popularity and promotion of a reconfigurable-computing hardware (e.g., refer to [8]) encouraged us to develop the implementation of the basecase-multiplication algorithm in MPA on a field-programmable gate array (FPGA). The method is implemented using the very high speed integrated circuit hardware description language (VHDL) and benchmarked on low-cost Xilinx Artix-7 FPGA [9]. Our results are compared with results reported in [10] demonstrating the advantages of the developed implementation. Due to application of the low-cost FPGA hardware in the research, the proposed multiplication method facilitates not only investigations in the area of scientific computing, but also can be useful for the embedded-system design with cryptographic solutions requiring MPA. For instance, modern encryption algorithms such as RSA and AES [11], [12], [13] employ keys whose length is higher than 256 bits. Currently, popular RSA keys are of length 1024 to 2048 bits, even though some experts believe that 1024-bit keys may become breakable in the near future [13]. Therefore, it can generally be assumed that RSA is secure if encryption keys are sufficiently long. As a result, critical systems employ dedicated cryptographic processors to protect information and to offload the related computational overhead (e.g., refer to [14]). To sum up, although the presented direction of the research is closely related to scientific computing, the results obtained might also be useful for development of electronic systems employing encryption.

II. DEVELOPED IMPLEMENTATION

The VHDL code for MPA multiplication is developed for Artix-7 FPGA using Vivado Design Suite [15]. The multiplication operation is executed on arrays of MPA numbers by the developed multiplier, which employs several multiplying units working in parallel. Each multiplying unit consists of a single DSP48E1 module available in this FPGA family [16]. The developed multiplier is scalable in terms of the number of multiplying units. It means that the multiplier can consist of a varying number of multiplying units, depending on the required processing power. In this contribution, we present the multiplication of two arrays of integer MPA numbers on a single multiplier consisting of a single multiplying unit for the sake of brevity. The multiplication operation for floating-point numbers can be obtained similarly, taking into consideration exponents and rounding, necessary to fit the output in fixed-precision (i.e., fixed-length) mantissa array of the result.

A. MPA Format of Integers

In the developed implementation, the format of integer numbers is the same as in the GMP library. It consists of an array of bytes representing the MPA number and its size. The absolute value of the size represents the length of the data array, whereas a sign of the size corresponds to a sign of

the MPA number. Therefore, in the case of multiplication, the sign of the result can be easily computed as the multiplication of operand signs. The developed digital circuit of the MPA multiplication works with precision varying in the range between 16 bits and 32 kbits. Such a range is not a limiting step for MPA applications because the basecase multiplication for very-high-precision MPA operands is not efficient and it is worth using in this case, e.g., the Karatsuba method of multiplication on standard CPU.

B. Basecase Multiplication Algorithm

The basecase multiplication of two n -bit and m -bit numbers is a straightforward rectangular set of cross-products, the same as long multiplication done by hand, and for that reason sometimes known as the schoolbook or grammar school method [5], [7]. To some extent, the basecase multiplication of two numbers in any numerical system is equivalent to the multiplication of polynomials, which can be considered as a convolution computation. Hence, this is an $O(n \times m)$ algorithm.

C. Design Considerations

When designing a module in FPGA, one should take into account factors such as frequency of operation, resource utilization, congestion, overall performance and power efficiency. For computing applications, the overall performance is usually the most important factor, and consequently all considerations here are focused on achieving that purpose. Since the performance of the multiplication is directly proportional to the frequency of operation, this frequency should be kept at the maximum level. By doing so, the maximum distance to travel by a signal between two registers is shortened, so practically the design requires additional registers, which extend the range at the expense of latency. To sum up, the purpose of the design process is to find the trade off between the frequency of FPGA operation, and the latency.

D. Application of DSP Module

The DSP48E1 module available in Artix-7 FPGA enables computations of several arithmetic functions, which can be programmed into it. However, for the purpose of this project, only operation $A \cdot B + C + \text{CARRY_IN}$ is employed. The maximum width of the operands is as follows:

- **A** - 25 bits
- **B** - 18 bits
- **C** - 48 bits
- **CARRY_IN** - 1 bit.

Symmetric operands are used, due to the fact that asymmetric operand multiplier (25×18 bits) is difficult in construction and data storage (i.e., partial results are stored in an accumulator). Therefore, the maximum width of operands in the multiplying unit should be set at 18×18 bits. However, such a non-electronic number (i.e., different than power of 2) lowers the utilization of the BRAM memory block available in the FPGA chip. Consequently, the operand width is set to 16×16 with operand C of size 32 bits. For high performance,

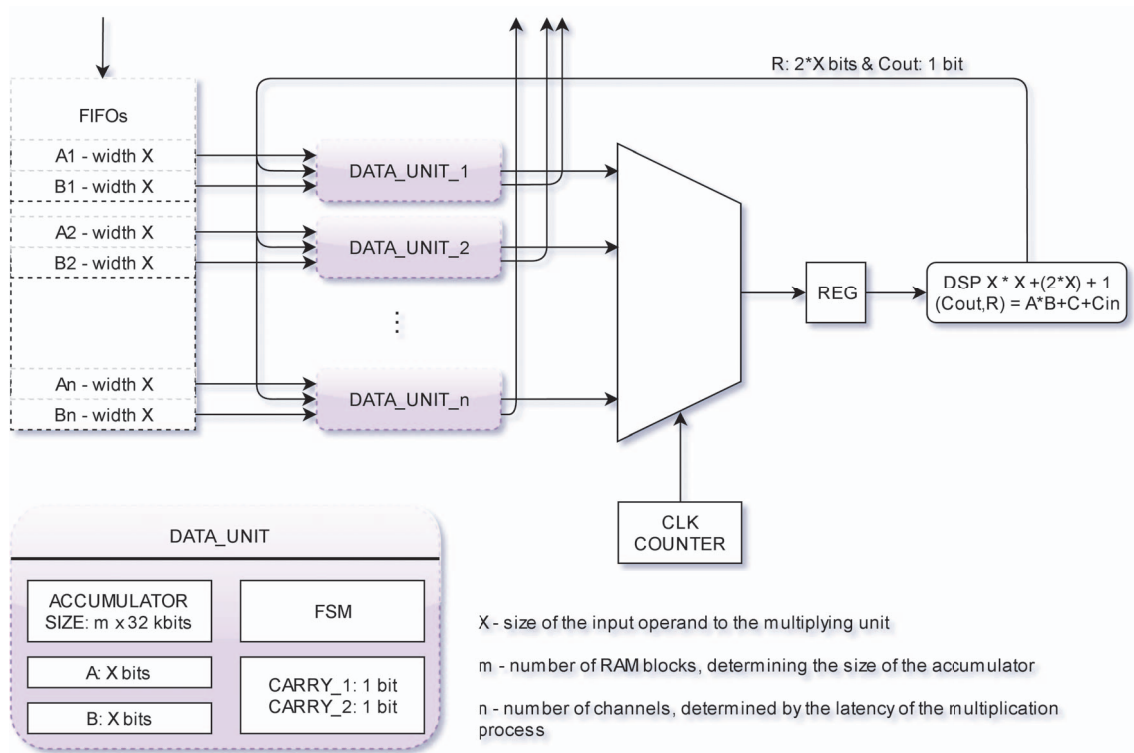


Fig. 1. Application of the DSP48E1 module in Artix-7 FPGA in the digital circuit implementing the MPA multiplication

TABLE II
EXEMPLARY EXECUTION TIMES OF MPA MULTIPLICATION IN
COMPARISON WITH RESULTS IN [10]

precision (bits)	[10] (μsec)	Artix-7 @ 500 MHz (μsec)		
		5 DSP	20 DSP	40 DSP
1024	0.41	1.6	0.41	0.205
2048	1.30	6.6	1.6	0.819

In our test, arrays of 1, 000, 000 random MPA numbers are multiplied, and the execution time of a single multiplication is calculated for a single output number. As seen, the developed solution outperforms reference results [10] for a sufficient number of DSP modules employed in the MPA-multiplication operation. Furthermore, the execution times decrease almost linearly with the number of DSP modules employed for the MPA multiplication.

For the basecase multiplication, an equation can be derived to calculate the average time needed to multiply two numbers, i.e.

$$t_{per_number} = \left(\frac{prec}{mult_prec} \right)^2 \cdot \frac{1}{mult_units \cdot freq} \quad (1)$$

$$t_{measured} = t_{per_number} \cdot table_size \quad (2)$$

where t_{per_number} is the average time for calculation of a single MPA-multiplication result, $prec$ is the precision of the

numbers to be multiplied, $mult_prec$ is the precision of the multiplying unit, $table_size$ is the number of multiplications, $mult_units$ is the number of multiplying units working in parallel performing multiplication operations, $freq$ is the frequency of operation of DSP modules and $t_{measured}$ is the time needed to multiply two arrays.

There is also an additional dependency between $mult_prec$, $mult_units$, $freq$ and $table_size$. The rule of thumb dictates that the higher the $mult_prec$, the lower the maximum number of multiplying units available in a single FPGA chip. This is due to the increase of latency for DSP modules, that forces an increase in the number of channels, which results in a higher resource utilization. Such an increase in the resource utilization may also limit the maximum speed of operation ($freq$). The higher the number of channels, the higher $table_size$ is required in order to limit the computation-time quantization error.

Our design efficiency per number of DSP modules is over two times higher than for the processor presented in [10]. This is mainly due to the fact that our design runs at two times higher frequency (500 MHz). Furthermore, in order to run 64-bit operations on DSP modules, the individual modules have to exchange data, which requires additional stage registers and this lowers the efficiency. The DSP modules in our solution work independently, hence, the higher efficiency. However, such a comparison has limited validity because consumption of other resources should also be taken into account.

Application of the proposed solution requires sending a data stream to the (e.g., memory-mapped) multiplier, whereas the solution [10] is based on a finite-state machine, which executes the multiplication operation depending on a micro-instruction set. Both solutions represent completely different approaches to the MPA multiplication and it limits the possibility of fair comparison.

The solution proposed here offers a fine scalability, and allows you to run MPA multiplication, even on the low-cost FPGA chips available currently on the market, i.e., any Artix, most of Spartan 7, and almost all Spartan 6 family chips. Being runnable on any Zynq based platform, our solution is suitable for embedded applications requiring the MPA multiplication. Such a need may stem from, e.g., an application of a demanding encryption method in a communication.

IV. CONCLUSION

Currently, there are still scientific problems that require higher numerical precision than provided by the standard 32/64-bit arithmetic. Availability of reconfigurable cards based on FPGAs in computing systems, allows one to implement MPA algorithms in hardware. In this contribution, our implementation of the basecase-multiplication algorithm in MPA on Xilinx Artix-7 FPGA is presented. In the developed implementation of the MPA multiplication, the multiplication of two integer 1024-bit numbers (2048-bit numbers) takes 205 nsec (819 nsec) with the use of 40 DSP modules. It gives an almost two-fold speedup in comparison to the reference results published in the literature. The developed digital circuit of the MPA multiplier works with integer numbers of precision varying in the range between 16 bits and 32 kbits with a step size of the multiplying unit precision (16 bits). Therefore, the results are hopefully useful not only in scientific computing, but can also be useful for design of embedded systems with cryptographic solutions requiring MPA.

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