Submitted to "Archives of Civil and Mechanical Engineering" 2007
Published in Vol. 8, Issue 4, 2008, Pages 61–72

DRAFT

Linear analysis of laminated multilayered plates with the application of zig-zag function

A. Sabik and I. Kreja

Gdańsk University of Technology, G. Narutowicza 11/12, 80-952 Gdańsk, Poland

An Equivalent Single Layer Model for linear static analysis of multilayered plates is considered in the paper. The displacements field of the First Order Shear Deformation Theory is supplemented with the Murakami's function to emulate the zig-zag effect. The applied Reissner's Mixed Variational Theorem allows to assume an independent shear stress field of parabolic distribution in each layer and to satisfy the equilibrium conditions at the layer interfaces. The Finite Element implementation of the model is based on the application of the 9-node Lagrangian element with a selective reduced integration. Two numerical examples of symmetrically laminated plates are presented to illustrate the accuracy of presented approach.

Keywords: *multilayered plates, zig-zag effect, interlaminar continuity*

1. Introduction

A high strength, a light weight, good thermal insulation and a superior durability are the most characteristic attributes of multilayered plates. These features are very desirable in many branches of industry. Initially, the technology of these heterogeneous structures was developed with intent to meet the particular requirements of the aeronautic engineering. However, decreasing cost of their production in recent years is the reason behind the popularity of multilayered plates in other branches, also in the civil engineering. Therefore, the problem of modelling such structures is a usual task of many structural engineers of today.

The formulation of mechanical model of multilayered plates causes however more difficulties in comparison with the case of homogeneous plates. For example, a question arises how to describe the changes of material properties over the cross-section. Is it necessary to assume independent displacement and stress fields for each layer, or maybe, it is possible to replace the sequence of the laminas by some equivalent single layer? In addition, due to low and different transverse stiffness of adjacent layers, the cross-section's deformation takes the form of a zig-zag. This is known as the *zig-zag effect* [1, 2], a recognized problem related to modelling of multilayered structures. Consequently, the assumption of linear deformation of the cross-section is inadequate here. Furthermore, due to the assumption of the perfect

bond between the layers, the fulfilment of equilibrium conditions at the layer interfaces is required. It means that the distribution of transverse stresses along the plate thickness should be described by continuous functions. In the literature, this is known as the *interlaminar continuity* [1].

The above facts complicate the formulation of computational models for multilayered structures. In the literature exists a multitude of approaches to overcome these difficulties. Some of them are: the three-dimensional formulations, the so called *Layer-Wise* (LW) [3] approach where independent displacement and stress fields for each layer are assumed, or *Equivalent Single Layer Models* (ESL) [3], where a single layer with a resultant stiffness is applied. Another classification follows from computational technique applied to solve the governing equations. Nowadays the most popular tool is the Finite Element Method (FEM). Here, the differences among various concepts result from the underlying weak formulation of the problem, for instance virtual displacements or multi-field variational principle. For the systematic review - see references [4, 5].

The aim of the present paper is to outline the formulation of the model (see, [6] for details) that belongs to the family of ESL formulations. Starting from the First Order Shear Deformation Theory (FSDT) additional assumptions are made in order to account for both the *zig-zag effect* and the *interlaminar continuity*. In the computations, the Finite Element Method is applied. To illustrate the performance of the model in linear static analysis, two numerical examples are presented.

2. Characteristic of the model

As mentioned above, the presented formulation belongs to the group of ESL models. This approach is based on reduction of a three-dimensional solid to a two-dimensional midsurface, as in case of homogeneous plates. This means that only one displacement field and one stress field are assumed for the whole cross-section.

2.1 Displacement field

The displacement distribution through the plate thickness is a sum of linear distribution (FSDT) and Murakami's zig-zag function (Figure 1). The Murakami's function is defined as follows:

$$f(z) = (-1)^k \cdot \xi_k, \qquad \xi_k = 2z_k / h_k, \qquad k = 1,2...\mathrm{N_L}, \tag{1}$$

where ξ_k, z_k are non-dimensional and dimensional coordinate of the layer k, respectively, h_k is the thickness of the layer k, and $\mathrm{N_L}$ stands for the number of layers.

MOST WIEDZY Downloaded from mostwiedzy.pl

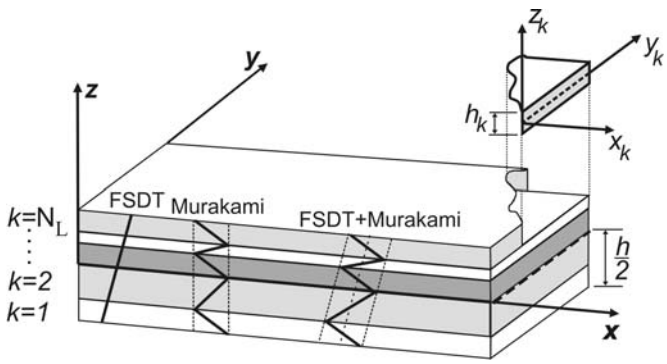

Figure 1. Displacement field

It can be clearly seen, that the function (1) changes the slope at the layer interfaces. Its introduction to the displacement field is a simple way to simulate the *zig-zag effect*. More details about properties of Murakami's function can be found in [1]. Other approaches used to describe the *zig-zag effect* are discussed in [7].

As a result, the standard five-parameter displacement field of FSDT, is enhanced by two additional variables:

$$
\begin{aligned}
u(x,y,z) &= u_o(x,y) + \varphi_x(x,y) + (-1)^k \cdot \xi_k \cdot D_x(x,y) \\
v(x,y,z) &= v_o(x,y) + \varphi_y(x,y) + (-1)^k \cdot \xi_k \cdot D_y(x,y) \\
w(x,y,z) &= w_o(x,y)\,,
\end{aligned}
\tag{2}
$$

where u_o, v_o, w_o are translations of midsurface points in *x*, *y* and *z-axis* direction, respectively (Figure 2), φ_x, φ_y are rotations in *x-z* and *y-z* surfaces. The additional parameters D_x, D_y can be interpreted as amplitudes of the Murakami's function. It can be seen from (2) that the transverse normal strain effect is omitted.

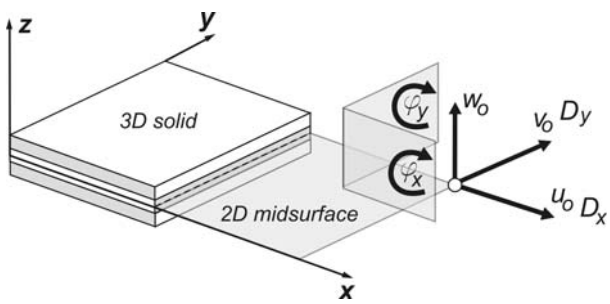

Figure 2. Coordinate system and degrees of freedom

MOST WIEDZY Downloaded from mostwiedzy.pl

2.2 Stress field

Bending stresses are evaluated following standard displacement formulation. However, there is a necessity to adopt an independent field of transverse stresses to satisfy the required equilibrium conditions at layer interfaces. Since the effect of transverse normal strain is not taken into account, the assumption of an independent field is made only with the reference to the transverse shear stresses. The shear stress field for each layer is determined by six unknowns assembled in vector $\mathbf{X}_{\boldsymbol{\sigma},k}$:

$$\mathbf{X}_{\boldsymbol{\sigma},k}^{\mathrm{T}} = \left\{ \sigma_{xz}^{t} \quad \sigma_{yz}^{t} \quad R_{xz} \quad R_{yz} \quad \sigma_{xz}^{b} \quad \sigma_{yz}^{b} \right\}_{k}^{\mathrm{T}}, \tag{3}$$

where indices t, b denote the top and bottom surface of layer k, respectively and R_{xz}, R_{yz} are stress resultants given by:

$$R_{iz} = \int_{h_k} \sigma_{iz} dz, \qquad i = x, y\,. \tag{4}$$

The distribution of shear stresses in layer k follows from interpolation scheme

$$\boldsymbol{\sigma}_{\mathbf{s},k} = \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix}_k = \begin{bmatrix} F_o & 0 & F_1 & 0 & F_2 & 0 \\ 0 & F_o & 0 & F_1 & 0 & F_2 \end{bmatrix}_k \cdot \mathbf{X}_{\boldsymbol{\sigma},k}\,, \tag{5}$$

where $\mathbf{X}_{\boldsymbol{\sigma},k}$ are given in (3) and the interpolating functions are defined as:

$$F_o(\xi_k) = -\frac{1}{4} + \frac{\xi_k}{2} + \frac{3}{4}\xi_k^2; \quad F_1(\xi_k) = \frac{3\cdot(1-\xi_k^2)}{2\cdot h_k}; \quad F_2(\xi_k) = -\frac{1}{4} - \frac{\xi_k}{2} + \frac{3}{4}\xi_k^2\,. \tag{6}$$

Due to the stress continuity requirements, the distribution of the stress field through the whole cross-section need to satisfy the following conditions:

$$\sigma_{xz,k}^{t} = \sigma_{xz,k+1}^{b} \qquad \sigma_{yz,k}^{t} = \sigma_{yz,k+1}^{b} \qquad k = 1,2,\ldots \mathrm{N_L} - 1\,. \tag{7}$$

Moreover, additional assumptions are made regarding the transverse shear stresses at the top and the bottom surfaces:

$$\sigma_{xz}(z = \pm h/2) = 0 \qquad \sigma_{yz}(z = \pm h/2) = 0\,. \tag{8}$$

MOST WIEDZY Downloaded from mostwiedzy.pl

Figure 3 illustrates the assumed shear stress distribution through the cross-section:

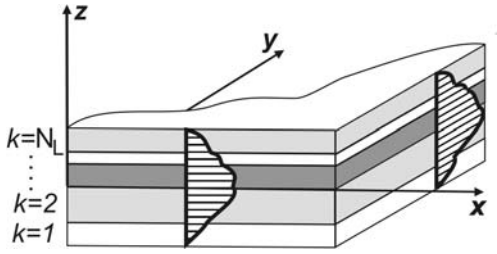

Figure 3. Assumed shear stresses distribution

2.3 Variational principle

A usual basis of any FEM model is some weak formulation of a particular mechanical problem. As the weak form the principle of minimum total potential energy [8] or virtual displacement method may be applied. The basic unknowns in these two cases are displacements. An alternative way can be adopting the principle of minimum complementary energy or virtual forces method. In this instance the stresses are initial unknowns [3]. There is also other way, namely by applying mixed variational principles, e.g. Hellinger-Reissner or Hu-Washizu principle [9].

The internal energy functional W_{in} used in the present formulation reads [3]:

$$W_{in} = \int_A \Big(\sum_{k=1}^{\mathrm{N_L}} \int_{h_k} \big(\delta\boldsymbol{\varepsilon}_{\mathbf{b}}^{\mathrm{T}} \cdot \boldsymbol{\sigma}_{\mathbf{b},H,k} + \delta\boldsymbol{\varepsilon}_{\mathbf{s}}^{\mathrm{T}} \cdot \boldsymbol{\sigma}_{\mathbf{s},k} + \delta\boldsymbol{\sigma}_{\mathbf{s},k}^{\mathrm{T}} \cdot (\boldsymbol{\varepsilon}_{\mathbf{s}} - \boldsymbol{\varepsilon}_{\mathbf{s},H}) \big) dz \Big) dA \tag{9}$$

where $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$ are strain and stress vectors, respectively; indices **b**, **s** denote correspondingly bending and shear terms; k is the layer number, A is midsurface area and H indicates terms obtained from the Hooke's law. Transverse shear stresses are computed as described in paragraph 2.2. The first and the second term are bending and shear energy, respectively. The last term in (9) represents the condition of shear stress compatibility at layer interfaces. The form of the relation in parenthesis results from simple transformations based on the assumption that

$$\boldsymbol{\sigma}_{\mathbf{s},k,H} = \mathbf{D}_k \, \boldsymbol{\varepsilon}_{\mathbf{s}} \quad \textit{and} \quad \boldsymbol{\varepsilon}_{\mathbf{s},H} = \mathbf{D}_k^{-1} \, \boldsymbol{\sigma}_{\mathbf{s},k} \,, \tag{10}$$

what allows to write

$$\boldsymbol{\sigma}_{\mathbf{s},k} = \boldsymbol{\sigma}_{\mathbf{s},k,H} \quad \Rightarrow \quad \boldsymbol{\varepsilon}_{\mathbf{s},H} = \boldsymbol{\varepsilon}_{\mathbf{s}} \quad \Rightarrow \quad \int_{h_k} \delta\boldsymbol{\sigma}_{\mathbf{s},k}^{\mathrm{T}} \cdot (\boldsymbol{\varepsilon}_{\mathbf{s}} - \boldsymbol{\varepsilon}_{\mathbf{s},H}) \, dz = 0 \,. \tag{11}$$

MOST WIEDZY Downloaded from mostwiedzy.pl

A proper classification of the variational principle (9) is somewhat problematic, because at first there are two kinds of unknowns, namely displacements and shear stresses, but due to the condition (11) stresses are eliminated from unknowns. This approach was proposed by Eric Reissner in the 80' and therefore is known as Reissner's Mixed Variational Theorem (RMVT) [3]. It should be noticed that the stress elimination must be carried out on cross-section level satisfying conditions (7) and (8) (compare [6]).

3. Numerical examples and discussion

Two numerical examples are given below to illustrate the performance of the considered model. As discussed in 2.3, the resulting finite element formulation based on the above assumptions possesses only seven kinematical degrees of freedom, given in (2). In the analysis, the 9-node Lagrangian element was applied. In order to avoid locking effect [8], the selective reduced integration [10] was used to obtain element's matrices. The results obtained within the framework of the present formulation are taken from [11].

3.1 Simple supported square sandwich plate

Following [5] a square, simply supported sandwich plate under uniform distributed load q is investigated (Figure 4).

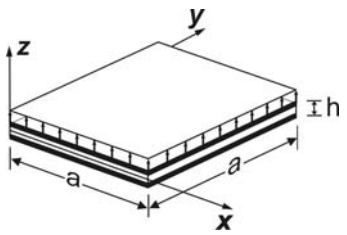

Figure 4. Square sandwich plate

The layers have following parameters: skins - $h_o = 0.1h$, $E_1/E_2 = 25$, $G_{12}/E_2 = G_{13}/E_2 = 0.5$, $G_{23}/E_2 = 0.2$, $\nu_{12} = 0.25$; inner layer - $h_w = 0.8h$, $E_1/E_2 = 1$, $G_{13}/E_2 = G_{23}/E_2 = 0.06$, $G_{12}/E_2 = 0.016$, $\nu_{12} = 0.25$. Maximal values of deflection, normal and shear stresses were obtained for various a/h ratio. Shear stresses are evaluated by integrating three-dimensional equilibrium equations as follows:

$$\sigma_{xz} = -\sum_{k=1}^{N_L} \int_{h_k} (\sigma_{xy,y} + \sigma_{xx,x}) dz; \qquad \sigma_{yz} = -\sum_{k=1}^{N_L} \int_{h_k} (\sigma_{xy,x} + \sigma_{yy,y}) dz \,. \tag{12}$$

MOST WIEDZY Downloaded from mostwiedzy.pl

Non-dimensional values are introduced as:

$$w^* = w \cdot 100 \cdot E_2 \cdot h^3 / (q \cdot a^4) \qquad \sigma^*_{xx} = \sigma_{xx} / (q \cdot (a/h)^2) \qquad \sigma^*_{xz} = \sigma_{xz} / (q \cdot (a/h)).$$

To discuss the effectiveness of the presented formulation the obtained solutions are compared in Tables 1, 2 and 3 with results calculated with other models. The error value in Tables 1, 2 and 3 ($\varepsilon = |w^* - w^*_{ref}| / w^*_{ref}$ 100%) is evaluated with respect to the reference solution obtained using the LW-type approach LM4 [5], which is basing on the variational formulation (9) but using the fourth order polynomial description of the displacement and transverse stress distribution through the layer thickness. ED4 and FSDT denotes ESL *only* displacement formulations with displacement field given by fourth and first order polynomial, respectively. It should be noticed that LM4 and ED4 take into account the transverse normal deformability of the plate. *Zig-zag effect* and *interlaminar continuity* are satisfied only by the LM4 and the present model.

Table 1. Comparison of deflection results for sandwich plate

Model	$w^*(x=a/2, y=a/2)$ [-]					
	a/h=4	ε [%]	a/h=10	ε [%]	a/h=100	ε [%]
LM4 [5]	10,682		3,083		1,262	
ED4 [5]	9,909	7,24	2,923	5,19	1,260	0,16
FSDT [5]	5,636	47,24	1,984	35,65	1,218	3,49
Present results	10,698	0,15	3,087	0,13	1,262	0,00

Table 2. Comparison of normal stresses results for sandwich plate

Model	$\sigma_{xx}^*(x=a/2, y=a/2, z=h/2)$ [-]					
	a/h=4	ε [%]	a/h=10	ε [%]	a/h=100	ε [%]
LM4 [5]	1,902		1,509		1,505	
ED4 [5]	1,929	1,42	1,519	0,66	1,506	0,07
FSDT [5]	1,168	38,59	1,391	7,82	1,476	1,93
Present results	1,905	0,13	1,503	0,40	1,506	0,07

Table 3. Comparison of shear stresses results for sandwich plate

Model	$\sigma_{xz}^*(x=0, y=a/2, z=0)$ [-]					
	a/h=4	ε [%]	a/h=10	ε [%]	a/h=100	ε [%]
LM4 [5]	0,4074		0,5276		0,5889	
ED4 [5]	0,3574	12,27	0,5104	3,26	0,5881	0,14
FSDT [5]	0,5249	28,84	0,5716	8,34	0,5876	0,22
Present results	0,3949	3,07	0,5245	0,59	0,5968	1,34

MOST WIEDZY Downloaded from mostwiedzy.pl

The contents of the Tables 1, 2 and 3 allow one to conclude that the accuracy of the present solutions is very satisfying. Only the shear stress result in the case of the thin plate (*a/h*=100) is less precise than values evaluated from ED4 and FSDT. However, the received error level in this case is still quite low. The significant differences between the present solutions and FSDT[5] emphasize the importance of consideration both the *zig-zag effect* and *interlaminar continuity* in modeling of multilayered structures. It can be also observed, that presented approach is more effective than ED4[5]. High accuracy (low relative error ε) in the case of thick plate could be a consequence of low transverse normal deformability of the cross-section.

3.2 Simple supported square cross-ply plate

This is a classical benchmark proposed by Pagano [12] and also investigated by many other authors (see e.g. [13]). Displacement and stress state of 9-layers (0/90/0/90/0/90/0/90/0), simply supported, square plate are analysed (Figure 5).

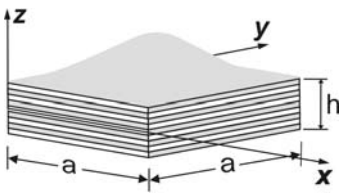

Figure 5. Square cross-ply plate

The parameters of each lamina are: E_L = 25E6 psi (1psi = 6.895 kPa), E_T = 10E5 psi, G_{LT} = 0.5E6 psi, G_{TT} = 0.2E6 psi, $\nu_{LT} = \nu_{TT} = 0.25$. The layers of the same orientation have equal thickness. The total thickness of 0º and 90º laminas is identical. The study was made with respect to several proportions *a/h*. The plate is loaded by bi-sinusoidal pressure:

$$p = p_o \cdot \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{a}\right)$$

The results are given in non-dimensional form according to:

$$w^* = \pi^4 Q w/(12 S^4 h p_o) \qquad \sigma_{xx}^* = \sigma_{xx}/(p_o S^2) \qquad \sigma_{xz}^* = \sigma_{xz}/(p_o S)$$
$$S = a/h \qquad Q = 4G_{LT} + [E_L + E_T(1+2\nu_{TT})]/(1-\nu_{LT}\nu_{TL})$$

Tables 4, 5 and 6 include present results confronted with the 3D solution given by Pagano and Hatfield [12].

MOST WIEDZY Downloaded from mostwiedzy.pl

Table 4. Comparison of deflection results for cross-ply plate

Model	$w^*(x=a/2, y=a/2)$ [-]					
	a/h=10	ε [%]	a/h=20	ε [%]	a/h=50	ε [%]
3D [12]	1,512		1,129		1,021	
Present results	1,441	4,70	1,112	1,51	1,018	0,29

Table 5. Comparison of normal stresses results for cross-ply plate

Model	$\sigma_{xx}^*(x=a/2, y=a/2, z=h/2)$ [-]					
	a/h=10	ε [%]	a/h=20	ε [%]	a/h=50	ε [%]
3D [12]	0,551		0,541		0,539	0,551
Present results	0,516	6,35	0,531	1,85	0,537	0,37

Table 6. Comparison of shear stresses results for cross-ply plate

Model	$\sigma_{xz}^*(x=0, y=a/2, z=0)$ [-]					
	a/h=10	ε [%]	a/h=20	ε [%]	a/h=50	ε [%]
3D [12]	0,247		0,255		0,258	
Present results	0,243	1,62	0,254	0,39	0,258	0,00

The comparison given in Tables 4, 5 and 6 seems to confirm the good accuracy of the presented formulation in the analysis of thin and moderately thick symmetrically laminated plates similarly, as observed in the previous example 3.1. Nevertheless, here the discrepancies between the present results and the reference solution are more pronounced. This may be attributed to the fact that the reference solution is a three-dimensional one. In addition, the differences may result from poor spatial approximation of the load in case of present computations. By comparing error values in Tables 4, 5 and 6, one can easily observe that the effectiveness of the model is lowest in the case of normal stresses. This is noteworthy, because normal stresses are evaluated according to the displacement field (from Hooke's law). Furthermore, following (12), shear stresses depend on normal stress results. Nevertheless, it should be reminded that the both stress fields in FEM-computations are obtained by using different derivatives of shape functions. Maybe this fact could be an explanation for this significant error distribution. Moreover, this example illustrates clearly that the accuracy of the presented model deteriorates with the decreasing ratio a/h. One can make conclusion that the correctness of this formulation in the case of thick plates depends strictly on the intensity of transverse normal strain effect (see 3.1).

MOST WIEDZY Downloaded from mostwiedzy.pl

4. Conclusions

The paper presents theoretical foundations and numerical investigations of two-dimensional FEM model for multilayered plates. The basis of the demonstrated approach is the First Order Shear Deformation Theory (FSDT). Two additional degrees of freedom of piecewise continuous distribution through the plate thickness are introduced to simulate the *zig-zag effect*. The formulation accounts for the *interlaminar continuity* requirement by means of the two-field variational principle, commonly known as the Reissner's Mixed Variational Theorem (RMVT). Due to the shear stress continuity condition the stress unknowns are eliminated at element level. Therefore, the resulting finite element possesses only kinematical degrees of freedom. Two numerical examples of symmetrically laminated plates are presented to demonstrate the efficiency of the model. A 9-node Lagrangian element with seven degrees of freedom in each node is used in computations. The analysis proves that the presented approach can be very effective tool in investigations of thin and moderately thick symmetrically laminated plates.

References

1. Demasi L., Refined multilayered plate elements based on Murakami zig-zag functions, *Composite Structures*, vol. 70, (2005), 308-316.
2. Carrera E., Historical review of Zig-Zag theories for multilayered plates and shells, *Appl Mech Rev* vol. 56 (2003), 287-308.
3. Carrera E., Developments, ideas, and evaluations based upon Reissner's Mixed Variational Theorem in the modeling of multilayered plates and shells, *Appl Mech Rev* vol. 54 (2001), 301-329.
4. Rohwer K., Friedrichs S., Wehmeyer J.N., Analyzing Laminated Structures from Fibre-Reinforced Composite Material – An Assessment, *Technische Mechanik*, vol. 25, (2005), 59-79.
5. Carrera E., Ciuffreda A., A unified formulation to assess theories of multilayered plates for various bending problems, *Composite Structures*, vol. 69, (2005), 271-293.
6. Brank B., Carrera E., Multilayered shell finite element with interlaminar continuous shear stresses: a refinement of the Reissner-Mindlin formulation, *International Journal for Numerical Methods in Engineering*, vol. 48, (2000), 843-874.
7. Di S., Rothert H., Solution of a laminated cylindrical shell using an unconstrained third-order theory, *Computers and Structures*, vol. 69, (1998), 291-303.
8. Reddy J.N., *Energy Principles and Variational Methods in Applied Mechanics,* John Wiley & Sons Canada, Ltd., Toronto (2001).
9. Ramm E., From Reissner plate theory to three dimensions in large deformations shell analysis, *Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 80, (2000), 61-68.
10. Zienkiewicz O.C., Taylor R.L., Too J.M., Reduced Integration Technique in general analysis of plates and shells, *International Journal for Numerical Methods in Engineering,* vol. 3, (1971), 275-290.

MOST WIEDZY Downloaded from mostwiedzy.pl

11. Witkowska(-Sabik) A., *Numeryczny model płyty warstwowej z zastosowaniem funkcji zig-zag*, magisterska praca dyplomowa (promotor I. Kreja), Wydział Inżynierii Lądowej i Środowiska Politechniki Gdańskiej, (*Numerical model of the multilayered plate with the use of zig-zag function (in Polish),* M.Sc. thesis, supervisor I. Kreja, Faculty of Civil and Environmental Engineering, Gdansk University of Technology), Gdańsk 2006.
12. Pagano N.J., Hatfield S. J., Elastic behaviour of multilayered bidirectional composites, *American Institute of Aeronautics and Astronautics Journal*, vol. 10, (1972), 931-933.
13. Bogdanovich A.E., Yushanov S.P., Three-dimensional variational analysis of Pagano's problems for laminated composite plates*, Composites Science and Technology*, vol. 60, (2000), 2407-2425.

Analiza liniowa laminowanych płyt warstwowych z zastosowaniem funkcji zig-zag

W pracy analizowany jest dwuwymiarowy model MES płyty warstwowej bazujący na teorii ścinania pierwszego rzędu, w którego sformułowaniu uwzględnione są dodatkowo efekt zig-zag oraz warunki równowagi międzywarstwowej. Zygzakowa deformacja przekroju modelowana jest poprzez wzbogacenie liniowego rozkładu przemieszczeń teorii ścinania pierwszego rzędu kawałkami ciągłą funkcją Murakami'ego. Pole przemieszczeń w prezentowanym sformułowaniu jest określone poprzez siedem stopni swobody. Spełnienie warunków równowagi na granicach warstw jest możliwe dzięki przyjęciu niezależnego, ciągłego w przekroju płyty rozkładu naprężeń poprzecznego ścinania. Efekt poprzecznego rozciągania jest w modelu pominięty. Dodatkowy warunek zgodności naprężeń na stykach warstw umożliwia wyeliminowanie niewiadomych naprężeniowych z zestawu niewiadomych. Eliminacja przeprowadzana jest na poziomie całego przekroju przy uwzględnieniu warunków ciągłości naprężeń na granicach warstw oraz ich braku na zewnętrznych powierzchniach płyty. Odpowiadający element skończony posiada jedynie kinematyczne stopnie swobody.

W artykule przedstawiono podstawy teoretyczne modelu z odniesieniem do traktujących o nim pozycji literatury. Zawarto dwa zaczerpnięte z dostępnych źródeł przykłady numeryczne, na podstawie których przeprowadzono dyskusję efektywności prezentowanego podejścia w zakresie statycznej analizy liniowej płyt symetrycznie uwarstwionych. Wyniki własne otrzymano przy zastosowaniu 9-węzłowego elementu Lagrange'a. Ponieważ elementy skończone zbudowane na bazie założeń prezentowanego modelu nie są wolne od efektu blokady, podczas całkowania macierzy sztywności elementów stosowano technikę całkowania selektywnie zredukowanego. Z porównania uzyskanych wyników z rozwiązaniami odniesienia wynika, że badany model bardzo skutecznie oddaje zachowanie cienkich i umiarkowanie grubych symetrycznie uwarstwionych płyt warstwowych.

MOST WIEDZY Downloaded from mostwiedzy.pl