

# Imperfection sensitivity of multilayered composite shells

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**ABSTRACT:** Stability of multilayered composite shells under axial compression is studied. To estimate the critical load level the linear eigenvalue problem as well as the nonlinear incremental analysis is performed. The focus of authors' interest is the sensitivity of analyzed panels to initial geometrical imperfections. Since the loss of stability can be qualified as a global phenomenon, the multilayered shell panel is treated as an equivalent single layer with a resultant stiffness. The computations were performed with the commercial code NX-Nastran.

## 1 INTRODUCTION

Shells are generally thin slender structures or light members of other constructions. Composites, characterized by high level of strength-to-weight and stiffness-to-weight ratio, can be seen as the most suitable materials for modern slender constructions. Typically composite shells are multilayered structures, in which laminas are made of isotropic materials or composites. Since most common composites are reinforced with uniaxial fibers, a single layer represents an orthotropic body. If the fiber orientation varies in the stacking sequence of layers, then the whole structure is an anisotropic medium. The anisotropy is an advantageous feature for design purposes but on the other hand it complicates the laminates' analysis and as a consequence the design process becomes complex.

The aim of this work is to analyze multilayered shells subjected to axial compression. Since the load capacity of such structures is to a high degree dependent on loss of stability, the crucial point of this work is to estimate the critical load level. Special attention is paid to the examination of the influence of initial geometrical imperfections on the structure response.

## 2 STABILITY OF SHELLS

The stability of structures depends on many aspects, especially on slenderness, boundary conditions and imperfections of various nature, like material, load

and geometrical imperfections. To evaluate the critical load level several methods can be used. The simplest one is the Linear Eigenvalue Problem.

$$(\mathbf{K}^{\text{con}} + \lambda \mathbf{K}^{\sigma}) \mathbf{v} = \mathbf{0} \quad (1)$$

where  $\mathbf{K}^{\text{con}}$  is the constitutive stiffness matrix,  $\mathbf{K}^{\sigma}$  stays for the geometrical stiffness matrix,  $\mathbf{v}$  is the eigenvector and  $\lambda$  is the critical load multiplier.

However, the solution obtained with this method can be incorrect if the structure undergoes large deformations in the pre-buckling range. In this case geometrically nonlinear incremental analysis should be performed:

$$(\mathbf{K}_T(\mathbf{q})) \Delta \mathbf{q} = \mathbf{R}(\mathbf{q}) \quad (2)$$

where  $\mathbf{K}_T(\mathbf{q})$  is the tangent stiffness matrix,  $\Delta \mathbf{q}$  is the displacement increment vector and  $\mathbf{R}(\mathbf{q})$  depicts the balanced force vector.

This approach is obviously much more expensive from the computational point of view and it requires a little bit of experience, but if the arc-length strategy is used in following of equilibrium path, then also the behavior of structure in post-buckling range can be examined.

As mentioned previously, the authors focus on the dependence of critical load level on geometrical imperfections. Understandable is the fact, that the realistic imperfection field would be the best one to include in the numerical model. However, if the structure is just to-be-designed, there is no knowledge about real imperfections and some assumptions have to be made. Of course, to preserve the structure safety, the worst imperfection pattern should be presumed in calculations, however, it is

not obvious. Arbocz & Starnes (2002) described the lower-bound approach to design of thin-walled structures that is basing on empirical data (see also Hühne et al., 2008), but it concerns only isotropic shells. Such a philosophy in design of composite shells does not exist (Arbocz & Starnes, 2002) and in this case other approaches should be applied.

One of the imperfect geometrical patterns can be the lowest eigenmode shape obtained in linear eigenvalue problem. However, if the structure response is nonlinear below the first critical load, then this method seems to be useless. Such an imperfection field was classified by Winterstetter & Schmidt (2002), Schmidt (2000) as the ‘worst’ beside the ‘realistic’ and ‘stimulating’ geometric imperfections. ‘Worst’ imperfections are understood as mathematically determined possibly worst imperfections like eigenmode shape or single buckles. It is worth to mention, that Deml & Wunderlich (1997) have proven that in some cases single buckles can be worse than eigenmode shapes covering larger parts of a structure. Nonetheless, this classification of imperfections and comparison between single buckles and eigenmode shape were related to steel shells.

In this work a stability analysis of composite panels is considered. Only numerical investigations are made. The disturbances of perfect shell geometry in a form of first eigenmode shape are assumed.

### 3 LAMINATES IN NX-NASTRAN

In the computations, the Equivalent Single Layer (ESL) concept is applied. It means that the laminated shell is treated as a single layer panel with resultant stiffness of a multilayered cross-section. Such a simplification can be justified by the global character of the analyzed phenomena, Altenbach & Altenbach (2001). The FEM calculations were performed with the commercial code NX-Nastran (Ver. 6.0). The fundamental assumptions of the computational model are summarized below. The layers are perfectly bonded together and no slip between them is possible. As one can read in the NX-Nastran User’s Guide, the formulation of the laminated finite element is basing on the Classical Lamination Theory (CLT). This statement is a little bit confusing, because the term CLT is well known as the simplest approach for laminates which bases on the Kirchhoff’s theory assumptions, i.e. transverse shear is not taken into account, see Jones (1975). However, in the laminates’ model in NX-Nastran transverse shear is included and from this point of view this approach corresponds rather with the Reissner-Mindlin type theory. Moreover the linear displacement distribution through the thickness is assumed. In authors’ opinion this formulation should be rather classified as the First

Order Shear Deformation (FOSD) theory. It is worth noticing, that no shear correction factors are presumed, but instead the equivalent transverse shear moduli for the whole cross-section are evaluated numerically by assuming the cylindrical bending of the shell. This approach seems to be very efficient and attractive in comparison to other, sometimes more complicated, formulations, see e.g. Sabik & Kreja (2008).

### 4 NUMERICAL EXAMPLE

This example was proposed by Becker et al. (1981) who analyzed behavior of laminated cylindrical panels axially compressed between two stiff plates. The straight edges of the shell are simply supported with possibility to move along the generatrix. Geometrical data are given in Figure 1. Since the load is applied to the top plate the rigid movement of the top curved edge of the shell is provided. All layers have equal thickness and are made of T300/5208 graphite epoxy composite with following material data:  $E_1=20.5E6$  psi,  $E_2=1.3E6$  psi,  $G_{12}=0.75E6$  psi,  $\nu_{12}=0.335$ . Three lamination schemes are studied: cross-ply  $(90,0)_{2s}$ , quasi-isotropic  $(90,\pm 45,0)_s$  and angle-ply  $(\pm 45)_{2s}$ .

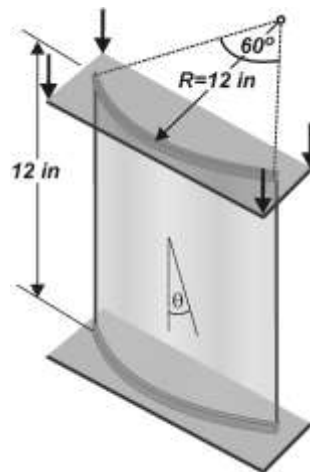


Figure 1. Axially compressed cylindrical panel.

Figure 2 illustrates the comparison between linear solution (linearized buckling analysis) and geometrically nonlinear incremental analysis. In all cases the shell was discretized with 4-node elements QUAD4. Convergence studies have confirmed that the mesh of 60x60 elements provided acceptable accuracy of results. Solid lines in Figure 2 represent the equilibrium paths for axial displacement of loaded edge ( $u$ ) versus normalized load ( $P=P_{act}L/t^3E_1$ , where  $L$  and  $t$  are the length and the thickness of the panel respectively and  $P_{act}$  is the actual applied force). Dashed lines depict the level of the lowest critical load obtained from the linear eigenvalue problem for each lamination scheme.

A visible disagreement can be observed in Figure 2 between the lowest critical load and the limit load

level determined in nonlinear incremental analysis in the case of angle-ply lay-up. This is a direct consequence of the nonlinear response of the shell in the pre-buckling range. With regard to the fact, that the disturbances of the perfect geometry are assumed in a form of the first eigenmode, the imperfection sensitivity is studied only for cross-ply and quasi-isotropic case. Additionally, Figures 3 and 4 illustrate the comparison between the first eigenmode shape and the deformation pattern at the maximum load level computed in the nonlinear incremental analysis. An evident similarity of the deformation patterns obtained in the nonlinear analysis for the perfect structure and the first eigenmodes seems to justify the presumption of the imperfection pattern in the form of the first eigenmode.

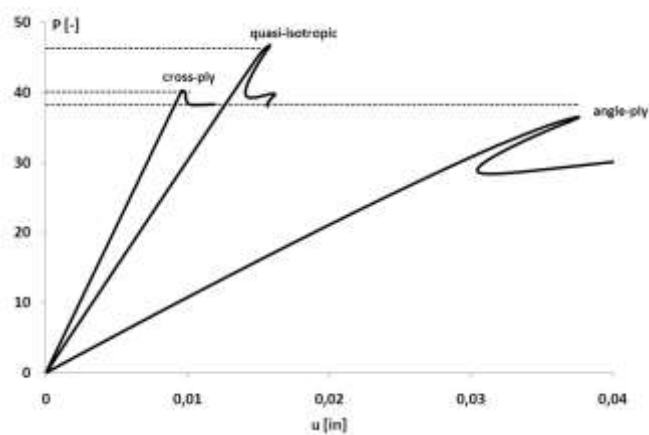


Figure 2. Axial displacement vs. normalized load. Comparison between the results of the linearized buckling analysis and nonlinear incremental analysis.

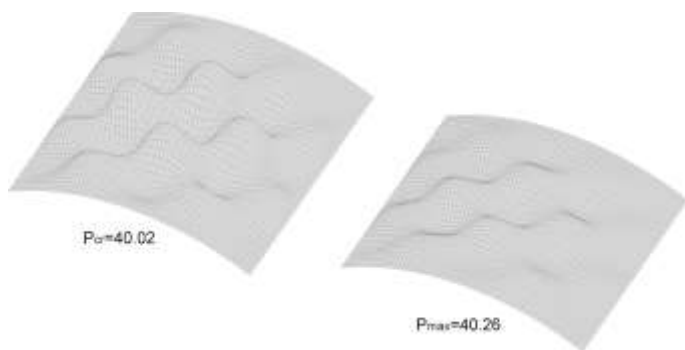


Figure 3. First eigenmode shape and deformation at maximum load level in nonlinear incremental analysis. Cross-ply shell.

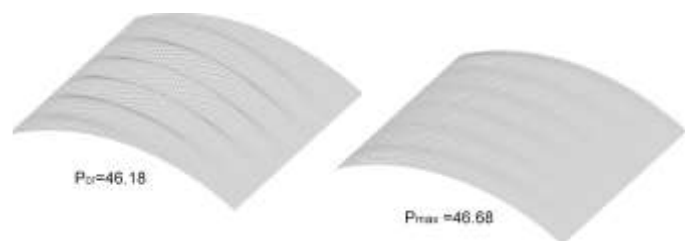


Figure 4. First eigenmode shape and deformation at maximum load level in nonlinear incremental analysis. Quasi-isotropic shell.

Figures 5 and 6 illustrate the imperfection sensitivity for the cross-ply and the quasi-isotropic panel, respectively. The equilibrium paths for the transverse deflection ( $w$ ) at the central point of the panel vs. the normalized load are presented for various imperfection levels which are defined as the proportional ratio between the central initial deflection and the shell thickness.

The bold solid lines in Figures 5 and 6 depict the equilibrium paths obtained for the perfect geometry. Other lines are related to the imperfect geometry modeled in a form of the first eigenmode with amplitude related with respect to the shell thickness as described above. Several levels of imperfection were studied, considering the initial central deflection directed inward as well as outward of the ideal shell curvature.

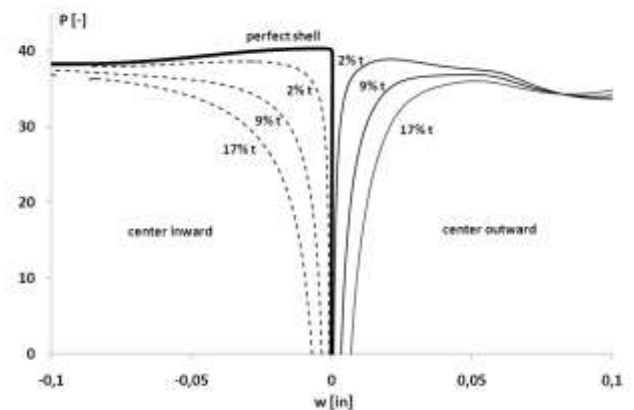


Figure 5. Influence of geometrical imperfections. Cross-ply shell.

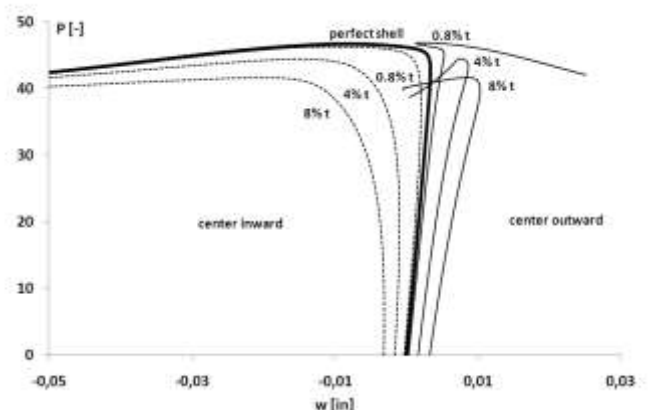


Figure 6. Influence of geometrical imperfections. Quasi-isotropic shell.

The increasing level of initial disturbances causes the decrease of the critical load level regardless of the imperfection sign. The equilibrium path for the perfect cross-ply panel (Fig. 5) corresponds to a classical solution for the structure with a symmetrical bifurcation point. After applying imperfections one can observe load limit points and turning points occurring on the paths. A response for

the quasi-isotropic shell (Fig. 6) is slightly different; here the equilibrium paths possess only load limit points and turning points. The character of the critical point depends on the level and sign of the imperfection.

Due to the symmetry of the geometry, boundary conditions and specific material features distribution in the case of cross-ply shell the obtained results are similar to the isotropic plate stability problem. However, because of the curvature of the shell, there is no symmetry in the graph. The structure undergoes little smaller displacements if the initial central deflection displaces outward. From another point of view, the load capacity is comparable for the same level but opposite sign of imperfection.

Nonsymmetrical distribution of material parameters in the case of quasi-isotropic lay-up is responsible for the significant difference of solution if compared with the results for cross-ply lamination scheme. The limit load is a little bit bigger if outward initial central deflection is presumed. Remarkably is the fact, that with increase of the outward initial central deflection finally after the turning point, the inward movement of the shell center takes place.

The obtained results show that the lamination scheme and global anisotropy of the shell play a very important role and influence significantly the response of structure. Firstly the changes of stiffness are observed, so that not all lay-up cases preserve linear behavior in the pre-buckling range. Secondly, depending on lamination scheme, the behavior of an imperfect shell is in a certain sense unpredictable if one compares the analyzed two lamination schemes.

## 5 CONCLUSIONS

The stability of composite multilayered shells under in-plane axial compression was studied. The critical load level was obtained in linear eigenvalue problem and geometrically nonlinear incremental static analysis. The role of geometrical imperfections, which were defined as the first eigenmode, was examined. The analysis was performed with commercial FEM code NX-Nastran Ver. 6.0.

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