IMPROVING THE ACCURACY OF BEARING IN ACTIVE SONAR WITH CYLINDRICAL ARRAY USING SPECTRUM ESTIMATION

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The articles presents a method for improving the accuracy of bearing in multibeam sonar with a cylindrical array. Based on a known spatial spectrum estimation technique, the method has been successfully used in linear array systems. Its accuracy of bearing is satisfactory and ensures a relatively low computational effort. The article discusses certain simplifications and assumptions to adapt the spatial spectrum estimation technique in linear arrays to the needs of cylindrical arrays. Simulation results are compared to those achieved in the monopulse method.

INTRODUCTION

It is assumed that the accuracy of bearing of a multibeam sonar with a delay-sum beamformer is approximately equal to a three decibel width of the receiving beam. In the case of sonars operating with a multi-element cylindrical array, beam width is usually close to the angular distance between the centres of adjacent array columns. The size of the array and the resulting number of columns and angular distance between them depend on some practical considerations. Because arrays are installed in the ship's hull, space is a limiting factor. On the other hand, columns with evenly spaced transducers should maintain a distance of at least half the wavelength for mid-channel frequency of the sounding signal [1]. Design requirements usually reduce the number of columns to a number in between 36 or 40, spaced at 9°,10°. This means that the system's bearing accuracy will only be around 10°. In many cases, this is not enough.

Although originally designed for beamformers with multi-element linear arrays, some methods can be used to improve bearing accuracy. One of them is based on spatial spectral estimation of the acoustic field distribution in array transducers. A plane wave incident at a specific angle on a linear array causes acoustic field distribution in the array. For a straight line that distribution is sinusoidal. The spatial frequency of the distribution is uniquely related to the angle of wave incidence and may be used for determining the bearing of an underwater sound source [1]. With a finite and usually low number of acoustic field distribution samples, spatial spectrum estimation becomes a problem, especially regarding the spectrum's frequency resolution. As we know, a discreet signal comprising N samples is represented by exactly the same number of spectral lines on the discrete frequency axis. In other words, for a system comprising N transducers, all that can be estimated is N directions of the incoming wave. There are two ways to improve system resolution. The first involves a higher number of acoustic field distribution samples, which means a bigger array and more transducers, as a result. It is both costly and sometimes even impossible for practical reasons. The other method is to produce a virtual array. This involves the generation of additional virtual transducers using information from hydrophones available at the time. Acoustic field distribution using virtual transducers can be estimated with parametric methods of spatial spectrum estimation, such as Burg's method, MUSIC, etc. [2,3].

In the case of a 360° observation, a cylindrical array will be more practical because it covers all angles. The beam pattern of a beam produced from N transducers placed on a section of the array's cylinder is not much different from the beam pattern of a linear array consisting of N transducers spaced evenly on the chord running across the extreme elements of the cylinder's sector section. This suggests that at least to some extent we can apply the high resolution spectrum estimation method, which is typical for a linear array in relation to transducers of a cylindrical array.

The article presents the assumptions made when adapting the spatial spectrum estimation method and the final results. The objective of the method is to improve the resolution of bearing in the beamformer with a cylindrical array at a possibly low computational cost and without the need for changing the design of an existing sonar system.

1. CONCEPT OF THE METHOD

Let us consider a straight line, incident on which in time t_0 is a plane wave of constant and known frequency f_0. Let the angle of incidence between the wave and array acoustic axis be θ_k. Figure 1 shows the situation. The distribution of acoustic field on the line of size x takes this form:

$$p(x,t_0) = p_k \cdot \sin\left(\frac{\omega_0}{c} \cdot \sin\theta_k + \varphi_0 + \varphi_{0k}\right) \tag{1}$$

where: $\omega_0 = 2\pi f_0$, c – speed of propagation of acoustic wave in water, θ_k angle of wave incidence on the array, φ_0 wave phase in time t_0, φ_{0k} initial phase of incoming wave. The relation (1) shows that the distribution of acoustic pressure on the array is sinusoidal. Considering the fact that:
$\omega_0 / c = 2\pi/\lambda_0$, relation (1) may be rewritten to:

$$p(x,t_0) = p_k \cdot \sin\left(2\pi \cdot \sin\theta_k \cdot \frac{x}{\lambda_0} + \varphi_0 + \varphi_{0k}\right) \tag{2}$$

Relation x/λ_0 is interpreted as a spatial variable and treated as time analogue. Relation (2) shows that the distribution of pressure on the straight line has spatial frequency $F_k = \sin\theta_k$. Let us note that the range of incidence angles between the wave and array is limited in the range (-90°, 90°). This means that spatial frequency changes in a limited range as well (-1,1). As a consequence, the band of spatial distribution frequencies is limited. A signal like that can be sampled following Nyquist criterion. The information contained in samples collected at constant and discrete intervals is sufficient and can be used to recreate the original signal. As a consequence, the shape of acoustic pressure distribution on a linear array can be reproduced from its samples collected at array transducers' outputs. Figure 1 shows discrete transducers evenly spaced at d, which corresponds to signal sampling in the domain of time, at constant time intervals. Because, as noted above, spatial frequency meets inequality $-1< F_k <1$, hence – in accordance with Nyquist theorem – the distance between neighbouring transducers should not be greater than:

$$\frac{d}{\lambda_0} \leq \frac{1}{2F_{kmax}} \qquad (3)$$

We determine spatial frequency F_k based on the samples collected and read the wave incidence angle from the relation:

$$\theta_k = \sin^{-1} F_k \qquad (4)$$

Before the above method can be used for a sonar with a multi-element cylindrical array, a virtual linear array must be generated. To that end phases of signals received in array columns must be compensated. For a beamformer generating receiving beams from N transducers, phase compensation should be conducted in a selected array sector on the acoustic axis as shown in Figure 2.

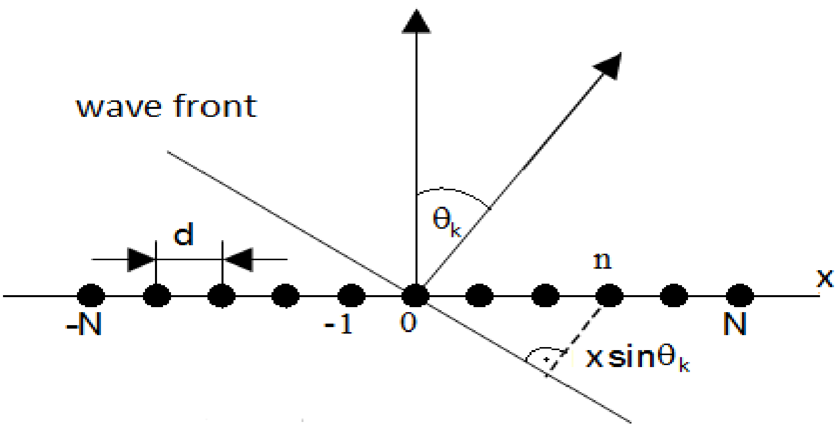

Fig.1. The position of centers of transducer radiating surface in linear antenna.

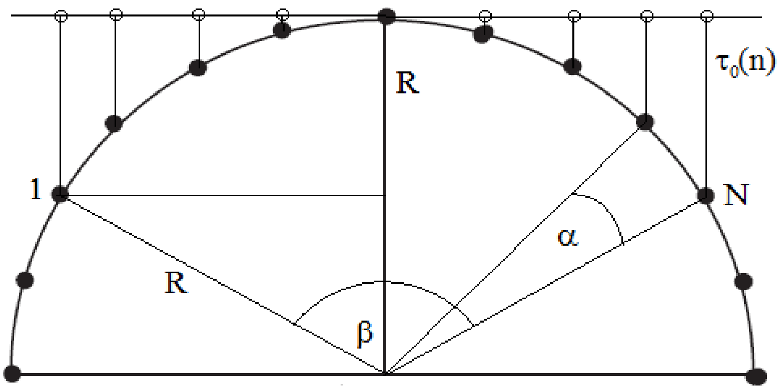

Fig.2. The position of centers of transducer (black points) radiating surface in cylindrical antenna and virtual linear array (white points).

The geometry in Figure 2 shows that delays compensating the phase on the main direction amount to:

$$\tau_0(n) = \frac{R}{c}\left(1 - \cos\left\{(n-1)\alpha - \frac{\beta}{2}\right\}\right) \tag{5}$$

Figure 3 shows beamforming for a linear array with N transducers and a cylindrical array with the same number of elements evenly spaced at d, after phase compensation following relation (5).

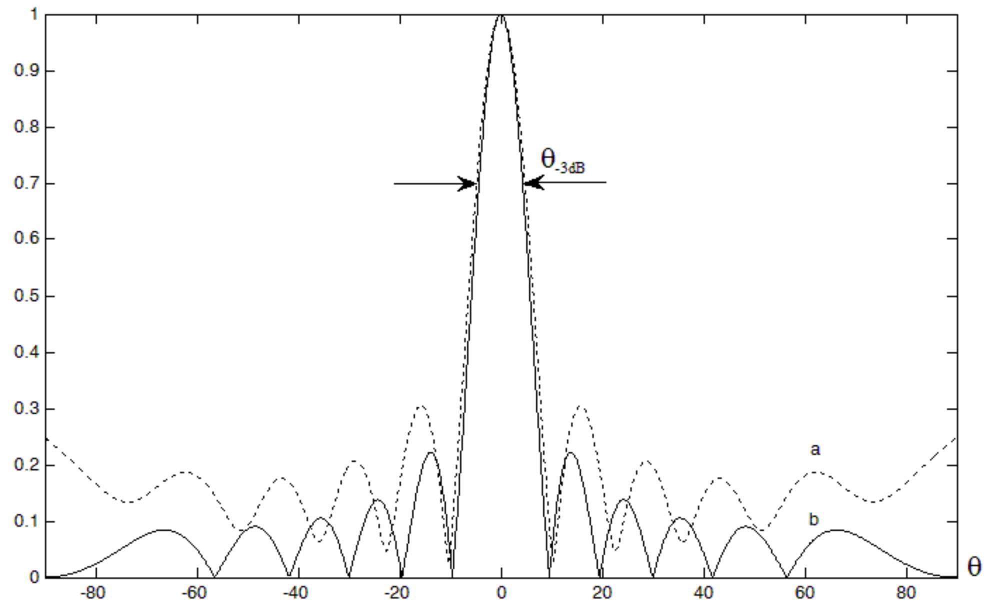

Fig.3. Directivity pattern : a - sector of cylindrical array, b - linear array.

Phase compensation for a cylindrical array creates a virtual linear array. While the transducers are not spaced evenly, the gaps are less than half a wavelength. The beam patterns as shown in Figure 3 suggest that at least to some extent, spatial spectrum estimation can be used to determine bearing.

2. SYSTEM SIMULATION

For the purposes of the simulation a cylindrical array was designed comprising 36 columns, spaced every 10° degrees on the cylindrical surface. The distances between the centres of neighbouring columns are exactly half the wavelength of a sounding signal's mid-channel frequency. To keep it simple, the transducers were contact transducers with a 360° beam pattern. A plane wave of frequency f_0 is incident on the array's transducers in the presence of white noise whose power is σ^2. The receiver simulation takes account of geometric delays which are the result of the incoming wave direction, as shown in the relation:

$$\tau_k(n) = \frac{R}{c}\left(1 - \cos\left\{(n-1)\alpha - \frac{\beta}{2} - \theta_k\right\}\right) \tag{6}$$

and phase compensation on the acoustic axis direction (formula 5). Signals received by array transducers undergo band-pass filtration in filters with linear phase characteristics. Once filtered, the signals are sampled from frequency $F_s = 4 \cdot f_0$, following the principles of second order sampling. During sampling a pair of samples is collected at each output, which are the real and imaginary components of a complex sample. In a single sampling cycle N complex samples are produced where N is the number of beamformer columns. Next, the complex samples are used to determine power spectrum density, using the parametric method of estimation. The next step is to find the spectral line of spatial distribution. But because the spectrum of a complex signal represents frequencies from 0 to F_s, spectral lines must be converted to those representing the frequency range from $-F_s/2$ to $F_s/2$, following this relation:

$$F_k = \begin{cases} \frac{2(k-1)}{M} & , k \leq \frac{M}{2} + 1 \\ \frac{-2(M-k+1)}{M} & , k > \frac{M}{2} + 1 \end{cases} \tag{7}$$

where k is the number of the line of a high resolution estimate of spatial spectrum and M is the number of spectral lines. The bearing angle is determined using relation (4). For a given angle of incidence the procedure is repeated 100 times to determine the average bearing value. The entire procedure was repeated for different signal-to-noise ratios and different numbers of array elements.

3. RESULTS OF THE SIMULATION

Figure 4 presents the results of bearing estimation using formula (4), depending on the wave incidence angle. The chart is made for three numbers of active array columns under analysis. The echo signal received by transducers has a signal-to-noise ratio equal to 40 dB.

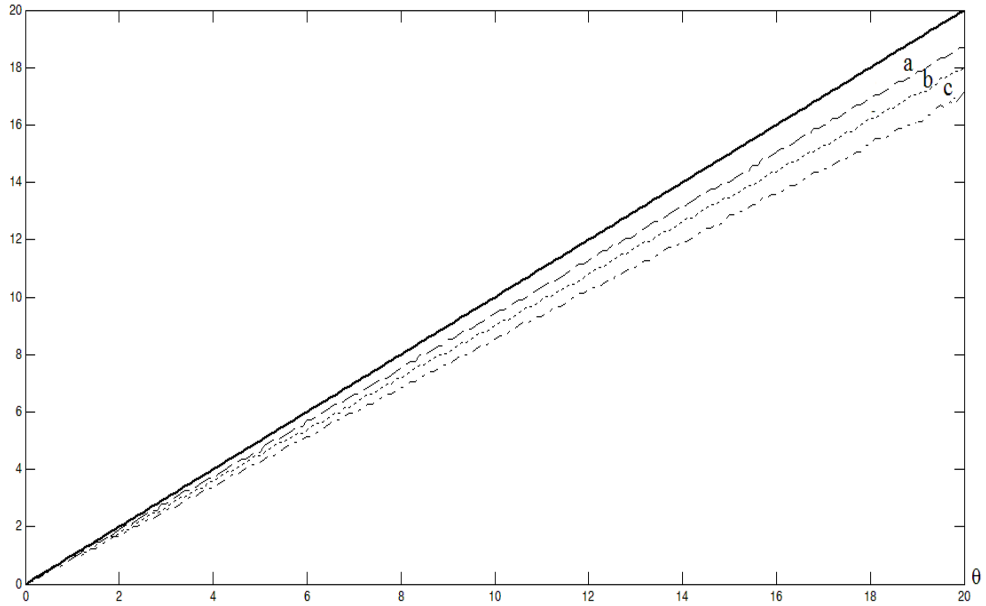

Fig.4. Relation between mean bearing and the wave incident angle for different transducer number N (a: N = 8, b: N = 10, c: N = 12, SNR = 40dB).

Figure 4 shows a linear deviation of the estimated bearing for all three cases. As we can see, the incidence angle of the bearing being estimated remains approximately constant in the wide range of angles. The range significantly exceeds 3dB of beam width which for the beam designed is 10°. Figure 5 shows the exact relation between inaccurate bearing and the incidence angle.

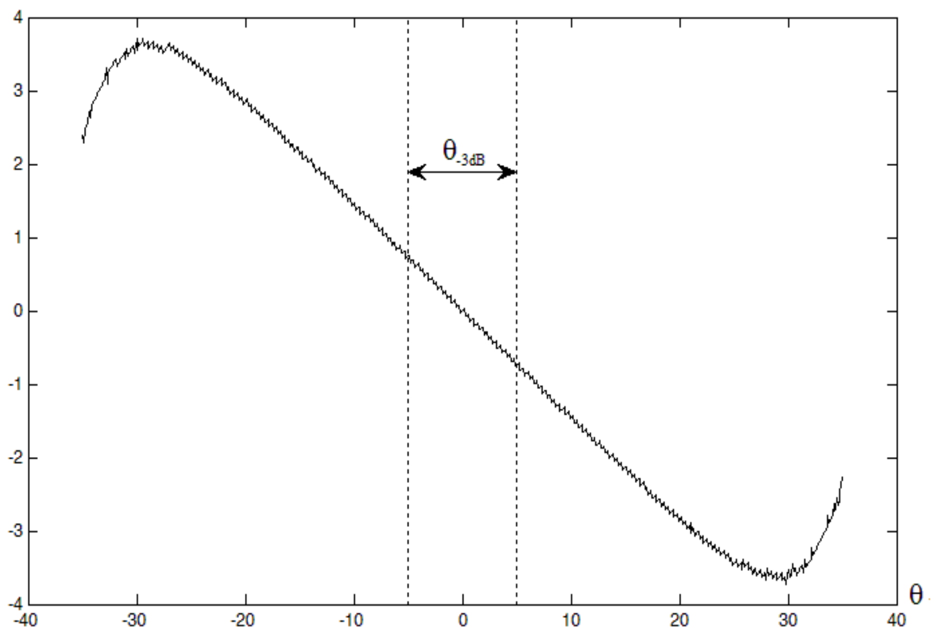

Fig.5. Bearing error in the wave incident angle. (N = 12, SNR = 40dB).

As we can see from the chart, estimation error stays linear in a limited range of wave incidence angles. For reasons of practicality, our area of interest is the nature of the error within the width of the receiving beam. As you can see in Figure 5, within this range the bearing error maintains the linear relation. The saw-like line of the error is the result of discretization of the spatial frequency axis, which also implies discrete bearing estimations. Despite that, we can estimate the linear regression coefficient using the least squares method in the range of wave incidence angles of interest to us. For each case separately regression coefficients were determined which are a_{12} = 0.8514 for the array sector with 12 array columns, a_{10} = 0.8935 for 10 columns and a_8 = 0.939 for 8 columns respectively. By multiplying the bearing estimate determined using relation (4), using inverse regression coefficient to match the active number of columns, we obtain a modified formula for determining bearing in the following form:

$$\theta_k = a \cdot \sin^{-1} F_k \qquad (8)$$

where a = $1/a_{12}$, $1/a_{10}$ or $1/a_8$ depending on the number of active cylindrical array columns.

The procedure is analogous to the one used for adapting the monopulse method for the cylindrical array, which also involved a modification of the formula for the linear array by a constant corrective coefficient [4]. With this simple measure we can reduce the bearing estimation error in passive sonar with a cylindrical array described in publication [5].

Figure 6 shows the relation between bearing standard deviation and signal-to-noise ratio for three variants of the number of active columns of the cylindrical array sector for a selected wave incidence angle.

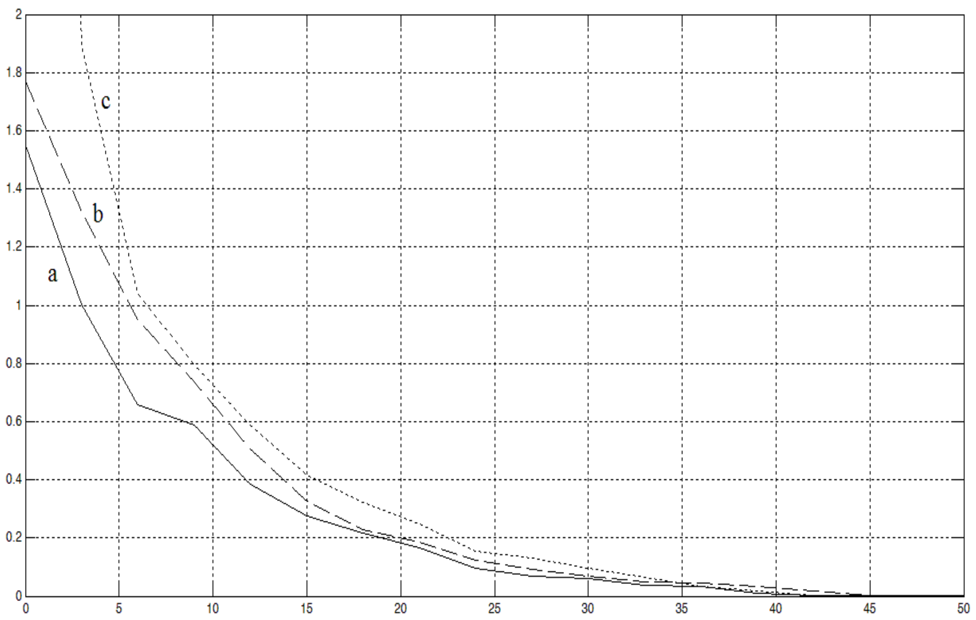

Fig.6. Standard deviation of bearing in the signal to noise ratio function for different transducer number N. (a: N = 12, b: N = 10, c: N = 8).

As we can see in Figure 6, the method is sensitive to signal-to-noise ratio. A possible conclusion is that the method makes sense for at least 10dB signal-to-noise ratio. It is important to notice that standard deviation increases when bearing estimation involves a smaller number of array columns. It is a fact despite a smaller difference between transducer distances in the virtual linear array. This is illustrated in Figure 2. On the other hand, a small number of input data affects the accuracy of spatial spectrum estimation.

4. CONCLUSION

Simulation results show that the spatial spectrum estimation method can be successfully applied in multibeam sonar with cylindrical array, even though the method is typically used with linear arrays. What is needed is a virtual linear array, generated using phase compensation for signals received on the acoustic axis direction of the receiving beam. The consequence of this approach is linear estimation error, which can be relatively easily reduced using a corrective coefficient.

Even though acoustic field sampling on the virtual array was uneven, the effective range for estimating the directions of incoming waves goes significantly beyond the width of a single beamformer receiving beam. This means that the method can be successfully used for determining precise bearing within the width of the beam with the highest echo level detected.

The results show that the method is sensitive to signal-to-noise ratio. Bearing becomes accurate for a relatively high value of this parameter. As a consequence, the method works best for auxiliary tasks when determining bearing in the classic delay-sum beamformer.

REFERENCES

[1] R. Salamon, Systemy Hydroakustyczne, Wydawnictwo Gdańskiego Towarzystwa Naukowego, Gdańsk 2006.

[2] S.L. Marple, Digital spectra analysis, Prentice Hall, Englewood Cliffs 1987.

[3] J.P. Burg, The relationship between maximum entropy spectra and maximum likelihood spectra, Geophysics, vol. 37, pp. 375-376, 1972.

[4] W. Leśniak, J. Marszal, R. Salamon, M. Rudnicki, A. Schmidt, Digital mono-pulse method in cylindrical antenna, Archives of Acoustics, Vol. 31, pp. 357-363, 2006.

[5] J. Marszal, W. Leśniak, R. Salamon, A. Jedel, K. Zachariasz, Passive sonar with cylindrical array, Archives of Acoustics, Vol. 31, pp. 365-371, 2006.