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INFLUENCE OF WRONGLY ASSUMED PROBABILITY DISTRIBUTION ON THE UNCERTAINTY OF RESISTANCE MEASUREMENT BY TECHNICAL METHOD

The paper presents studies on the influence of probability distributions on the expanded uncertainty of the resistance measurement. Choosing the correct probability distribution is very important to estimate of measurement uncertainty. The most commonly used distribution is the rectangular distribution. The paper presents the results of analysis of the resistance measurement uncertainty using the technical method of two resistances: 1 Ω and 100 GΩ using different measuring equipment. The analysis of the uncertainty measurement of resistance was carried out repeatedly, each time assuming a different probability distribution of measuring instruments (normal, rectangular, triangular or trapezoidal).

The results of the research presented in the article show that the influence of the assumed probability distributions on the result of the measurement uncertainty analysis is significant and results discrepancies can reach up to 30%.

KEYWORDS: uncertainty of measurement, technical method, measurement of resistance.

1. INTRODUCTION

The modern development of metrology requires engineers to have knowledge on the estimation of measurement uncertainty. Therefore, it is important to learn the principles of its designation.

Understanding the fundamentals of the uncertainty theory is important because measurement uncertainty is a component of the presented measurement result. It is crucial not only to carry out the given experiment, but also to correctly present the obtained value of the measured value together with the qualitative measure - measurement uncertainty, because only complete results can be compared with each other.

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Although more than 20 years have passed since the introduction of the Guide to the Expression of Uncertainty in Measurements [1, 2] (acronym GUM) and International Metrology Dictionary [3] (VIM, Vocabulary International Metrology), in which documents the methodology of determination of measurement uncertainty and metrological terminology was introduced. Some engineers still have problems with determining measurement uncertainty, especially when it concerns indirect measurements.

The aim of the research is to check the influence of wrongly assumed probability distribution of instruments on the result of uncertainty estimation of the resistance measurement. In many cases, the selection of an appropriate probability distribution is difficult, and, as shown by the authors' research, significantly influences the result of the measurement uncertainty analysis.

2. MEASUREMENT METHOD

Extended uncertainty of resistance measurement by technical method $U_p(R)$ is determined in accordance with the guidelines presented in the GUM guide [1, 2]. In order to estimate it, the coverage factor k_p should be determined for the assumed probability of expansion and the uncertainty of the measurement $u_c(R)$.

$$U_p\left(R\right)=k_p\cdot u_c\left(R\right) \tag{1}$$

Combined uncertainty of resistance measurement $u_c(R)$, assuming no correlation between the uncertainties of the input quantities, according to the law of uncertainty propagation, specifies the following formula:

$$\begin{aligned}u_c\left(R\right)&=\sqrt{\left(c_1\right)^2\cdot u^2\left(U\right)+\left(c_2\right)^2\cdot u^2\left(I\right)}=\\&=\sqrt{\left(\frac{\partial f(U,I)}{\partial U}\right)^2\cdot u^2\left(U\right)+\left(\frac{\partial f(U,I)}{\partial I}\right)^2\cdot u^2\left(I\right)}=\\&=\sqrt{\left(\frac{1}{I}\right)^2\cdot u^2\left(U\right)+\left(\frac{-U}{I^2}\right)^2\cdot u^2\left(I\right)}\end{aligned} \tag{2}$$

where: c_1, c_2 – sensitivity coefficients determined on the basis of partial derivatives of the measurement function $f(U,I)$ (Ohm's law), $u(U)$ – uncertainty of voltage measurement U, $u(I)$ – uncertainty of current measurement I.

Uncertainty of voltage $u(U)$ and current $u(I)$ measurement were determined using the type A and type B methods [4].

Uncertainty type A $u_A(x)$ was determined by the statistical analysis method from a series of single observations.

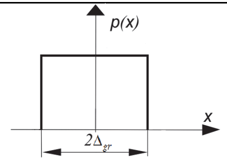
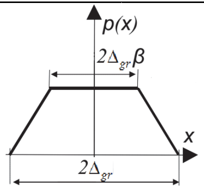
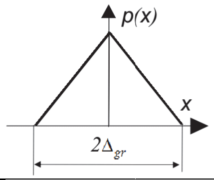
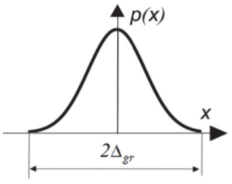
$$u_A(U)=\sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(U_i-\overline{U})^2} \tag{3}$$

$$u_A(I) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(I_i - \overline{I})^2} \tag{4}$$

where: n – number of observations, U_i, I_i – successive values of properly measured voltage U and current I, $\overline{U}, \overline{I}$ – average values of the measured voltage U and current I.

Uncertainty type B is estimated on the basis of known or assumed probability density function measurements, which require the observer to have knowledge of the instruments that were used in the experiment. The main problem in estimating type B $u_B(x)$ uncertainty is the selection of the probability density function. There are many possibilities, in the studies limited to four: normal, rectangular, trapezoidal and triangular distribution. The dependencies on the basis of which the uncertainty type B $u_B(x)$ was determined for voltage and current measurements are presented in Table 1 [5, 6].

Table 1. Dependences describing type B uncertainty.

No.	Probability distribution	Type B uncertainty $u_B(x)$
1		$u_B(x) = \frac{\Delta_{gr}}{\sqrt{3}}$
2		$u_B(x) = \frac{\Delta_{gr}\sqrt{1+\beta^2}}{\sqrt{6}}$
3		$u_B(x) = \frac{\Delta_{gr}}{\sqrt{6}}$
4		$u_B(x) = \frac{\Delta_{gr}}{\sqrt{4}}$

Δ_{gr} is the maximum permissible error (limit error) of the measured value x (in the analysed case: voltage and current). The value of the maximum permissible error is variously characterized depending on whether the value is measured by an analogue or digital meter. It is determined on the basis of parameters defined by the manufacturer of the measuring device [4].

3. EXPERIMENTAL RESEARCH

At the Gdańsk University of Technology, Faculty of Electrical and Control Engineering, studies are being carried out as part of the FAIR [7] project. Their goal is to design and manufacture a measurement and diagnostic system for testing the proper functioning of superconducting magnets' electrical circuits. One of the key parameters that will be the subject of the diagnostics is the measurement of the insulation resistance of individual circuits. Based on preliminary tests, it has been determined that the resistance can vary from individual GΩ to several TΩ. An important part of the research aimed at the implementation of the measurement and diagnostic system is to determine the uncertainty of the resistance measurement of this system.

The measurement system is complicated and consists of measuring cables, switching circuits and a measuring instrument, which is the Megger S1-568.

Determining the measurement uncertainty of the measurement and diagnostic system is complex and many factors should be taken into account. One of them is the selection of appropriate probability distributions of measuring instruments. The authors, using the experience gained during the performing of many measurements of resistance of the order of hundreds of GΩ, examined the influence of incorrectly selected probability distribution of measuring instruments on the result of the analysis. The test object was a 100 GΩ reference resistor. For comparison, the authors decided to investigate a resistor with a resistance of 1 Ω.

Basic parameters of both these objects are presented in the Table 2.

Table 2. Parameters of the reference resistor [8].

1	Resistance	100 GΩ	1 Ω
2	Class	2.5	0.01
3	Voltage	5 kV	1 V
4	Current	50 nA	1 A
5	Power	0.25 mW	1 W

The measurements were made using the Megger S1-568 insulation meter and two Hameg HM8112-3 multimeters. The Megger S1-568 measures the resistance using the technical method, while allowing the user to read the current and voltage measurements. The basic parameters of the meters are presented in Table 3.

Table 3. Parameters of the used measuring instruments [9].

	Parameter	Megger S1-568	Hameg HM8112-3
1	Current measurement range	0.01 nA – 6 mA	1 A
2	Accuracy of current measurement	5% ± 0.2 nA	0.002% ± 1 μA
3	Voltage measurement range	30 V – 5 kV	600 V
4	Accuracy of voltage measurement	3% ± 3 V	0.003% ± 60 μV
5	Measuring voltage	3 kV	9 V

The 100 GΩ resistor was measured by a Megger S1-568. The device supplies the tested resistor with a known voltage and measures the current. Resistance is determined by the technical method. Measurements using the Megger S1-568 meter were made remotely using the "Remote Control" mode of the meter. It allows to run the device and start measuring with commands sent via the USB interface from a PC. Sending commands and receiving data was carried out using an application prepared by the authors in the MATLAB environment. Resistance measurement was performed at 3 kV test voltage generated by the meter. Each measurement was performed after two minutes from application of the test voltage to the resistor, then the voltage was disconnected for 30 s and this procedure was repeated 50 times.

To measure the resistance of 1 Ω resistor, two Hameg HM8112-3 multimeters were used. This resistance measurement system was connected using the technical method with correctly measured voltage. The system was supplied with 100 mV DC voltage.

Each time the measured voltage and current values were recorded and saved in a Microsoft Excel spreadsheet. Such prepared data was imported into the R-Tech program.

In order to investigate the impact of an wrongly chosen probability distribution on the uncertainty of resistance measurement using the Megger S1-568 instrument, it is necessary to know the actual distribution of the measurement error of this meter. For this purpose, a series of measurements was carried out with this device, which showed that the probability distribution of voltage and current measurements, in practically every case, is a triangular distribution - which is confirmed by exemplary histograms of results obtained with this meter (Figure 1).

A series of measurements consisting of 90 samples was used to plot the histograms shown in Figure 1. The samples were collected while measuring resistance of 100 GΩ.

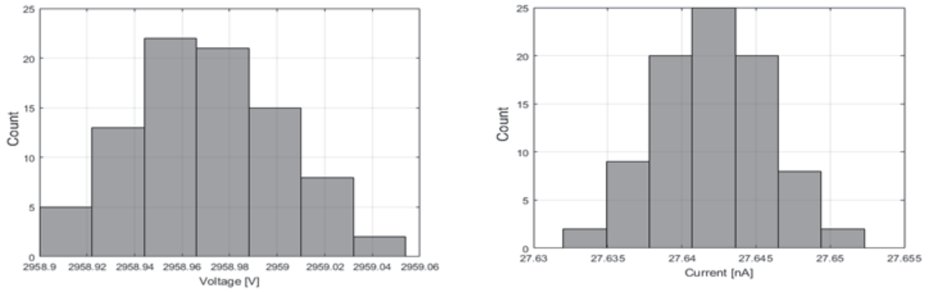


Fig. 1. Histogram of voltage and current measurements made with the meter Megger S1-568

4. ESTIMATING UNCERTAINTY OF RESISTANCE MEASUREMENT

The results of the research described in this article were carried out using proprietary R-Tech software prepared in the MATLAB environment. This software allows to determine the expanded uncertainty of the resistance measurement using the technical method and takes into account the error of the method (measurement with correctly measured: voltage or current).

4.1. R-Tech application

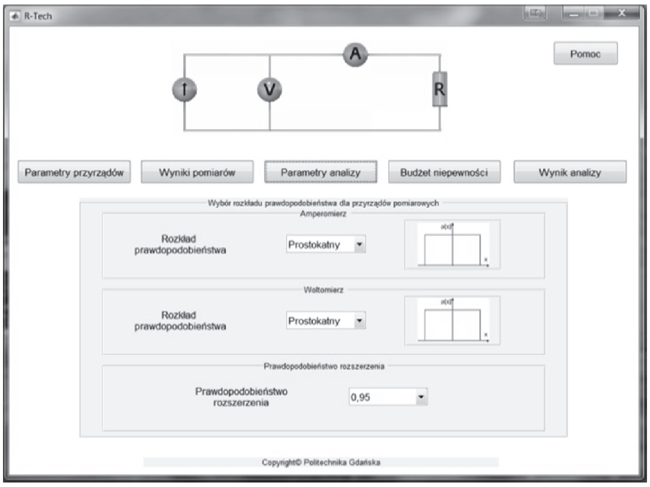
The R-Tech application has been prepared by the authors in two versions: basic and advanced. The basic version of the software presented in [4] was designed in such a way that the user would not need to have advanced metrological knowledge related to the measurement uncertainty analysis. The user is only required to enter measurement results and information about the measuring instruments used in the experiment. The application of this version may be didactics.

The advanced version R-Tech allows the user to choose the probability distribution of the measuring instrument and coverage factor.

In the advanced version R-Tech application, the functionality of the program has been divided into five parts through the use of switchable tabs to increase the transparency of the graphical user interface:

1) Parametry przyrządów – information about measuring instruments,
2) Wyniki pomiarów – entering collected measurement data,
3) Parametry analizy – setting the parameters of the measurement uncertainty analysis,
4) Budżet niepewności – presentation of the measurement uncertainty budget,
5) Wynik analizy – presentation of the analysis results.

„Parametry analizy” tab (in Figure 2) allows the user to select probability distributions for the ammeter and voltmeter.



Wybór rozkładu prawdopodobieństwa dla przyrządów pomiarowych

Amperomierz

Rozkład prawdopodobieństwa: Prostokatny

Woltomierz

Rozkład prawdopodobieństwa: Prostokatny

Prawdopodobieństwo rozszerzenia

Prawdopodobieństwo rozszerzenia: 0,95

Fig. 2. Program window showing the „Parametry analizy” tab

The user has the choice of one of four options for each instrument: normal, rectangular, trapezoidal or triangular. On the panel shown in Figure 2, the user also has the option to choose the probability of expansion. The following values were adopted: 0.68, 0.95 and 0.99.

The uncertainty budget presentation in the R-Tech application gives quickly obtain information on the contribution of components of the resistance uncertainty to the measurement uncertainty result.

The final result of the expanded uncertainty estimation of the resistance measurement is presented in the last tab „Wynik analizy”. The result of the measurement is: estimation of the resistance value (determined as the arithmetic mean value of the resistance calculated for each pair of voltage and current) and expanded uncertainty $U_p(R)$.

Tabs functionality: „Parametry przyrządów”, „Wyniki pomiarów”, „Wynik analizy” is the same in both the basic and advanced versions of the application. Their appearance has been presented in [4].

4.2. The influence of probability distributions on the uncertainty of resistance measurements

After importing the measurement data from the Excel spreadsheet to the advanced version of the R-Tech application, the uncertainty of the measurement was analysed. The experiment was repeated, changing the adopted probability distributions of measuring instruments. Table 4 and Table 5 present their impact on the measurement uncertainty of resistance.

Table 4. Impact of probability distribution on the budget of uncertainty of measurements resistance 100 GΩ made with the meter Megger S1-568.

	Distribution *I*	Distribution *U*	U_{CU} [V]	U_{CI} [nA]	U_R [GΩ]	Participation *U* [10^{15}Ω]	Participation *I* [Ω]
1	Rec.	Rec.	1.8	2.0	0.70	2.1	2.5
2	Rec.	Tri.	1.3	2.0	0.70	1.5	2.5
3	Rec.	Trap.	1.4	2.0	0.70	1.7	2.5
4	Rec.	Norm.	1.5	2.0	0.70	1.8	2.5
5	Tri.	Rec.	1.8	1.5	0.50	2.1	1.8
6	Tri.	Tri.	1.3	1.5	0.50	1.5	1.8
7	Tri.	Trap.	1.4	1.5	0.50	1.7	1.8
8	Tri.	Norm.	1.5	1.5	0.50	1.8	1.8
9	Trap.	Rec.	1.8	1.6	0.56	2.1	2.0
10	Trap.	Tri.	1.3	1.6	0.56	1.5	2.0
11	Trap.	Trap.	1.4	1.6	0.56	1.7	2.0
12	Trap.	Norm.	1.5	1.6	0.56	1.8	2.0
13	Norm.	Rec.	1.8	1.8	0.61	2.1	2.2
14	Norm.	Tri.	1.3	1.8	0.61	1.5	2.2
15	Norm.	Trap.	1.4	1.8	0.61	1.7	2.2
16	Norm.	Norm.	1.5	1.8	0.61	1.8	2.2

Table 5. Impact of probability distribution on the budget of uncertainty of measurements resistance 1 Ω made with the meter Hameg HM8112-3.

	Distribution *I*	Distribution *U*	U_{CU} [10^{-5}V]	U_{CI} [10^{-5}A]	U_R [10^{-4}Ω]	Participation *U* [10^{-3}Ω]	Participation *I* [10^{-3}Ω]
1	Rec.	Rec.	3.7	1.1	4.3	4.6	1.4
2	Rec.	Tri.	2.6	1.1	3.2	3.3	1.4
3	Rec.	Trap.	2.9	1.1	3.5	3.7	1.4
4	Rec.	Norm.	3.2	1.1	3.8	4.0	1.4
5	Tri.	Rec.	3.7	0.77	4.2	4.6	0.98
6	Tri.	Tri.	2.6	0.77	3.1	3.3	0.98
7	Tri.	Trap.	2.9	0.77	3.4	3.7	0.98
8	Tri.	Norm.	3.2	0.77	3.7	4.0	0.98
9	Trap.	Rec.	3.7	0.86	4.2	4.6	1.1
10	Trap.	Tri.	2.6	0.86	3.1	3.3	1.1
11	Trap.	Trap.	2.9	0.86	3.4	3.7	1.1
12	Trap.	Norm.	3.2	0.86	3.7	4.0	1.1
13	Norm.	Rec.	3.7	0.94	4.3	4.6	1.2
14	Norm.	Tri.	2.6	0.94	3.1	3.3	1.2
15	Norm.	Trap.	2.9	0.94	3.4	3.7	1.2
16	Norm.	Norm.	3.2	0.94	3.7	4.0	1.2

The participation each of source of uncertainty on combined standard uncertainty is determined by the product of the sensitivity coefficient and the standard combined uncertainty – from the formula (2).

When analysing the uncertainty of resistance measurements of 100 GΩ, the voltage sensitivity factor is 15 orders of magnitude smaller than the current sensitivity coefficient. For this reason, the participation of combined uncertainty of voltage is close to zero (Table 4). Therefore, special attention should be paid to the selection of a current measuring instrument, because the parameters of this device have a great impact on the result of the measurement uncertainty analysis.

The tests were also performed by analyzing the measurements of the resistor with the value of 1 Ω (Table 5). In this case, the sensitivity coefficients are of similar order and there are no such large discrepancies as in the case of the 100 GΩ resistor.

In both cases (Table 4 and Table 5), the largest measurement uncertainty values occur with the assumption of a rectangular distribution, and the smallest with the assumption of a triangular distribution. These discrepancies reach 30%.

Figures 3 and 4 (for 100 GΩ and 1 Ω respectively) graphically present the combined uncertainty of resistance measurement depending on the selected probability distribution of measuring instruments.

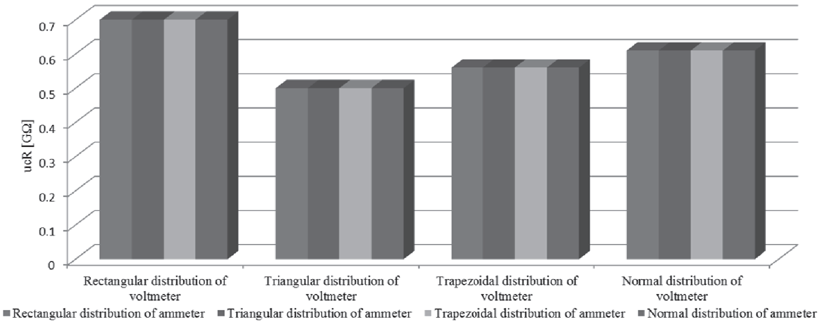


Fig. 3. Combined uncertainty of the resistance measurement depending on the selected probability distribution when testing the resistor 100 GΩ

On the basis of the presented data, it can be noticed that the highest uncertainty values are obtained for a rectangular distribution, and the smallest for a triangular one. As shown in Figure 3, assuming a rectangular distribution for the ammeter, the effect of the voltmeter distribution is imperceptible. The smallest uncertainty of measurement is obtained assuming a triangular current probability distribution.

In the case of the results presented in Figure 4, the largest measurement uncertainty values are obtained assuming a rectangular probability distribution of the voltmeter. However, the effect of the probability distribution of the second instrument is noticeable.

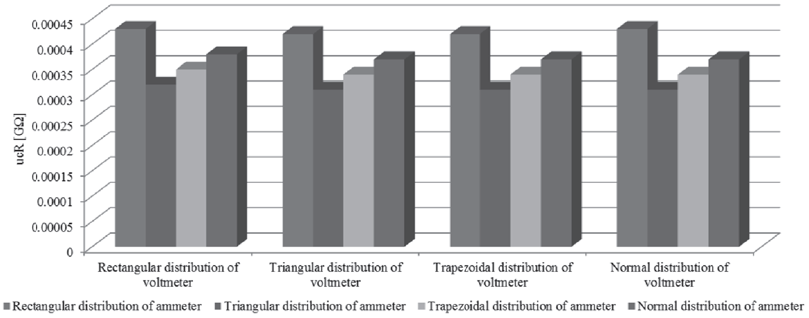


Fig. 4. Combined uncertainty of the resistance measurement depending on the selected probability distribution when testing the resistor 1 Ω

This is because, when testing a resistor of 1 Ω, the sensitivity coefficients have similar values. Knowing that the probability distribution of measurement results for the Megger S1-568 meter is a triangular distribution, and for the Hameg HM8112-3 meter it is close to the normal distribution, it is possible to analyse the impact of assumed probability distributions on the measurement uncertainty result.

Knowledge of the actual distributions of the results of the meters (for Megger S1-568 meter is a triangular distribution, for the Hameg HM8112-3 meter it is close to the normal distribution) allows to perform to analyse the impact of assumed probability distributions on the measurement uncertainty result.

If during resistance measurement 100 GΩ (using the Megger S1-568) instead of the triangular distribution the trapezoidal, normal or rectangular distribution would be chosen, the discrepancies in the estimated expanded uncertainty *Up*(*R*) would be 7%, 20% and 33%, respectively. Similarly, in the case of 1 Ω measurement by the Hameg HM8112-3 device, assuming a triangular, trapezoidal and rectangular probability distribution, the difference in $U_p(R)$ estimates is accordingly: underestimated by 18%, 9%, and overestimated by 17%.

As one can observe from the above tests results, it is very important to properly choose the probability distribution of the instrument's measurement results, taking into account good knowledge of the measuring device.

5. SUMMARY

The correct selection of probability distributions for measuring instruments used to make indirect measurements is important and has a significant influence on the measurement uncertainty result. The erroneous selection of the probability distribution of the measurement results leads to an incorrect estimation of the measurement uncertainty associated with the uncertainty of the measuring instrument used, and thus an incorrect estimation of the combine uncertainty of a resistance measurement $u_c(R)$.

In the cases analysed in the paper, the largest values of the combined uncertainty of a resistance measurement were obtained by choosing a rectangular distribution. In order to reliably evaluate the uncertainty of measurement, it is necessary to get to know the measuring instrument first. Based on the historical results obtained with this device, the probability distribution can be appropriately assumed to estimate the type B uncertainty. Most often, if one does not know the instrument, a rectangular probability distribution is assumed, and as it turns out, this choice may lead to a significant expansion of the confidence interval - as was in the case of the resistance measured with the Megger S1-568 meter. The above described is acceptable, but one greatly overestimates the value of uncertainty. In the second analysed case, when the resistance measurements were made with the Hameg HM8112-3 meter, the erroneous selection of the probability distribution, triangular instead of normal, can lead – to underestimation of the uncertainty which from the metrological point of view is unacceptable.

Therefore, it is important to correctly recognize and select the correct probability distribution of the measurement results of the measuring instrument used to measure the resistance. Only this approach guarantees obtaining the correct uncertainty of the resistance measurement.

LITERATURE

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