Identification of quasi-periodically varying systems using the local basis function approach

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***Abstract*—In this paper we propose a solution to the problem of tracking quasi-periodically varying systems based on the local basis function (LBF) approach. Within this framework, parameter trajectories are locally approximated using linear combinations of specific functions of time known as basis functions. We derive both bias and variance characteristics of LBF estimators. Additionally, we demonstrate that the computational burden associated with LBF estimation algorithms can be significantly reduced, without sacrificing high estimation accuracy, by employing the computationally fast, approximate version of the LBF scheme.**

***Index Terms*—quasi-periodically varying systems, identification, adaptive algorithms, local basis function method**

I. Introduction

This work addresses the identification and tracking of quasi-periodically varying systems, prevalent in various fields. One example is mechanical systems [1], [2], where the cyclic nature of parameter changes is attributed to the presence of rotating elements. Another example is underwater acoustic (UWA) communication systems [3], [4], [5], where channel parameters, representing reflections of the emitted sound waves from surrounding objects (ship, sea bottom, sea surface, etc.) undergo periodic-like changes due to the Doppler effect. The Doppler effect, primarily induced by the relative transmitter-object motion and sea surface movement, varies across different propagation paths [7], [8]. Given that both the amplitudes and frequencies of parameter changes may fluctuate over time due to relative speed variations and environmental factors, accurate tracking is crucial for the proper functioning of the UWA communication system.

The methods presented in this paper are noncausal, hence, can be implemented only in applications allowing almost real-time operation. An example of such application from the UWA communication domain is self-interference (SI) cancellation in UWA communication systems operating in full-duplex mode. In this application both communication devices transmit and receive signals simultaneously, using the same bandwidth [9], [10], [11]. The operational principle of these devices leads to contamination of the signal received from the far-end transmitter with the signal produced by a near-end transmitter and its reflections from underwater objects. This self-interference should be canceled, which requires accurate estimates of quasi-periodically varying SI channel.

The signals in the UWA communication systems are typically complex-valued. Hence we will adopt the following notation: $\mathbf{x}^*$ denotes complex conjugate and $\mathbf{x}^{\mathrm{H}}$ is a Hermitian (conjugate) transpose of a matrix or vector.

II. Problem formulation

Usually the UWA communication channel is modeled using the finite impulse response (FIR) model [3], [5], [6]

$$y(t) = \sum_{i=1}^{p} \beta_i^*(t) u(t - s_i) + e(t), \tag{1}$$

where $t = \ldots, -1, 0, 1, \ldots,$ is a discrete time, $\{u(t)\}$ represents transmitted signal, $\{e(t)\}$ is a measurement noise, and $\{y(t)\}$ denotes received signal. The parameter $\beta_i^*(t) = \theta_i^*(t) e^{j\omega_0 n_i t}$ is quasi-periodically time-varying where $\theta_i(t)$ is the slowly time-varying delay-Doppler-spread function, and $(s_i, n_i) \in \mathcal{P} = \{(s_1, n_1), \ldots, (s_p, n_p)\}$ are pairs corresponding to p different scatterers at different time delay and Doppler shift. Their values can be found prior to identification [12]. Active time delays can be also estimated effectively during identification using the preestimation technique [11]. The frequency is defined as $\omega_0 = 2\pi\Delta\nu$, where $\Delta\nu$ is the Doppler spacing. For more details on the interpretation of these quantities, see [4]. Many dedicated techniques have been developed for the identification of such systems, with the best-known method being the time-updated recursive least squares (TU-RLS) method [4], [13]. Also, classical general-purpose methods for the identification of nonstationary systems, such as the local basis function (LBF) method [14], can be employed.

In this study we improve the approach based on the LBF method by exploiting the available prior knowledge of the "structure" of parameter changes.

First, we rewrite the model equation (1) as

$$y(t) = \sum_{i=1}^{p} \theta_i^*(t) e^{j\omega_0 n_i t} u(t - s_i) + e(t) = \boldsymbol{\theta}^{\mathrm{H}}(t)\boldsymbol{\varphi}(t) + e(t). \tag{2}$$

Computer simulations were carried out at the Academic Computer Centre CI TASK in Gdańsk.

In the equation above, $\boldsymbol{\theta}(t) = [\theta_1(t), \ldots, \theta_p(t)]^{\mathrm{T}}$ is a vector of time-varying scaling factors, and $\boldsymbol{\varphi}(t) = [e^{j\omega_0 n_1 t}u(t - s_1), \ldots, e^{j\omega_0 n_p t}u(t - s_p)]^{\mathrm{T}}$ is the regression vector.

III. Identification using the local basis function (LBF) method

Assume that each scaling factor $\theta_i(t),\ i = 1, \ldots, p$ can be modeled, within the analysis window $T_k(t) = [t - k, t + k]$ centered on the time instant t, by a linear combination of known, linearly independent functions of time, called basis functions

$$\begin{gathered}\theta_i(t + \tau) = \sum_{l=1}^{m} f_l^*(\tau) a_{il}(t) = \mathbf{f}^{\mathrm{H}}(\tau)\boldsymbol{\alpha}_i(t),\\ \tau \in I_k = [-k, k], i = 1, \ldots, p,\end{gathered} \tag{3}$$

where $\mathbf{f}(\tau) = [f_1(\tau), \ldots, f_m(\tau)]^{\mathrm{T}}$ is the vector of basis functions, and $\boldsymbol{\alpha}_i(t) = [a_{i1}(t), \ldots, a_{im}(t)]^{\mathrm{T}}$ is the vector of basis function coefficients. It is assumed that the basis function coefficients are constant within the analysis window. However, since the estimation is performed in a sliding window mode, their values can change over time, hence the time dependence. Within this work we adopt the orthonormal discrete-time Legendre polynomials (ODLPs) [15].

Under the assumption above, the output signal can be locally modeled as

$$y(t + \tau) = \boldsymbol{\alpha}^{\mathrm{H}}(t)\boldsymbol{\psi}(t, \tau) + e(t + \tau), \tau \in I_k, \tag{4}$$

where $\boldsymbol{\psi}(t, \tau) = \boldsymbol{\varphi}(t + \tau) \otimes \mathbf{f}(\tau)$ is the generalized regression vector, and $\otimes$ denotes the Kronecker product. The LBF estimates of system parameters can be computed as

$$\begin{aligned}\widehat{\boldsymbol{\alpha}}_{\mathrm{LBF}}(t) &= \arg\min_{\boldsymbol{\alpha}} \sum_{\tau=-k}^{k} |y(t + \tau) - \boldsymbol{\alpha}^{\mathrm{H}}\boldsymbol{\psi}(t, \tau)|^2\\ &= \mathbf{P}^{-1}(t)\mathbf{p}(t)\\ \widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t) &= \mathbf{F}(0)\widehat{\boldsymbol{\alpha}}_{\mathrm{LBF}}(t),\end{aligned} \tag{5}$$

where

$$\begin{aligned}\mathbf{P}(t) &= \sum_{\tau=-k}^{k} \boldsymbol{\psi}(t, \tau)\boldsymbol{\psi}^{\mathrm{H}}(t, \tau)\\ \mathbf{p}(t) &= \sum_{\tau=-k}^{k} \boldsymbol{\psi}(t, \tau) y^*(t + \tau),\end{aligned} \tag{6}$$

and $\mathbf{F}(\tau) = \mathbf{I}_p \otimes \mathbf{f}^{\mathrm{H}}(\tau),\ \tau \in I_k$, where $\mathbf{I}_p$ is a $p \times p$ identity matrix.

Note that if the basis functions are recursively computable, i.e., if there exists a $m \times m$ matrix $\mathbf{A}$ such that $\mathbf{f}(i-1) = \mathbf{A}\mathbf{f}(i)$, then the generalized regression matrix $\mathbf{P}(t)$ and the vector $\mathbf{p}(t)$ can be recursively updated using the formulas given in [14].

IV. Properties of the LBF method

The properties of the LBF estimator will be derived under the following assumptions

(A1) $\{u(t)\}$ is a sequence of zero-mean, independent circular complex random variables with variance σ_u^2 and fourth statistical moment $\mathrm{E}[|u(t)|^4] = c_u^4$.

(A2) $\{e(t)\}$, independent of $\{u(t)\}$, is a sequence of zero-mean, independent and identically distributed (i.i.d.) circular complex random variables of variance σ_e^2.

The first assumption is typically met in UWA communication, where the input signal can be modeled as an i.i.d. sequence of the form $u(t) = \pm 1 \pm j$.

Under (A1) and (A2), the generalized regression matrix tends to its expected value in a mean squared sense

$$\lim_{k \to \infty} \mathbf{P}(t) = \sigma_u^2 \mathbf{I}_{mp} \quad \text{m.s.} \tag{7}$$

This fact can be easily proven using a technique similar to the one presented in [16], after noting that the values of the ODLPs can be easily bounded for growing k.

The values of ODLPs can be found using the following formulae [15]

$$\begin{gathered}f_{l+1}(\tau) = b_{l,K} \sum_{s=0}^{l} (-1)^s \binom{l}{s}\binom{l+s}{s} \frac{(\tau + k)^{(s)}}{(2k)^{(s)}},\\ \tau \in I_k, l = 0, \ldots, m - 1,\end{gathered} \tag{8}$$

where

$$\tau^{(s)} = \begin{cases} 1 & \text{if} \quad s = 0 \\ \tau(\tau - 1)\ldots(\tau - s + 1) & \text{if} \quad s > 0 \end{cases}$$

is the backward factorial function of order s, and $b_{l,K}$ is the normalizing constant [15]

$$b_{l,k} = \sqrt{\frac{(2l + 1)(2k)^{(l)}}{(2k + l)^{(l+1)}}}. \tag{9}$$

The aforementioned bound, which follows directly from (8) and (9), has the form

$$\exists_{c>0} \forall_{\tau \in I_k} \forall_{l \in \{1,2,\ldots,m\}} |f_l(\tau)| \leq \frac{c}{\sqrt{k}}. \tag{10}$$

The asymptotic result (7) justifies the approximation that holds when k becomes sufficiently large

$$\begin{aligned}\widehat{\boldsymbol{\alpha}}_{\mathrm{LBF}}(t) &\cong \frac{1}{\sigma_u^2}\mathbf{p}(t)\\ \widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t) &\cong \frac{1}{\sigma_u^2}\mathbf{F}(0)\mathbf{p}(t) = \frac{1}{\sigma_u^2} \sum_{\tau=-k}^{k} g(\tau)\boldsymbol{\varphi}(t + \tau) y^*(t + \tau),\end{aligned} \tag{11}$$

where $g(\tau) = \mathbf{f}^{\mathrm{H}}(0)\mathbf{f}(\tau),\ \tau \in I_k$. The last transition follows from the properties of the Kronecker product.

A. Bias

Note that

$$\begin{aligned}\widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t) \cong\ & \frac{1}{\sigma_u^2} \sum_{\tau=-k}^{k} g(\tau)\boldsymbol{\varphi}(t + \tau)\boldsymbol{\varphi}^{\mathrm{H}}(t + \tau)\boldsymbol{\theta}(t + \tau)\\ &+ \frac{1}{\sigma_u^2} \sum_{\tau=-k}^{k} g(\tau)\boldsymbol{\varphi}(t + \tau) e^*(t + \tau).\end{aligned} \tag{12}$$

Hence, if the measurement noise is independent of the input signal, it holds that

$$\mathrm{E}[\widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t)] \cong \sum_{\tau=-k}^{k} g(\tau)\boldsymbol{\theta}(t+\tau), \tag{13}$$

which means that the mean path of LBF estimates is approximately equal to the result of filtering the true parameter trajectory using a noncausal FIR filter with basis-dependent impulse response $g(\tau)$. It is easy to check that the filter based on ODLPs is low-pass.

If the parameter trajectories inside the analysis window $T_k(t)$ are exactly linear combinations of the chosen basis functions, then the LBF estimator is unbiased. Otherwise, the average estimated trajectory can be seen as an orthogonal projection of the true trajectory onto a subspace spanned by the basis functions.

B. Covariance matrix

Assume that parameter trajectories are linear combinations of the chosen basis functions within the analysis window $T_k(t)$ (i.e., the LBF estimator is unbiased). Note that this condition is approximately met if the number of basis functions m is sufficiently large. Using (3), the estimation error can be written in the form

$$\begin{aligned}\Delta\boldsymbol{\theta}(t) &= \widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t) - \boldsymbol{\theta}(t)\\ &\cong \mathbf{F}(0)\left[\frac{1}{\sigma_u^2}\sum_{\tau=-k}^{k}\boldsymbol{\psi}(t,\tau)\boldsymbol{\psi}^{\mathrm{H}}(t,\tau) - \mathbf{I}_{mp}\right]\boldsymbol{\alpha}(t)\\ &+ \frac{1}{\sigma_u^2}\sum_{\tau=-k}^{k} g(\tau)\boldsymbol{\varphi}(t+\tau)e^*(t+\tau) = \mathbf{v}_1(t) + \mathbf{v}_2(t).\end{aligned} \tag{14}$$

Since $\mathrm{E}[\mathbf{v}_1(t)\mathbf{v}_2^{\mathrm{H}}(t)] = \mathrm{E}[\mathbf{v}_2(t)\mathbf{v}_1^{\mathrm{H}}(t)] = 0$ due to the mutual independence of the measurement noise and input signal, the covariance matrix is given by

$$\begin{aligned}\mathrm{cov}[\widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t)] &= \mathrm{E}[\Delta\boldsymbol{\theta}(t)\Delta\boldsymbol{\theta}^{\mathrm{H}}(t)]\\ &= \mathrm{E}[\mathbf{v}_1(t)\mathbf{v}_1^{\mathrm{H}}(t)] + \mathrm{E}[\mathbf{v}_2(t)\mathbf{v}_2^{\mathrm{H}}(t)],\end{aligned} \tag{15}$$

It can be shown [16] that the elements of $\mathrm{E}[\mathbf{v}_1(t)\mathbf{v}_1^{\mathrm{H}}(t)]$ are $O\left(\frac{1}{k}\right)$. So, for k large enough and under (A2), the following approximation holds

$$\begin{aligned}\mathrm{cov}[\widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t)] &\cong \mathrm{E}[\mathbf{v}_2(t)\mathbf{v}_2^{\mathrm{H}}(t)]\\ &= \frac{1}{\sigma_u^4}\sum_{\tau=-k}^{k}|g(\tau)|^2\mathrm{E}\left[\boldsymbol{\varphi}(t+\tau)\boldsymbol{\varphi}^{\mathrm{H}}(t+\tau)\right]\sigma_e^2\\ &= \frac{\sigma_e^2}{\sigma_u^2 L_{\mathrm{LBF}}}\mathbf{I}_p,\end{aligned} \tag{16}$$

where $L_{\mathrm{LBF}} = \left[\sum_{\tau=-k}^{k}|g(\tau)|^2\right]^{-1}$ is called the equivalent memory of the LBF estimator.

V. Fast local basis function (fLBF) method

The LBF method provides high accuracy estimates at the cost of high computational burden. This computational cost can be reduced by using the fast version of the LBF approach - the fLBF method [17]. This approach consists of two steps: preestimation and postfiltering. The first step allows obtaining approximately unbiased estimates, regardless of the type and speed of parameter changes. The unbiasedness of preestimates comes at the cost of a high variance of preestimation errors. Therefore, additional filtering is necessary, and for this purpose, the Savitzky-Golay filtering [18] is usually chosen [17].

The computation of the preestimates starts by finding exponentially weighted least squares (EWLS) estimates of system parameters

$$\widehat{\boldsymbol{\theta}}_{\mathrm{EWLS}}(t) = \arg\min_{\boldsymbol{\theta}}\sum_{\tau=1}^{t}\lambda^{t-\tau}[y(\tau) - \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{\varphi}(\tau)]^2, \tag{17}$$

where $\lambda \in (0,1)$ is the so-called forgetting constant. The steady-state formula for preestimates boils down to the "inverse filtering" of the EWLS estimates

$$\widetilde{\boldsymbol{\theta}}(t) = \frac{1}{1-\lambda}[\widehat{\boldsymbol{\theta}}_{\mathrm{EWLS}}(t) - \lambda\widehat{\boldsymbol{\theta}}_{\mathrm{EWLS}}(t-1)]. \tag{18}$$

The next step, postfiltering, is based on the model (3) and leads to the following fLBF (fast LBF) estimates, approximating the LBF estimates

$$\begin{aligned}\widehat{\boldsymbol{\alpha}}_{\mathrm{fLBF}}(t) &= \arg\min_{\boldsymbol{\alpha}}\sum_{\tau=-k}^{k}|\widetilde{\boldsymbol{\theta}}(t+\tau) - \mathbf{F}(\tau)\boldsymbol{\alpha}|^2\\ \widehat{\boldsymbol{\theta}}_{\mathrm{fLBF}}(t) &= \mathbf{F}(0)\widehat{\boldsymbol{\alpha}}_{\mathrm{fLBF}}(t).\end{aligned} \tag{19}$$

It can be shown that this procedure simplifies to the filtering operation [17]

$$\widehat{\boldsymbol{\theta}}_{\mathrm{fLBF}}(t) = \sum_{\tau=-k}^{k} g(\tau)\widetilde{\boldsymbol{\theta}}(t+\tau). \tag{20}$$

It was shown in [17] that, in spite of its computational simplicity, the fLBF scheme produces estimates that closely approximate LBF estimates, i.e., $\widehat{\boldsymbol{\theta}}_{\mathrm{fLBF}}(t) \cong \widehat{\boldsymbol{\theta}}_{\mathrm{LBF}}(t)$.

VI. Adaptive choice of m and k

The number of basis functions m and the length of the analysis window $K = 2k+1$ are crucial hyperparameters for the LBF and fLBF methods. They control two components of the mean squared parameter estimation error (MSE) - the bias component and the variance component. Increasing m and decreasing k increases the modeling capabilities and thus reduces the bias at the cost of increasing the variance. The opposite effect occurs when decreasing m and increasing k. Therefore, the values of m and k should be carefully chosen to achieve a trade-off between the bias and variance components of the MSE. This can be done by parallel computation. Suppose that M estimators equipped with different values of $(m,k) \in \mathcal{M} = \{(m_1,k_1),\ldots,(m_M,k_M)\}$ are run in parallel. At each time instant we choose the estimator that minimizes the local sum of squares of the leave-one-out interpolation errors

$$m(t), k(t) = \underset{(m,k)\in\mathcal{M}}{\arg\min}\ J_{\mathrm{LBF}}(t,m,k), \tag{21}$$

where

$$J_{\mathrm{LBF}}(t,m,k) = \sum_{\tau=-N}^{N} |\varepsilon^{o}_{\mathrm{LBF}}(t+\tau,m,k)|^2, \tag{22}$$

N denotes the half-width of the local decision window and

$$\varepsilon^{o}_{\mathrm{LBF}}(t,m,k) = y(t) - [\widehat{\boldsymbol{\theta}}^{o}_{\mathrm{LBF}}(t,m,k)]^{\mathrm{H}}\boldsymbol{\varphi}(t). \tag{23}$$

The estimates $\widehat{\boldsymbol{\alpha}}^{o}(t,m,k)$ are obtained by excluding the central observation from the analysis window

$$\begin{aligned}\widehat{\boldsymbol{\alpha}}^{o}_{\mathrm{LBF}}(t,m,k) &= \arg\min_{\boldsymbol{\alpha}} \sum_{\substack{\tau=-k\\ \tau\neq 0}}^{k} |y(t+\tau) - \boldsymbol{\alpha}^{\mathrm{H}}\boldsymbol{\psi}(t,\tau)|^2 \\ \widehat{\boldsymbol{\theta}}^{o}_{\mathrm{LBF}}(t,m,k) &= \mathbf{F}(0)\widehat{\boldsymbol{\alpha}}^{o}_{\mathrm{LBF}}(t,m,k).\end{aligned} \tag{24}$$

Using the matrix inversion lemma [19], one can derive the formula for the leave-one-out interpolation error $\varepsilon^{o}(t,m,k)$ which does not require solving (24)

$$\varepsilon^{o}_{\mathrm{LBF}}(t,m,k) = \frac{\varepsilon_{\mathrm{LBF}}(t,m,k)}{1-\delta(t,m,k)}, \tag{25}$$

where $\delta(t,m,k) = \boldsymbol{\psi}^{\mathrm{H}}(t,0)\mathbf{P}^{-1}(t)\boldsymbol{\psi}(t,0)$, and $\varepsilon_{\mathrm{LBF}}(t,m,k) = y(t) - \widehat{\boldsymbol{\theta}}^{\mathrm{H}}_{\mathrm{LBF}}(t,m,k)\boldsymbol{\varphi}(t)$ is the interpolation error.

As demonstrated in [20], the quantity $\delta(t,m,k)$ can be approximated using the following formula

$$\bar{\delta}(m,k) = \mathrm{E}[\delta(t,m,k)] \cong p\,\mathbf{f}^{\mathrm{H}}(0)\mathbf{f}(0). \tag{26}$$

For the fLBF estimators, a similar procedure based on the sums of the squared leave-one-out interpolation errors can be constructed, resulting in [17]

$$\varepsilon^{o}_{\mathrm{fLBF}}(t,m,k) = \frac{\varepsilon_{\mathrm{fLBF}}(t,m,k) - g(0)\widetilde{\varepsilon}(t)}{1-g(0)}, \tag{27}$$

where $\varepsilon_{\mathrm{fLBF}}(t) = y(t) - \widehat{\boldsymbol{\theta}}^{\mathrm{H}}_{\mathrm{fLBF}}(t)\boldsymbol{\varphi}(t)$, and $\widetilde{\varepsilon}(t) = y(t) - \widetilde{\boldsymbol{\theta}}^{\mathrm{H}}(t)\boldsymbol{\varphi}(t)$.

VII. Computer simulations

For testing the proposed algorithms, we utilized a simulated UWA communication channel with three parameters. The experiments, as elaborated in [12], demonstrate the practical utility of even such a simple model. The received (output) signal is modeled as

$$\begin{aligned} y(t) &= \theta_1^*(t)e^{j\omega_0 t}u(t-2) + \theta_2^*(t)e^{2j\omega_0 t}u(t-3) \\ &\quad + \theta_3^*(t)e^{4j\omega_0 t}u(t-10) + e(t). \end{aligned} \tag{28}$$

The parameters $\theta_i(t)$, $i = 1,2,3$, were generated as lowpass-filtered zero-mean circular Gaussian noises with a variance of 2, sampled at frequency of 1000 Hz with a bandwidth of 1 Hz. We set $\omega_0 = 0.002\pi$.

The input signal was a circular random sequence of the form $u(t) = \pm 1 \pm j$. The noise was a random circular Gaussian sequence with variance equal to $\sigma_e^2 = 3.8$, or $\sigma_e^2 = 0.38$, corresponding to signal-to-noise ratios (SNRs) of 5 and 15 dB, respectively. To mitigate boundary effects, the signal generation started 500 time steps before identification and concluded 500 time steps after the identification process was halted.

The TU-RLS algortihm was implemented according to [13], and its parameters μ and λ were chosen heuristically for each SNR value to minimize the MSE score. They were set to $\mu = 0.0006$, $\lambda = 0.96$ for SNR equal to 5 dB and $\mu = 0.002$, $\lambda = 0.92$ for SNR equal to 15 dB.

All the results presented below were obtained by comparing $\beta_i(t) = \theta_i(t)e^{j\omega_0 n_i t}$, $i = 1,2,3$ with their estimates.

Table 1 presents the averaged MSE values (in decibels) over 100 independent realizations of measurement noise for the "old" LBF method, based on the model (1), and the "new" one proposed in this paper, based on the model (2). The letter "A" denotes the adaptive algorithm, combining results provided by the algorithms equipped with $m \in \{1,3,5\}$ and $k \in \{50,100,200\}$, and "A0" denotes the adaptive algorithm obtained after replacing $\delta(t,m,k)$ with $\bar{\delta}(m,k)$. For both adaptive algorithms $N = 30$.

TABLE I
THE MSE SCORE (IN DECIBELS) AVEREGED OVER 100 INDEPENDENT REALIZATIONS OF THE MEASUREMENT NOISE, FOR THE LBF ALGORITHM DESCRIBED IN [14] (OLD LBF) AND THE LBF APPROACH DESCRIBED IN THIS PAPER (NEW LBF).

SNR 15 dB

Algorithm	$m\backslash k$	50	100	200
old LBF	1	-7.30	1.66	6.14
	3	-18.57	-12.67	2.36
	5	-16.68	**-19.71**	-6.64
new LBF	1	-21.05	-18.70	-9.41
	3	-18.76	-21.86	**-24.75**
	5	-16.68	-19.86	-22.94
	A		-22.19	
	A0		-22.11	
TU-RLS			-14.51	

SNR 5dB

Algorithm	$m\backslash k$	50	100	200
old LBF	1	-6.23	1.73	6.15
	3	-8.73	-9.47	2.43
	5	-6.68	**-9.83**	-5.80
new LBF	1	-12.26	-14.10	-8.99
	3	-8.76	-11.86	**-14.89**
	5	-6.68	-9.86	-12.94
	A		-12.37	
	A0		-12.30	
TU-RLS			-7.62	

The results for the "old" fLBF approach [17], based on the model (1), and the "new" fLBF approach, based on the model (2), are gathered in Table 2. The forgetting constant $\lambda = 0.9$ (EWLS) was chosen according to the rule proposed in [11]. As shown in the tables above, the LBF and fLBF algorithms proposed in this paper for identifying systems with quasi-

TABLE II
THE MSE (IN DECIBELS) SCORE AVEREGED OVER 100 INDEPENDENT REALIZATIONS OF MEASUREMENT NOISE FOR THE FLBF ALGORITHM DESCRIBED IN [17] (OLD FLBF) AND THE FLBF ALGORITHM PROPOSED IN (17) - (19) (NEW FLBF).

SNR 15 dB

Algorithm	$m\backslash k$	50	100	200
old fLBF	1	-7.83	1.61	6.08
	3	-18.04	-12.90	2.31
	5	-16.12	**-19.12**	-6.74
new fLBF	1	-21.93	-19.88	-9.70
	3	-18.73	-21.72	**-24.61**
	5	-16.80	-19.78	-22.78
	A		-21.93	

SNR 5 dB

Algorithm	$m\backslash k$	50	100	200
old fLBF	1	-6.61	1.69	6.10
	3	-8.68	-9.52	2.39
	5	-6.76	**-9.72**	-5.86
new fLBF	1	-12.25	-14.35	-9.22
	3	-8.76	-11.74	**-14.74**
	5	-6.84	-9.81	-12.80
	A		-12.25	

periodically changing parameters consistently outperform the algorithms discussed in [14], [17], and [4]. The adaptive algorithms consistently match or outperform the best algorithms employed in parallel computations, highlighting the adaptive scheme as a crucial practical tool. An example of estimates obtained for SNR equal to 5 dB using the TU-RLS method, the "old" LBF method, and the "new" LBF method is illustrated in Figure 1.

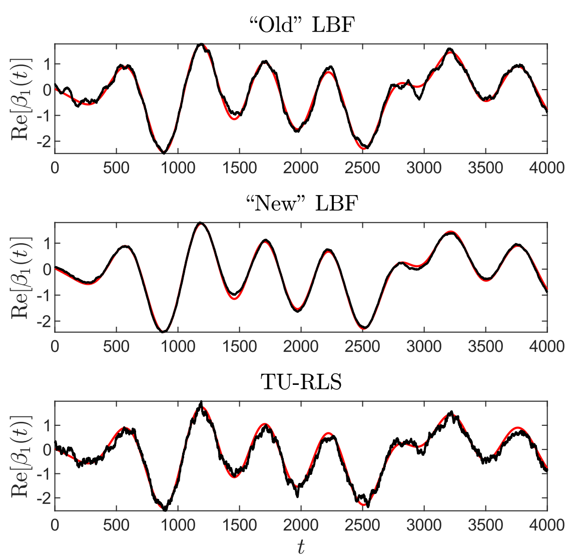

Fig. 1. Comparison of the real part of the estimated parameter trajectory (black line) obtained for three different identification methods, superimposed on the real part of the true parameter trajectory (red line).

VIII. CONCLUSION

This paper introduced a new method for identifying systems with quasi-periodically changing parameters, utilizing insights into the structure of these parameter changes to improve estimation outcomes. We explored the characteristics of the proposed LBF method and introduced its fast, computationally efficient version. For the selection of hyperparameters in both LBF and fLBF algorithms, we suggested an adaptive scheme relying on leave-one-out cross-validation. The algorithms developed in this study were effectively benchmarked against the currently available state-of-the-art algorithms.

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