# Fracture mechanics model of cutting power versus widespread regression equations while wood sawing with circular saw blades

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**Abstract:** *Fracture mechanics model of cutting power versus widespread regression equations while wood sawing with circular saw blades.* A comparison of the theoretical cutting power consumption results forecasted with the model (FM_CM model) which include work of separation (fracture toughness) in addition to plasticity and friction, and two widespread regression equations while wood sawing with circular saw blades has been described. in and cutting power consumption forecasted. In computations of the cutting power consumption during rip sawing of Scots pine wood (*Pinus sylvestris* L.) of the Baltic Natural Forest Region (PL) provenance values of fracture toughness and shear yield stresses were taken from previous empirical works. The carried out analyses revealed a lack of conformity of the results obtained with regression equations if compared with the FM_CM model.

*Keywords*: circular sawing machine, cutting power, fracture mechanics model, regression equation

## INTRODUCTION

Reliable estimates for the power requirements necessary to cut wood are essential for the proper design of cutting tools/machines to assure safety of operation and to optimize production quality (Chuchala et al. 2014). A large amount of logs are being machined in sawmills every day. Therefore, prediction of power consumption is becoming an increasingly important economic factor in all processing stages. Understanding of the effect of wood cutting parameters on power consumption would enable increase energy efficiency, thus, reduce operating costs and enhance profitability (Cristóvão et al 2013). In the latter work Cristóvão et al. (2013) tried to compare measured in industrial conditions values of cutting power during sawing of pine wood (*Pinus sylvestris* L.) on double shaft rip circular sawing machines, with predicted values of cutting power with the regression equation developed by Axelsson et al. (1993). The mentioned comparison has revealed that the predicted "model" had given lower values of power consumption than the experimental results (Cristóvão et al. 2013).

Porankiewicz et al. (2011) attempted to evaluate statistical, non-linear, and multi-variable dependencies of the (main) $F_C$ cutting force, during rotational cutting (Figure 1, circular sawing simulating) of the wood of *Pinus sylvestris*. In their laborious computations they applied also a set of experimental results which were obtained by Axelsson et al. (1993). The authors of the paper by Porankiewicz et al. (2011) have stated that their regression equation for the cutting force $F_C$ has the quality of approximation much more better than the regression equation developed by Axelsson et al. (1993), and later slightly modified by (Cristóvão et al 2013).

On the other hand, cutting power (forces) could be considered from a point of view of modern fracture mechanics (Orlowski et al. 2013). As it was presented previously by Orlowski et al. (2012; 2013), cutting power values, in case of sawing of dry pine wood (originated from the Baltic Natural Forest Region, PL) on the circular sawing machine, obtained with the Manžos method (the classical approach, Pc_Man) and the cutting model that includes work of separation in addition to plasticity and friction (Pc_Frac) have been

more or less the same. Nevertheless, the latter method allow the user to predict the cutting power for the sawing process more precisely because the wood derivation ought to be taken into account (Chuchala et al. 2014). In the models for a circular sawing machine kinematics described in works by Orlowski et al. (2012; 2013), similarly to metal milling, the sum of all uncut chip thicknesses of the simultaneously teeth engaged represented the mean uncut chip thickness.

The aim of this study was to compare the fracture mechanics model of cutting power, which include work of separation (fracture toughness) in addition to plasticity and friction (Orlowski et al. 2012, 2013) versus widespread regression equations developed by Axelsson et al. (1993) and Porankiewicz et al. (2011) while wood sawing with circular saw blades.
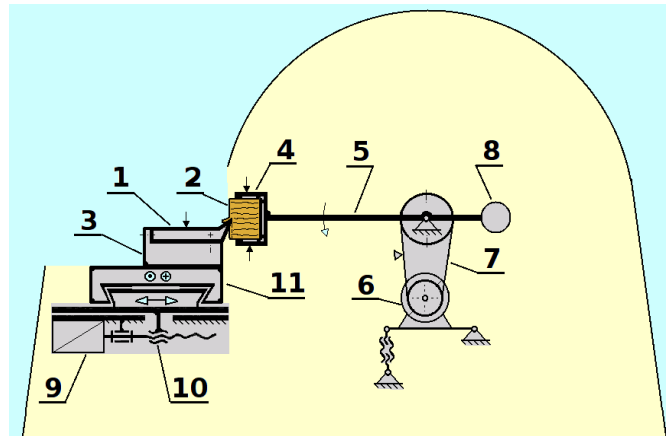


**Figure 1.** Schematic diagram of the measurement system applied by both Axelsson at al. (1993) and Porankiewicz et al (2011), where: 1 - cutting tool; 2 - workpiece; 3 - 3D piezoelectric load cells; 4 - workpiece holder; 5 - rotating arm; 6 - motor; 7 - transmission belt; 8 - balance mass; 9 - servomotor; 10 - feed screw mechanism; 11 - tool support (Porankiewicz et al. 2011)

THEORETICAL BACKGROUND

In Figure 2 kinematics of sawing on circular sawing machines is shown. In that kind of cutting the cutting speed direction in regard to the wood grain direction changes its position. It could be noticed that in the measurement stand (Figure 1) and in a regular sawing machine (Figure 2) relative motions of tool and workpiece are the same.
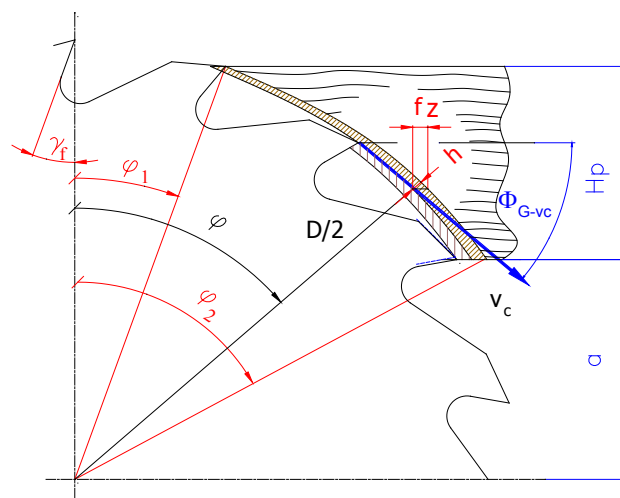


**Figure 2.** Sawing kinematics on circular sawing machine: $f_z$ – feed per tooth, $D$ – circular saw blade diameter, $h$ – uncut chip thickness, $H_p$ – workpiece height (depth of cut), $a$ – position of the workpiece, $\varphi$ – angular tooth position, $\Phi_{G\text{-}vc}$ – an angle between grains and the cutting speed direction

*Fracture mechanics approach*

In case of cutting with circular saw blades (for the simplest model FM_CM) uncut chip thickness $\overline{h}$ (an average value) ought to be taken into account, hence, the cutting power may be expressed as:

$$\overline{P}_{cw} = F_c v_c + P_{ac} = \left[ z_a \cdot \frac{\tau_\gamma S_t \gamma}{Q_{shear}} v_c \overline{h} + z_a \cdot \frac{RS_t}{Q_{shear}} v_c \right] + P_{ac} \qquad (1)$$

where: $z_a = \left( \dfrac{\varphi_2 - \varphi_1}{\varphi_t} \right)$ is a number of teeth being in the contact with the kerf (average), $\varphi_1$ is

an angle of teeth entrance which is given by $\varphi_1 = \arccos \dfrac{2(H_p + a)}{D_{cs}}$, $\varphi_2$ is an exit angle which

can be determined as $\varphi_2 = \arccos \dfrac{2a}{D_{cs}}$, $D_{cs}$ is a diameter of circular saw blade, an average

uncut chip thickness is given by $\overline{h} = f_z \sin \overline{\varphi}$, and an average angle of tooth contact with

a workpiece $\overline{\varphi}$ is calculated from $\overline{\varphi} = \dfrac{\varphi_1 + \varphi_2}{2}$, $\tau_\gamma$ is the shear yield stress, $R$ is specific work

of surface separation/formation (fracture toughness). The chip acceleration power $P_{ac}$ variation as a function of mass flow and tool velocity. The shear strain along the shear plane $\gamma$ and the friction correction $Q_{shear}$, which is a function of the shear angle $\Phi_c$ (defines the orientation of the shear plane with respect to cut surface), can be calculated with formulae given in the work (Orlowski et al. 2013). However, the shear angle $\Phi_c$ (material dependent) can be obtained numerically. Since the cutting speed has indirect positions in relation to grains, fracture toughness $R$ and the shear yield stress $\tau_\gamma$ may be calculated from formulae known form strength of materials (Orlicz 1988; Orlowski et al. 2013).

*Regression equation developed by Axelsson et al. (1993)*

The original regression equation has been modified by Cristóvão et al. (2013) (multiplication by the ratio of an overall set (kerf) $S_t$ to 4.25). The latter number was a cutting edge width in Axelsson's experiments. The modified Axelsson's regression equation is as follows:

$$F_c = \left( -7.37 + A_1 + 15.61\Phi_{G-vc} - 2.6\Phi_{G-vc}^3 + 1.31\rho_{CE} + 0.2v_c + A_2 \right) \frac{S_t}{4.25} \qquad (2)$$

Where:

$$A_1 = \left( 0.38\rho - 224.5\gamma_f \right) h_j(\varphi) \qquad (3)$$
$$A_2 = \left( 0.3 \cdot \Phi_{G-vc} - 0.01T \right) \cdot MC \qquad (4)$$

$\Phi_{G-vc} = \varphi_j$ in radians (Figure 2), $\gamma_f$ is a rake angle in radians, $\rho_{CE}$ is a radius of the cutting edge ($\mu$m), $\rho$ is wood density at 8% moisture content (kg·m$^{-3}$), $MC$ is moisture content (%), $T$ is wood temperature (°C).

If equation (2) is multiplied by $v_c$ and $z_a$, the product is cutting power $P_c$.

*Regression equation developed by Porankiewicz et al. (2011)*

The first attempt of the with the regression equation published in the work by Porankiewicz et al. (2011) has not given expected results. In the next step we have communicated with Dr. Porankiewicz. Thanks to him (Porankiewicz 2014) we have obtained the proper regression equation together with the appropriate coefficients. Hence, the correct regression developed by Porankiewicz (2014) can be given by:

$$F_c = (a_1 + P_{A1}) \cdot a_2 \cdot \left| \cos(\overline{\varphi} + a_3) \right|^{a_4} + (a_5 + P_{A2}) \cdot a_6 \cdot \left| \sin(\overline{\varphi} + a_7) \right|^{a_8} + a_{27} \qquad (5)$$

$$P_{A1} = \frac{\overline{h}^{a_9} \gamma_f^{a_{10}} \rho_{CE}^{a_{11}} v_c^{a_{12}} \rho^{a_{13}}}{\left( a_{14} - e^{MC \cdot a_{15}} \right) \left( a_{16} - e^{T \cdot a_{17}} \right)} \qquad (6)$$

$$P_{A1} = \frac{\overline{h}^{a_{18}} \gamma_f^{a_{19}} \rho_{CE}^{a_{20}} v_c^{a_{21}} \rho^{a_{22}}}{\left( a_{23} - e^{MC \cdot a_{24}} \right) \left( a_{25} - e^{T \cdot a_{26}} \right)} \qquad (7)$$

where: $a_1 = 224914.765$, $a_2 = 0.00006412$, $a_3 = -0.04709$, $a_4 = 18.90097$, $a_5 = 2.57193$, $a_6 = 10.57092$, $a_7 = 0.0955$, $a_8 = 0.30406$, $a_9 = 1.1726$, $a_{10} = -2.15735$, $a_{11} = -0.05021$, $a_{12} = -0.07183$, $a_{13} = 2.43628$, $a_{14} = 0.99865$, $a_{14} = 0.99865$, $a15 = 0.00006883$, $a_{17} = 0.02962$, $a_{16} = -27269.686$, $a_{18} = 0.41571$, $a_{19} = -0.17439$, $a_{20} = 0.23563$, $a_{21} = 0.19386$, $a_{22} = 1.21835$, $a_{23} = -0.57335$, $a_{24} = -0.24134$, $a_{25} = -878.17982$, $a_{26} = 0.01668$, $a_{27} = -27.65757$, $\overline{\varphi}$ an average angle of tooth contact with a workpiece in radians, $\overline{h}$ average uncut chip thickness in mm (0.05; 0.5 mm), $\gamma_f$ is a rake angle in radian (0.17; 0.51993 rad), (9.74°; 29.79°), $\rho_{CE}$ is a radius of the cutting edge in μm (5; 20 μm), $v_c$ is cutting speed in m·s$^{-1}$ (14.916; 39.741 m·s$^{-1}$), $\rho$ is wood density at 8% moisture content in kg·m$^{-3}$ (372; 735 kg·m$^{-3}$), $MC$ is moisture content in % (8; 133%), $T$ is temperature of wood (-15; 20ºC). Numbers given in parentheses define the lower and superior limits of changes of independent variables which were applied in the experiments (Axelsson et al. 1993; Porankiewicz et al. 2011).

If the equation (6) is multiplied by the ratio of an overall set (kerf) $S_t$ to 4.25 (value of the cut width used by Axelsson et al. (1993) it is possible to calculate a value of cutting force $F_c$ for other values of the kerf. Moreover, if the equation (6) is additionally multiplied by $v_c$ and $z_a$, the product is cutting power $P_c$ of the sawing process.

MATERIALS AND METHODS

Predictions of cutting powers have been made for the case of sawing on the circular sawing machine (HVS R200, f. HewSaw), which is used in Polish sawmills. However, in conducted computations it was impossible to apply feeding speeds from the upper values range because of limits of the regression equations (Axelsson et al. 1993; Porankiewicz et al. 2011; Porankiewicz 2014). The machine settings were as follows: number of saw blades $n_b = 1$, spindle rotational speed 3490 rpm, cutting speed $v_c = 63.95$ ms$^{-1}$, average of uncut chip thickness $h < 0.5$ mm), clearance of a circular saw blade over the workpiece 0 mm, an average angle of the tooth contact with a workpiece $\overline{\varphi} = 28.56°$ (0.4974 rad), cutting kinematics – up-sawing, electric engine power $P_{EM} = 90$ kW. One circular saw blade was applied with data as follows: 350 mm ($D$) × 70 mm ($d$) × 2.4 mm ($s$), overall set $S_t = 3.6$ mm, number of carbide tipped teeth $z = 24$, and side rake angle $\gamma_f = 25º$ (f. Gasstech, PL). In computations for the regressions by Axelsson et al. (1993) and Porankiewicz (2014) it was assumed that $\rho_{CE}$ is equal to 5 μm. According to Orlicz (1988) that kind of cutting edge could be classified as a sharp edge.

Table 1. Raw material data (Orlowski et al. 2013)

| Region | $\rho$ | $R_\perp$ | $\tau_{\gamma\perp}$ | MOR* |
|---|---|---|---|---|
| | kgm$^{-3}$ | Jm$^{-2}$ | kPa | MPa |
| Baltic | 520 | 1295.33 | 20861 | 41,6 |
| $\rho$ – density, MOR – modulus of rupture in bending (* values were taken from Krzosek (2009)) | | | | |

Scots pine (*Pinus sylvestris* L.) samples originating from the Baltic Natural Forest Region (PL) were used as hypothetical experimental samples. It was assumed that samples were in the shape of rectangular blocks with dimensions of 80 mm (*H*) with temperature of 20°C. Raw material data of pine wood, which was taken for numerical computations for the fracture mechanics model, for MC 12% is presented in Table 1. It ought to be emphasised, that part of the samples investigated by Krzosek (2009) has been explored within the previous cutting research, in which the raw material data given in Table 1 has been determined according to the procedure described in the papers by Orlowski and Atkins (2007), and Orlowski and Palubicki (2009).

RESULTS AND ANALYSES

In Figure 2 the results of predictions of cutting power for the circular sawing machine with one circular saw blade, in the case of the rip sawing process of pine wood (*Pinus sylvestris* L.) of the Baltic Natural Forest Region (PL) provenance, obtained with the use of: the FM_CM cutting model that include work of separation in addition to plasticity and friction (A – Fracture mechanics), the modified regression equation of Axelsson et al. (1993) (A – Axelsson), and the regression equation by Porankiewicz (2014) (A – Porankiewicz) are presented.
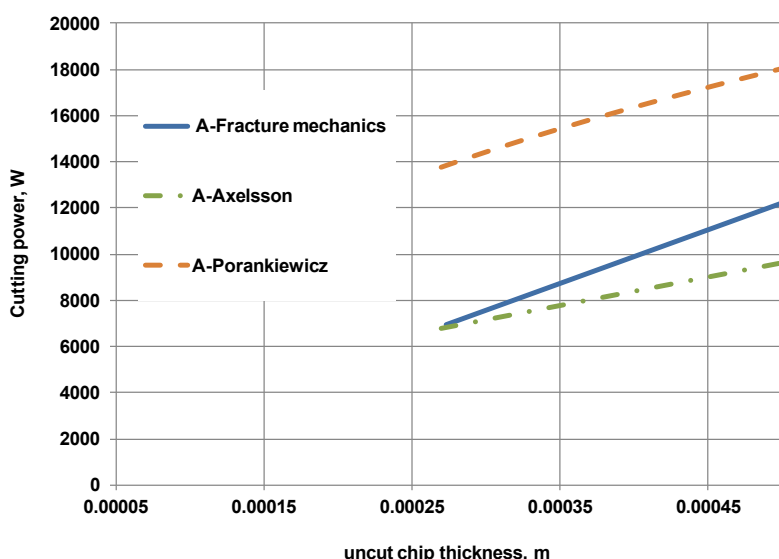


**Figure 3.** Predictions of cutting power for the circular sawing machine with one circular saw blade in the case of pine sawing from the Baltic Natural Forest Region (PL) ($v_{f\max} = 87$ m·min$^{-1}$) obtained with the use of the cutting model that include work of separation in addition to plasticity and friction (A – Fracture mechanics); the modified regression equation of Axelsson et al. (1993) (A – Axelsson), and the regression equation by Porankiewicz (2014) (A – Porankiewicz)

While the classic approach FM_CM is used, it meant that the sum of all uncut chip thicknesses of the simultaneously teeth engaged represented the mean uncut chip thickness, the computed theoretical value of the cutting power has a similar value to the results obtained with the modified regression equation of Axelsson et al. (1993) (A – Axelsson) for small values of uncut chip thicknesses. Unfortunately, with the increase of the uncut chip thickness the difference of the results is larger and simultaneously lower than the results from the FM_CM model. The latter phenomenon was observed by Cristóvão et al. (2013).

If the regression equation by Porankiewicz (2014) (A – Porankiewicz) is applied, the obtained results of the forecasted cutting power consumption are in the whole range of uncut chip thicknesses fairly larger than the results from the FM_CM model and simultaneously the difference is at the same level.

CONCLUSIONS

The carried out results analyses revealed a lack of conformity of the results obtained with regression equations if compared with the FM_CM model. For that reason, it is not recommended to apply the analysed widespread regression equations for forecasting of cutting power consumption while wood rip sawing with circular saw blades. Nevertheless, the both analysed widespread regression equations could be appropriate to analyse phenomena and the effect of the factors such as wood density, moisture content, cutting edge radius, rake angle, wood temperature etc. upon energetic effects (forces) during cutting pine wood (*Pinus sylvestris* L.) with a single cutting edge.

REFERENCES

1. AXELSSON B. O. M., LUNDBERG Å. S., GRÖNLUND J. A. (1993): Studies of the main force at and near cutting edge. *Holz as Roh-und Werktoff*, **51**(1), 43-48.
2. CHUCHAŁA D., ORLOWSKI K. A., SANDAK A., SANDAK J., PAULINY D., BARAŃSKI J. (2014): The effect of wood provenance and density on cutting forces while sawing Scots pine (*Pinus sylvestris* L.). *BioRes*. **9**(3), 5349-5361.
3. CRISTÓVÃO L., EKEVAD M., GRÖNLUND A. (2013): Industrial sawing of *Pinus sylvestris* L.: Power consumption. *BioRes*. **8**(4): 6044-6053.
4. KRZOSEK S. (2009): Wytrzymałościowe sortowanie polskiej tarcicy konstrukcyjnej różnymi metodami (in Polish: Strength grading of Polish structural sawn timber with different methods), Wydawnictwo SGGW, Warszawa. 127 p.
5. MANŽOS F. M. (1974): Derevorežušie Stanki. (In Russian: Wood Cutting Machine Tools), Izdatel'stvo "Lesnaâ promyšlennost'", Moskva.
6. ORLICZ T. (1988): Obróbka drewna narzędziami tnącymi. (In Polish: Wood machining with cutting tools). Skrypty SGGW-AR w Warszawie, Wydawnictwo SGGW-AR, Warszawa.
7. ORLOWSKI K. A., ATKINS A. (2007): Determination of the cutting power of the sawing process using both preliminary sawing data and modern fracture mechanics. pp. 171–174. In: Navi, P., Guidoum, A. (Eds.) Proc. of the Third Inter. Symposium on Wood Machining. Fracture Mechanics and Micromechanics of Wood and Wood Composites with regard to Wood Machining., 21–23 May, Lausanne, Switzerland. Presses Polytechniques et Universitaires Romandes, Lausanne.
8. ORLOWSKI K. A., OCHRYMIUK T., CHUCHAŁA, D. (2012). On some approaches to cutting power estimation while wood sawing. Ann. WULS-SGGW, *Forestry and Wood Technology* 79, 129-134.
9. ORLOWSKI K., OCHRYMIUK T., ATKINS A., CHUCHALA D. (2013): Application of fracture mechanics for energetic effects predictions while wood sawing. *Wood Sci Technol*, **47**: 949–963.
10. ORLOWSKI K., OCHRYMIUK T. (2013): Dynamics of cutting power during sawing with circular saw blades as an effect of wood properties changes in the cross section. Ann. WULS-SGGW, *Forestry and Wood Technology* **83**: 322-328.

11. ORLOWSKI K. A., PAŁUBICKI B. (2009): Recent progress in research on the cutting process of wood. A review COST Action E35 2004-2008: Wood machining-Micromechanics and fracture. *Holzforschung* **63**(2), 181-185.

12. PORANKIEWICZ B., AXELSSON B., GRÖNLUND A., MARKLUND B. (2011). "Main and normal cutting forces by machining wood of *Pinus sylvestris*," *BioRes.* 6(4), 3687-3713.

13. PORANKIEWICZ B. (2014): Personal contact.

**Streszczenie**: *Model bazujący na mechanice pękania wobec szeroko rozpowszechnionych równań regresji dla przypadku przecinania drewna za pomocą pił tarczowych.* W niniejszym artykule przedstawiono porównanie wyników prognozowania mocy skrawania za pomocą modelu zawierającego elementy współczesnej mechaniki pękania (energię właściwą na tworzenie nowej powierzchni (wiązkość), naprężenia tnące w strefie ścinania oraz tarcie), z wynikami prognozowania za pomocą opublikowanych równań regresji. W obliczeniach mocy skrawania w procesie rozpiłowywania wzdłużnego na pilarce tarczowej drewna sosnowego (*Pinus sylvestris* L.) stosowano wartości wiązkości i naprężeń tnących w strefie ścinania uzyskane podczas wcześniejszych badań doświadczalnych z wykorzystaniem próbek pochodzących z tej samej Krainy Przyrodniczo-Leśnej. Stwierdzono, brak zgodności wyników prognozowanych mocy skrawania

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