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METHOD FOR SHIP'S ROLLING PERIOD PREDICTION WITH REGARD TO NON-LINEARITY OF GZ CURVE

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The paper deals with the problem of prediction of the rolling period. A special emphasis is put on the practical application of the new method for rolling period prediction with regard to non-linearity of the GZ curve. The one degree-of-freedom rolling equation is applied with using the non-linear stiffness moment and linear damping moment formulas. A number of ships are considered to research the discrepancies between the pending GM-based IMO--recommended method and the results of conducted numerical simulations performed for a wide range of operational loading conditions. Since the research shows some drawbacks of the IMO formula for the ship rolling period, a new formula is worked out and proposed instead.

Keywords: ship natural roll period, GZ approximation, rolling prediction, safety against capsizing

1. Introduction

A ship performing in rough sea conditions experiences complex motions which are the outcome of the combination of linear displacements and rotations considering each axis of the reference system. With regard to a ship safety, the greatest concern is related to its rolling oscillations. The main parameters of the rolling equation are: inertia, damping, stiffness and excitation. Each of them reveals a significant nonlinearity and its proper application has an impact on the obtained results.

Generally, potentially dangerous situations that may cause capsizing of a ship that remains intact, can be divided into resonant and non-resonant ones (Błocki, 2000). The non-resonant situations are mainly the outcome of two possible scenarios. First of all, they are caused by a combination of the ship rolling motion and a dynamic gust of wind. The scenario is well described by weather criteria of stability in accordance with the IMO IS-Code (IMO, 2009). Secondly, they are caused by the loss of stability on following or quartering seas when the wave crest is amidships. Furthermore, broaching and surfriding may be classified as non-resonant phenomena which may lead to capsizing (IMO, 2007).

The resonant situations may be divided into parametric resonance and synchronous rolling. The first one occurs mainly on following and quartering seas and is caused by inducing instability and an increase in the rolling amplitude due to the periodic variation of ships stability characteristics, not as a result of direct roll excitation. The synchronous rolling takes place usually at beam seas when the encounter period is close to the natural period of ship roll or half of this period (IMO, 2007).

The above-mentioned adverse phenomena represent a real threat to the ship safety. Also they are very complex and have significantly nonlinear characteristics. As a result, there is a great amount of research that has been undertaken worldwide for many years. The literature review shows a list of works taking into consideration various aspects of ships rolling motion. Some of the older works on the topic of damping of rolling are worth mentioning, see Cardo *et al.*

(1994), Haddara and Bennett (1989), Himeno (1981). Among newer papers, there is a significant number of worthwhile works devoted to the synchronous rolling and parametric rolling in the context of development of the second generation of intact stability criteria (Belenky *et al.*, 2011; Holden *et al.*, 2007; Neves and Rodriguez, 2006, 2009; Shin *et al.*, 2004; Umeda, 2013). Also, there is a noteworthy quantity of works dedicated to various mathematical models of rolling, mainly with regard to nonlinearities existing in particular components of the roll equation or a system of motion equations in the case of the coupling analysis (Bulian, 2005; Contento *et al.*, 1996; Francescutto and Contento, 1999; Taylan, 2000). The importance of the influence of the mathematical model used was presented by Spanos and Papanikolaou (2009) who presentes a comparison of results of calculations of rolling on waves using 14 different, recognized simulations programs. The consistency of some results left much to be desired giving the room for further researches in this field.

There is a group of works where one of the goals was to describe relatively simple analytical formulas enabling one to calculate singular rolling characteristics, for example the maximum rolling amplitude that a ship may reach in parametric rolling conditions (Shin *et al.*, 2004).

In many of the above-mentioned papers the natural roll frequency, which strongly determines the resonance mode of motion, is used as one of the rolling equation parameters. Unfortunately, this frequency is calculated on the basis of the initial metacentric height of a ship, most often with the use of a simple formula recommended in the IMO IS Code. However, this metacentric height is valid for small angles of heel, normally up to about 7 degrees, which do not pose a threat to the ship stability. The rolling frequency varies for greater amplitudes. The influence of the rolling amplitude on its frequency and, consequently, on the rolling period was investigated by Contento *et al.* (1996). In the paper by Kruger and Kluwe (2008), the authors directly suggest that variations of the rolling period should be taken into account and the energy balance method is proposed for the equivalent metacentric height calculation. This current paper is a continuation of the former work regarding the period prediction of ships natural rolling.

2. Applied model of rolling

The most commonly used model of rolling, which neglects any couplings from motions taking place in other than rolling degrees of freedom may be given by the following formula

$$(I_x + A_44)\ddot{\phi} + B_e\dot{\phi} + K(\phi) = M_w \cos(\omega_e t) \tag{2.1}$$

where I_x denotes the transverse moment of ship inertia; A_{44} is the moment of added mass due to water dragging by the rolling hull; B_e is the equivalent linear roll damping coefficient, $K(\phi)$ describes the righting moment, e.g. stiffness of the ship, M_w is the external heeling moment exciting rolling and ω_e is the encounter frequency of waves. Although, the roll damping is significantly nonlinear and the equivalent linear roll damping coefficient B_e depends on the amplitude, the frequency and the ship speed (Himeno, 1981; Uzunoglu and Guedes Soares, 2015), for the sake of simplicity the coefficient B_e is frequently assumed to be constant regardless the parameters of roll motion (Himeno, 1981; Shin *et al.*, 2004). In the case of still water conditions and the lack of other exciting moments the right hand side of equation (2.1) is equal to zero. In such a case, the ship motion is called the free rolling and is often used in the roll amplitude decay test. If so, the rolling equation is the following

$$(I_x + A_44)\ddot{\phi} + B_e\dot{\phi} + K(\phi) = 0 \tag{2.2}$$

The righting moment equals

$$K(\phi) = D\,GZ(\phi) \tag{2.3}$$

where GZ is the righting arm curve and D is ship weight.

After substitution of formula (2.3) into equation (2.1) we divide both sides by $(I_{xx} + A_{44})$ and introduce the commonly used notation. Then the roll equation becomes

$$\ddot{\phi} + 2\mu\dot{\phi} + \frac{\omega^2}{GM}GZ(\phi) = \xi_w \cos(\omega_e t) \tag{2.4}$$

where ω denotes the natural roll frequency of the ship, GM is the initial metacentric height, μ is the damping coefficient and ξ_w means the exciting moment coefficient.

Formula (2.4) is used quite often, however apart from the GZ characteristic which is valid for the full range of angles of the ship heel, it also contains the initial metacentric height and the natural roll frequency that are only related to the initial stability of the ship. Thus, the two fore mentioned variables reflecting the stiffness characteristics for small angles of heel restrict the potential applicability of equation (2.4). To avoid such restrictions and eliminate the initial stability characteristics from equation (2.4) it is transformed into a more versatile form by substituting $\omega = \sqrt{g\,GM}/r_x$:

— for excited rolling

$$\ddot{\phi} + 2\mu\dot{\phi} + \frac{g}{r_x^2}GZ(\phi) = \xi_w \cos(\omega_e t) \tag{2.5}$$

— for free rolling

$$\ddot{\phi} + 2\mu\dot{\phi} + \frac{g}{r_x^2}GZ(\phi) = 0 \tag{2.6}$$

where g is the gravity acceleration and r_x is the gyration radius of the ship and added masses (which is assumed to be constant for the sake of simplicity).

Formula (2.6) constitutes the mathematical model of rolling applied in the course of the conducted research. All further simulations of the ship motion are based on this equation.

The rolling equation given by formula (2.6) requires modeling the stiffness related to transverse stability of the ship. The most straightforward approach would be calculation of the righting arm based on the actual shape of the underwater part of the ship hull performed in every time step of the rolling simulation. However, such a solution is excessively time consuming and requires too much processing power for practical use. Thus, the next approach is to apply a $GZ(\phi)$ function obtained prior the start of rolling simulation for the actual loading condition of the ship. However, there is a lack of analytical formulas describing the righting arm GZ due to the fact that the shape of KN arm strictly depends on the shape of the ship hull. To address this problem, most researchers apply various approximations of the GZ curve. The choice of the GZ modeling method is a kind of trade-off decision because of the contradictory goals like accurate GZ approximation and reasonable time of each simulation.

The simplest approximation formulas like for instance $GZ(\phi) = GM\phi(1 - \phi^2)$, see Belenky *et al.* (2011), produces so rough estimation of the GZ characteristic that applied in the rolling equation they cannot achieve a sufficient accuracy of the results of rolling simulation. On the basis of the literature review, it seems that the most popular approach is the application of a polynomial power series. Many authors use the fifth to ninth order polynomials (Contento *et al.*, 1996; Surendran and Venkata Ramana Reddy, 2003; Taylan 2000) with only odd powers of the angle of heel due to a symmetrical character of the GZ curve. A limited number of authors apply higher order polynomials like seventh or ninth, and rarely even higher (Bulian, 2005).

For the purpose of this study, two approaches toward GZ approximation have been tested, e.g. the ninth-order polynomial power series with odd powers only and the trigonometric polynomial as an interesting alternative (Wawrzyński, 2015). Finally, the ninth-order polynomial power series are applied to approximate the value of the righting arm in each time-step of the performed rolling simulations. The GZ formula is the following

$$GZ(\phi) = C_1\phi + C_3\phi^3 + C_5\phi^5 + C_7\phi^7 + C_9\phi^9 \tag{2.7}$$

where C_1 to C_9 are the coefficients of odd orders obtained with the use of the least squares method.

3. Considered ships and their stability conditions

The intended scope of the study comprises researching a wide variety of ships in terms of their size, type and loading conditions. Thus, the number of significantly varying ships are selected with the length ranging from 76 m up to 320 m. The types of vessels are: motor tanker, bulk carrier, 5000 TEU Panamax container ship, 7500 TEU container ship, LNG carrier, general cargo ship and an offshore support vessel.

The stability characteristics of these vessels are also notably varying and they cover a wide range of stability options applicable in operational loading conditions. To obtain the wished-for variety of typical shapes of the GZ curve, the ships loading conditions are systematically selected. However, only loading conditions reflecting the practically applicable draft of ships and weight distributions are taken into account. The earlier work by Wawrzyński (2015) enables one to distinguish three typical shapes of the righting arm curve for loading conditions with obligatory positive GM value. The sketches of such GZ shapes are shown in Fig. 1. In the case of the pending research, stability characteristics are preselected to cover all three shapes marked A, B and C in Fig. 1.

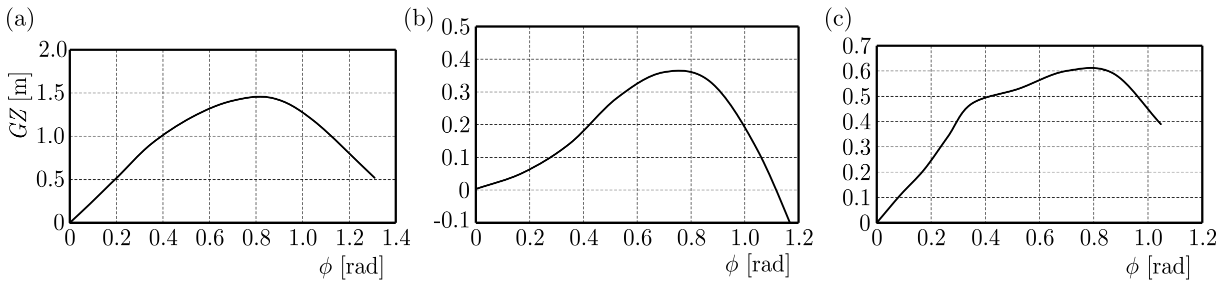

Fig. 1. Typical characteristics of the GZ curve for operational loading conditions with a positive value of GM (Wawrzyński, 2015)

To obtain a set of stability characteristics corresponding to typical cases given in Fig. 1. the initial metacentric height of considered vessels needs to significantly vary. Although some values of drafts and GM may subsist only in the ballast condition (due to location of typical ballast tanks in the ship double bottom) and others rather than in the fully loaded condition. Thus, the range of considered drafts and metacentric heights is also relatively wide. However, all the ships have a positive value of the metacentric height and not less than required by the IMO Intact Stability Code. The main particulars of the considered vessels and the variety of draft and stability of the ships are shown in Table 1. Since seven ships are studied and a couple of loading condition per each ship, the number of considered cases is pretty large.

4. Results of rolling simulations vs. GM-based IMO-recommended formula ones

The natural rolling period of the ship remains the matter of concern of many researches and practical issues. The notion of the natural rolling period is also mentioned in a number of documents issued by the International Maritime Organization, like Intact Stability Code (IMO, 2009) or MSC.1/Circ.1228 (IMO, 2007) as the second generation ship stability criteria which are under development (Umeda, 2013). The simple method for calculation of the rolling period τ is given in the Intact Stability Code and it is recommended for on-board use in absence of

Table 1. Main particulars of ships considered in the course of the research

Type of vessel	Length [m]	Breadth [m]	Considered draft T		Metacentric height GM	
			min [m]	max [m]	min [m]	max [m]
General cargo ship	140.00	22.00	6.00	9.00	0.40	1.00
5000 TEU Panamax container ship	283.20	32.20	7.50	13.50	0.50	3.00
7500 TEU container ship	285.00	45.60	7.50	12.50	2.00	5.00
Bulk carrier	156.10	25.90	6.50	10.50	1.00	3.50
Motor tanker	320.00	58.00	10.00	22.00	5.00	10.00
LNG carrier	278.80	42.60	7.50	12.00	1.00	5.00
Offshore support vessel	76.20	17.00	3.60	6.10	1.50	2.50

sufficient information. Although, the lack of reliable "sufficient information" takes place almost permanently. The formula based on the initial metacentric height of a ship is (IMO, 2009)

$$\tau = \frac{2cB}{\sqrt{GM}} \qquad c = 0.373 + 0.023\frac{B}{T} - 0.043\frac{L}{100} \tag{4.1}$$

where c is the coefficient describing ships transverse gyration radius; B means ship breadth and L ship length at waterline; T denotes mean ship draft. Actually, the radius of gyration equals cB where c is the dimensionless coefficient to be multiplied by the ship breadth given in meters.

The values obtained on the basis of simple formula (4.1) are compared in the course of the study with the rolling periods calculated in a more sophisticated way. The research and further reasoning are based on the solution of the rolling equation given by formula (2.6). We consider four loading conditions per each ship except for the offshore support vessel due to exceptional variety of shapes of the GZ curve. For this OSV, six loading conditions are taken into account. Thus, the set consists of 30 cases. For each case, numerous simulations have been carried out in order to obtain the rolling characteristics, mainly the period of roll for the full range of rolling amplitudes. The consecutive by considered amplitudes vary by one degree from 1 up to 60 degrees, so the total number of executed simulations reaches 1800.

The numerical simulations have been performed with the use of two separate software tools. One was the CAS software and the Runge-Kutta build-in method was applied. The limited number of cases solved this way was used for the purpose of comparison and validation of the results obtained with the use of the main tool which was the own-developed Matlab script, prepared to run the roll simulations. The results obtained using these two tools truly converge so the Matlab script is the preferred one as a main software giving the unlimited control on the computation practice and data processing.

When the free rolling of ships is considered (the right side of equation (2.6) equals zero), the assumed initial angle of the ship heel is the cause of rolling. Generally, the idea of this research is to analyze the period of roll for a specified rolling amplitude to work out a prediction method. Thus, there is a need to keep the amplitude almost steady for at least few ship's swings. To achieve this, the value of the damping coefficient has been set relatively small. However, the damping coefficient affects mainly the rolling amplitude while the change of the rolling period may be almost neglected, so the simulations results reflect free rolling of a ship fairly enough.

The solution obtained in each separate case related to one ship, one loading condition and one rolling amplitude is just a time history of roll motion which is shown in Fig. 2. The rolling period is one of the variables obtained in the course of data post processing. The fast Fourier transform is applied to acquire this period. Gathering the values of rolling period for the full range of amplitudes we obtain a set of data enabling comparison of numerical simulation results with the IMO formula based calculation, which is shown in Fig. 3 for one exemplary case. The description "IMO" used in the legend of the graph denotes the estimation of the rolling period according to IMO-recommended formula (4.1) based on the initial GM, and the description "num." refers to the results of own simulation.

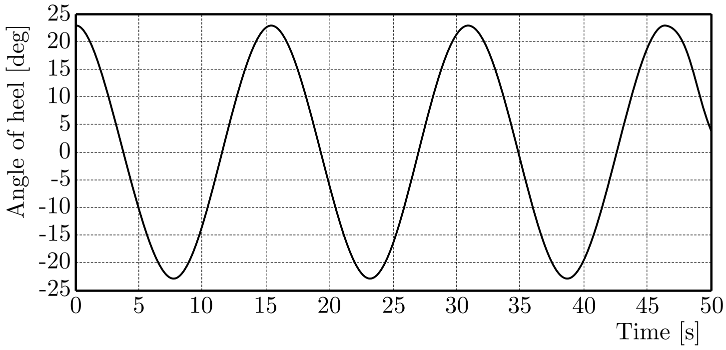

Fig. 2. Exemplary free rolling history of the general cargo ship for an considered rolling amplitude

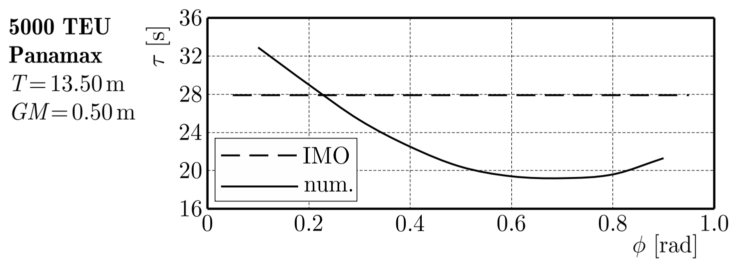

Fig. 3. Sample plot of the rolling period vs. rolling amplitude for 5000 TEU Panamax container ship

The sample plot of the rolling period vs. rolling amplitude for one selected ship (5000 TEU Panamax container ship in this case) and one loading condition (draft 13.50 m and initial metacentric height 0.50 m) clearly shows that the rolling period varies from 19 up to 33 seconds while the IMO formula returns the value close to 28 seconds. This example reveals the need for the elaboration of a new method.

5. Proposal of a simplified method for the rolling period prediction intended for practical use

To calculate the rolling period, very simple approximate formula (4.1) is recommended in the Intact Stability Code and it is used in the course of the assessment of the weather criterion of ship stability. Actually, small variations of the estimation of the rolling period influence the criterion to a limited degree as they slightly affect the angle of ships heel windward in the simplified model of rolling adopted in the weather criterion.

A much more serious matter is related to the IMO publication numbered MSC.1/Circ.1228 where simple formula (4.1) is also in use on-board. This is a *Revised guidance to the master for avoiding dangerous situations in adverse weather and sea conditions* intended to give some help to ship masters when sailing in stormy conditions. A relatively up to date publication (in comparison to dr. Rahola's findings from 1939 being the foundation of the Intact Stability Code) contains a set of remarks and advices regarding the avoidance of following dangerous dynamical phenomena at sea like surf-riding and broaching-to; reduction of intact stability when riding wave crest amidships; synchronous rolling motion; parametric roll motions (IMO, 2007). The recommendation expressed in the guidance is to estimate the natural roll period of the ship by observing roll motions in calm sea. Although such observations are rather difficult in many cases, so formula (4.1) is often used.

According to the IMO Circ.1228, some combinations of the wave length and wave height under certain operation conditions may lead to dangerous situations for ships complying with the IS Code (IMO, 2007). As the susceptibility of a ship to dangerous phenomena depends on the actual stability parameters, hull geometry, ship size and ship speed, the vulnerability to dangerous responses, including capsizing and its probability of occurrence in a particular sea state, may differ for each ship. Thus, the potentials for dangerous behavior of the ship shall be assessed.

One of the most important group of dangerous dynamic phenomena taken into account in IMO guidance 1228 is related to the resonance gain of rolling motion. This may occur due to nonlinearity of the ship response in resonance conditions, i.e. when the encounter wave frequency is similar to the first or second harmonic frequency of natural roll motion of a ship (Landrini, 2006). The prevention against synchronous rolling motion consists of avoiding such combinations of the ship speed and course which result in the encounter wave period T_E nearly equal to the natural rolling period of the ship τ ($T_E \approx \tau$) (IMO, 2007).

In the light of the briefly mentioned recommendations published in the *Revised guidance to the master for avoiding dangerous situations in adverse weather and sea conditions* and the degree of their importance related to safety of navigation, it seems to be fully justified to undertake the study on the rolling period prediction.

As the planned new rolling period approximation method is intended for the on-board use, it shall be well trade-off between the simplicity and the accuracy. The consequences of significant discrepancies between the predicted roll period of the ship and actual behavior at sea can be catastrophic in terms of potential involving the synchronous roll motion leading even to capsizing of the ship. Therefore, the planned method for roll period calculation should be as accurate as reasonably possible.

On the other hand, the new method cannot be too complex and complicated. Both authors of this study used to serve on-board seagoing ships and, thanks to this experience, they are aware of some limitations of practical use of any difficult formulas. Frankly, the best solution giving the greatest chance for real application in everyday sea practice is just the software realization with an easy to use interface. Thus, the proposed method of calculation should be simple enough to implement it and operate with no need for specialist knowledge of the crew, so any solver dealing with differential equations is just impractical on-board.

Keeping in mind the assumptions related to the method to be worked out, we focused our effort on the major elements governing the rolling period. On the basis of performed numerical simulations for all considered cases described in Section 3, we found two essential factors influencing variations of the rolling period. They are:

- the area under the GZ curve from the angle of heel equal to zero up to the rolling amplitude;
- the average inclination of the tangent line to the GZ curve from zero up to the rolling amplitude (such average inclination equals $GZ_{\varphi A}/\varphi_A$ for the amplitude of roll φ_A).

The key point is the relation of the rolling period characteristic and the appropriate GZ curve. The area under the GZ curve refers to the energy balance approach which was earlier suggested in Kruger and Kluwe (2008). The pace of change of the inclination of the GZ-tangent line describes the nonlinearity of the righting arm curve. The literature review reveals that this element was omitted in former works.

The elaborated simplified method for the rolling period prediction is based on both mentioned features which is far distinguishable from the contemporary method. On the other hand, the proposed new method should not be found by mariners as too complicated. As deck officers are familiar with IMO formula (4.1), the new one looks very similarly

$$\tau(\phi_A) = \frac{2cB}{\sqrt{GM_{eq}(\phi_A)}} \tag{5.1}$$

where the value of c coefficient remains the same as given in formula (4.1) $GM_{eq}(\phi_A)$ denotes the equivalent value of the transverse metacentric height of the ship for a specified rolling amplitude ϕ_A.

The approach proposed in form of formula (5.2) remains as simple as the original IMO version, however it requires proper calculation of $GM_{eq}(\phi_A)$ value which strictly depends on the area under the GZ curve and the average inclination of the tangent line to GZ. It shall be emphasized that the equivalent metacentric height is a function of the rolling amplitude ϕ_A and no longer can be given by one number.

The proposed formula for the equivalent metacentric height calculation is following

$$GM_{eq}(\phi_A) = w_1 \frac{2}{\phi_A^2} \int_0^{\phi_A} GZ(\phi)\, d\phi + w_2 \frac{GZ_{\phi_A}}{\phi_A} \tag{5.2}$$

where w_1 and w_2 are the weights adjusting the influence of two components on the equivalent metacentric height.

The weight coefficients w_1 and w_2 can be the subject of further consideration, however their values tuned on the basis of this research equals 0.5 each. In such a case, formula (5.2) becomes

$$GM_{eq}(\phi_A) = \frac{1}{\phi_A^2} \int_0^{\phi_A} GZ(\phi)\, d\phi + \frac{GZ_{\phi_A}}{2\phi_A} \tag{5.3}$$

As it is mentioned earlier, the proposed method for the rolling period calculation shall be simple enough to develop undemanding software for the on-board use. Such software can be prepared simply as an amendment to the standard stability program available on many ships. The GZ curve is obtained as a routine procedure to perform the intact ship stability assessment on the basis of the stability standards issued by the classification society. This curve could be the starting point for simple calculation according to formula (5.3). The result of computation would be a curve showing the rolling period versus the rolling amplitude for a particular ship in actual loading condition. Such a graph may be helpful to the captain when sailing in adverse weather to assess the possibility of occurrence of the synchronous roll motion. As the rolling amplitude is just an observable fact, the captain is able to enquire the natural roll period of the ship relevant to the noticed situation.

The elaborated proposal of the new method of the natural roll period calculation, expressed in formula (5.3), shall be verified on the basis of the results of numerical simulations. The required verification has been done for all 30 cases considered within this study in the full range of angles of heel.

The results of comparison are given as a set of plots shown in Figs. 4 to 10. The description "IMO" in the legend denotes the estimation of the rolling period according to the IMO-recommended formula based on the initial GM. The next description "num." refers to the results of numerical simulation and "GMeq" means the rolling period calculated according to newly proposed formula (5.2) with the equivalent value of GMeq calculated according to formula (5.3). The left side of each Figs. 4 to 10 shows the graph presenting the natural rolling period versus rolling amplitude, and the right side presents the corresponding GZ curve with the metacentric triangle.

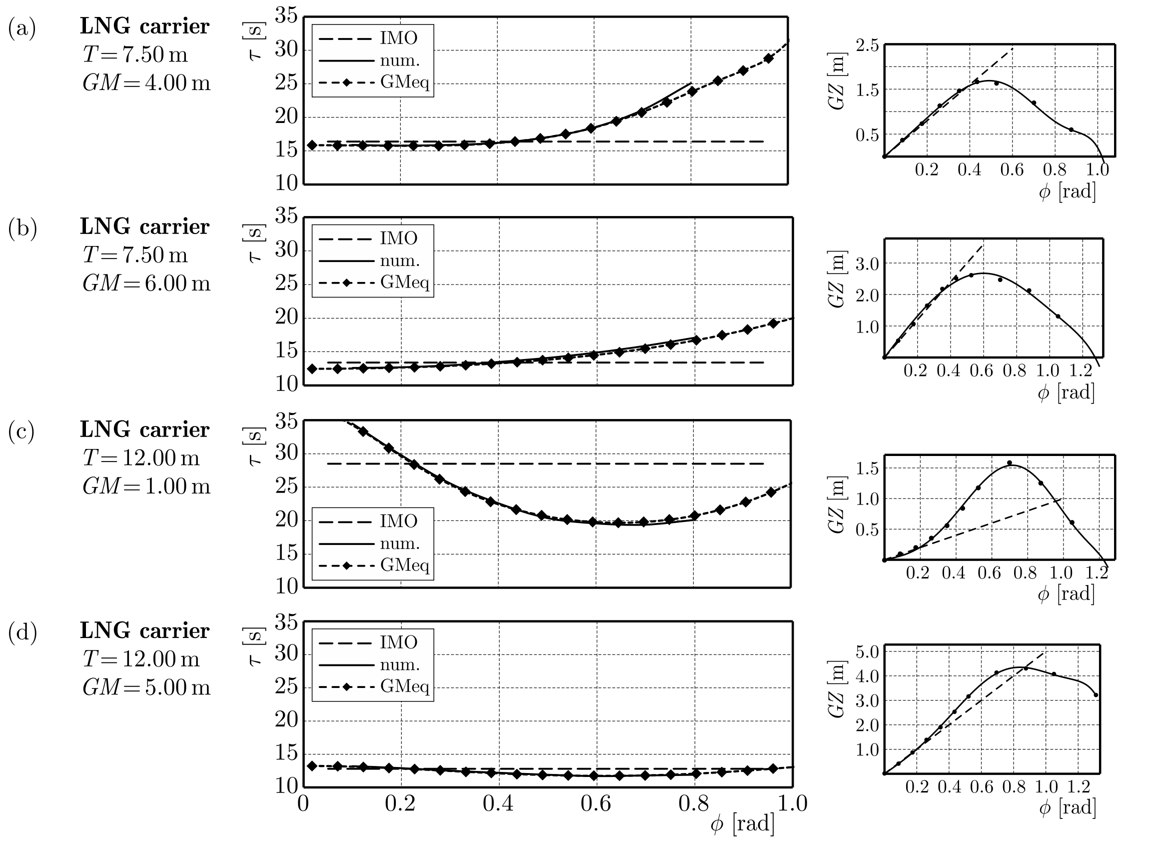

Fig. 4. Rolling period vs. rolling amplitude for the LNG carrier (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

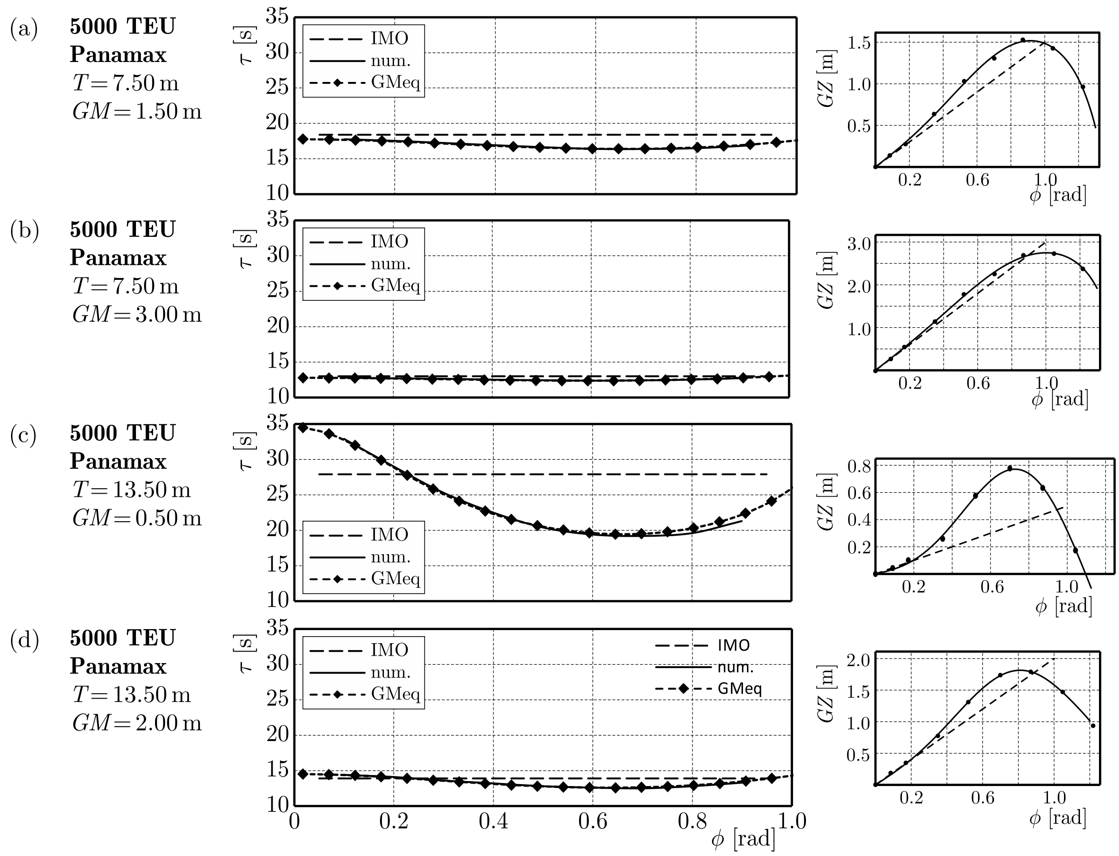

Fig. 5. Rolling period vs. rolling amplitude for the 5000 TEU Panamax container ship (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

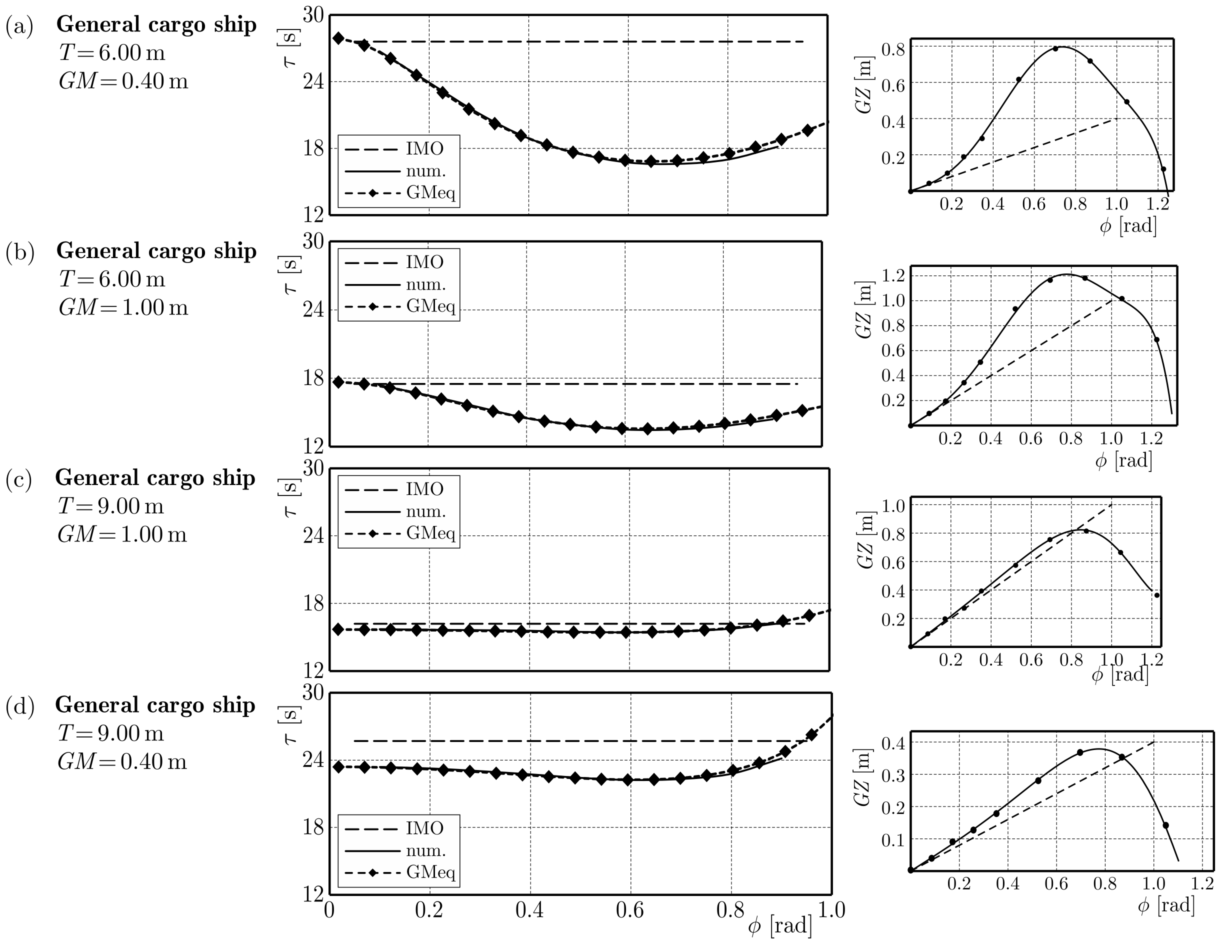

Fig. 6. Rolling period vs. rolling amplitude for the general cargo ship (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

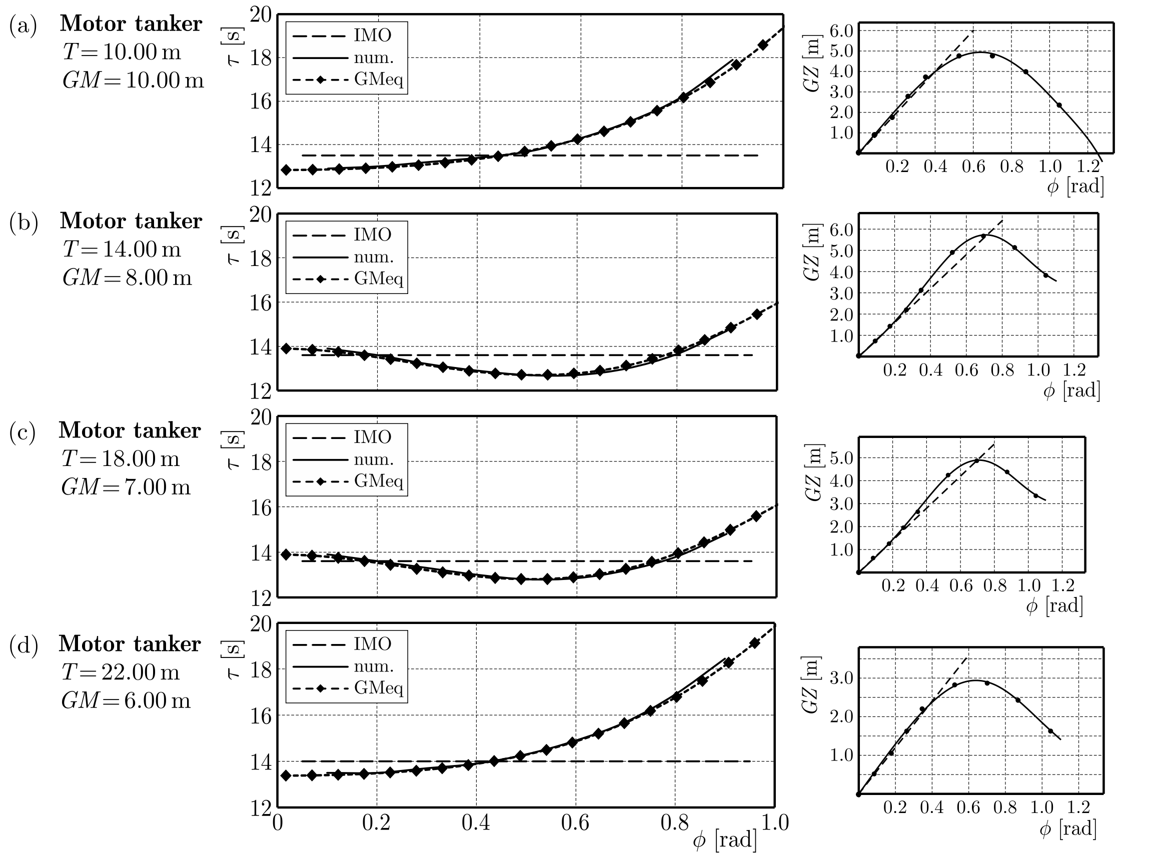

Fig. 7. Rolling period vs. rolling amplitude for the motor tanker (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

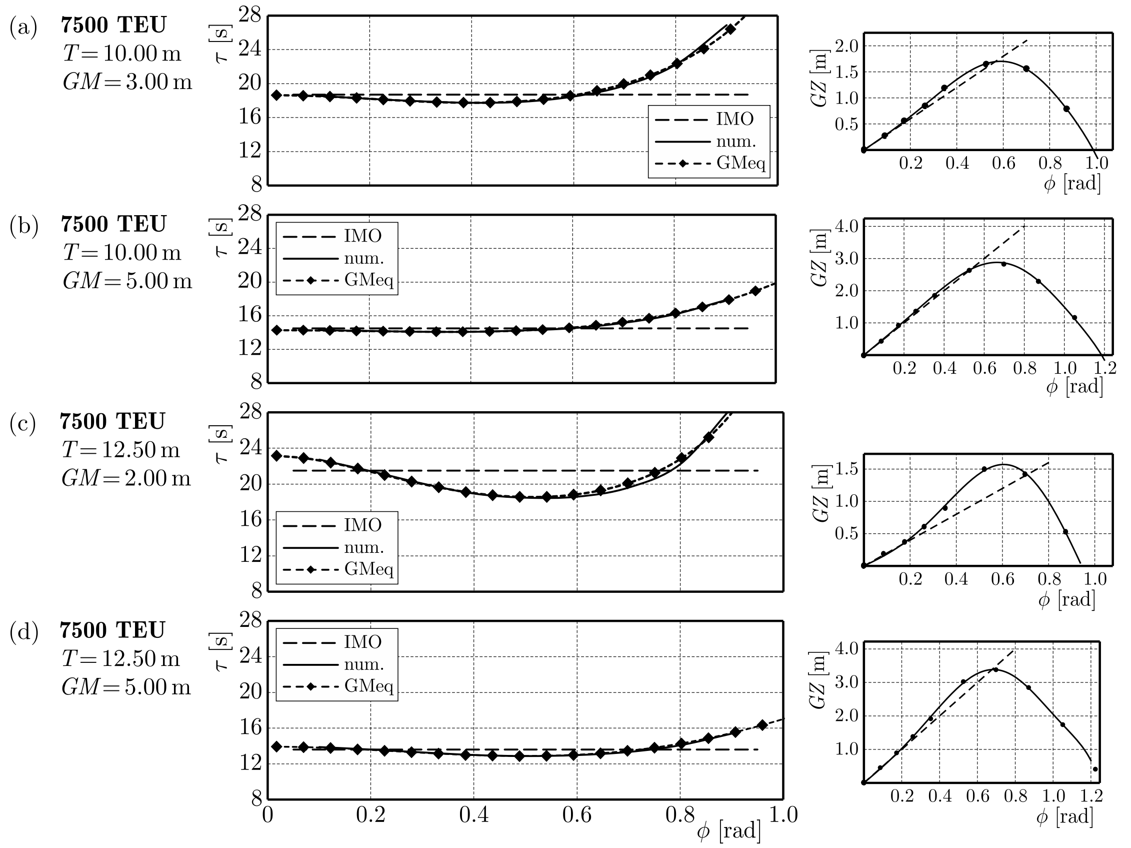

Fig. 8. Rolling period vs. rolling amplitude for the 7500 TEU container ship (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

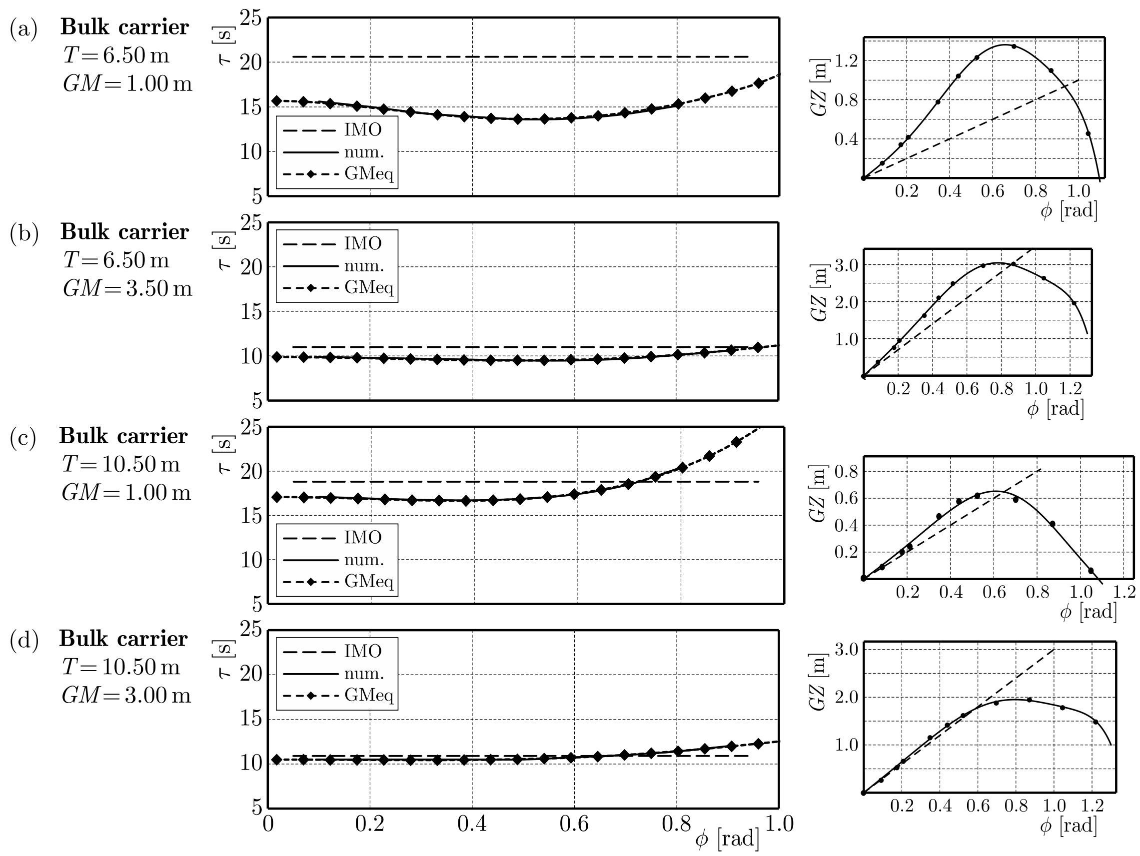

Fig. 9. Rolling period vs. rolling amplitude for the bulk carrier (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

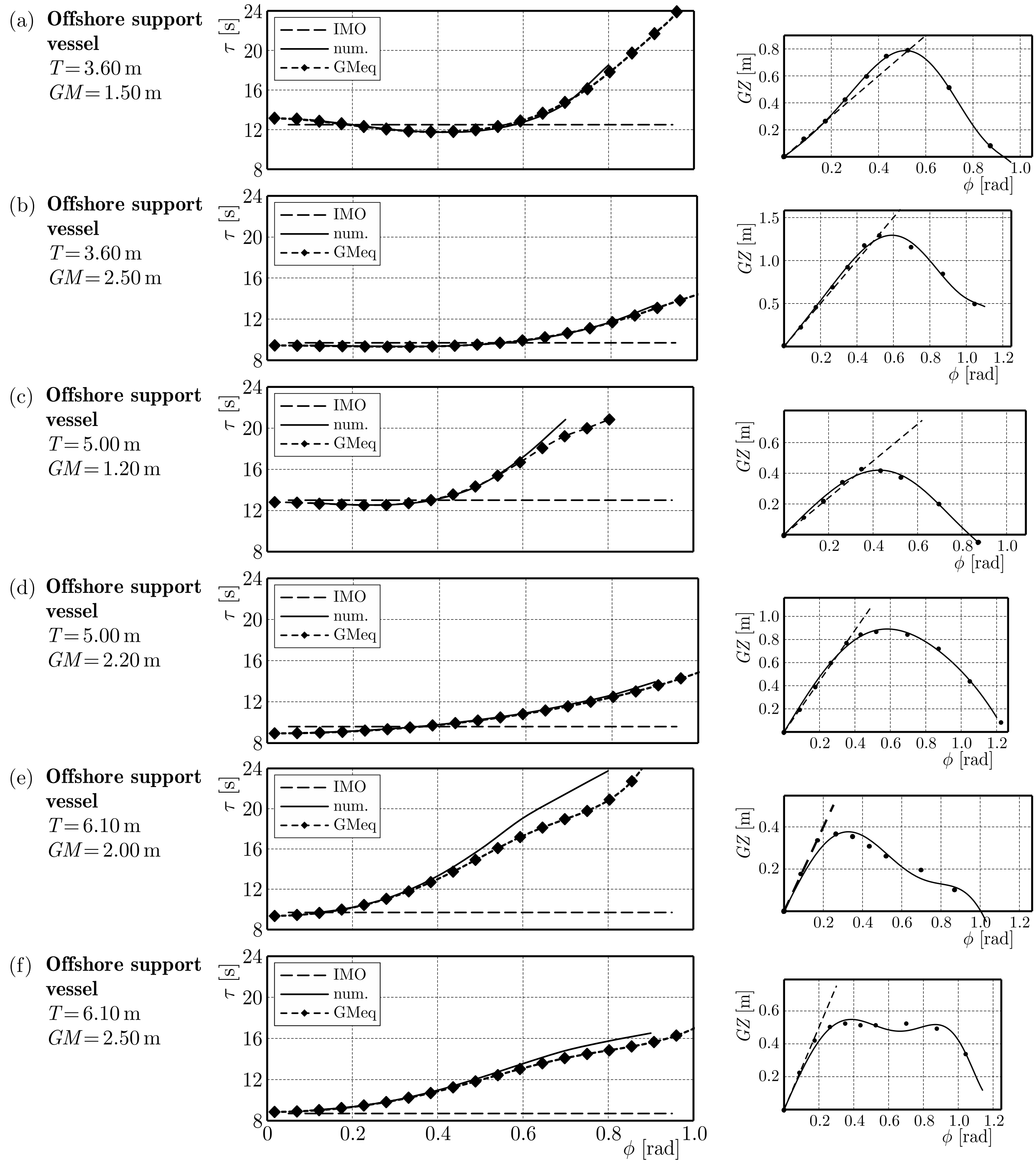

Fig. 10. Rolling period vs. rolling amplitude for the offshore support vessel (left plot) and the corresponding GZ curve and the metacentric triangle (right plot)

The verification of the proposed method of calculation of the rolling period, which is presented in Figs. 4 to 10, reveals very good consistency with to the results of numerical simulations in a wide range of angles of heel. Comparing to the IMO-recommended formula, the new one reaches the unattainable earlier level of accuracy. The most difficult and demanding cases are some loading conditions of the offshore support vessel. The results shown in Fig. 10 prove that the proposed method manages to predict the rolling period with a precision sufficient for on-bard use even dealing with such complicated shapes of the righting arm curves.

Formulas (5.2) and (5.3) are rather simple in terms of practical calculations. Anyway, it is worth to consider whether the proposed method could be more simplified to enable calculation even without any software supporting the computation. The first element $(\int_0^{\phi_A} GZ(\phi)\,d\phi)/\phi_A^2$ contains a definite integral $\int_0^{\phi_A} GZ(\phi)\,d\phi$. Its value can be easily obtained with the use of a common trapezoidal rule of numerical integration used in the same way as in the case of ship dynamical stability calculation. This approach is well known by officers on-board. The second element $GZ_{\phi_A}/(2\phi_A)$ requires no difficult computation at all. An example of such calculation is presented in Table 2.

Table 2 shows a typical pattern used for ship dynamical stability calculation (e.g. the area under the GZ curve) which is supplemented by the formula enabling calculation of the simplified equivalent metacentric height

$$GM_{eq\_simple}(\phi_A) = \frac{1}{\phi_A^2}\Big(S_{\phi_A} + \frac{1}{2}GZ(\phi_A)\phi_A\Big) \tag{5.4}$$

where S_{ϕ_A} is the area under the GZ curve integrated from zero up to the rolling amplitude.

Table 2. Exemplary simplified calculation for the natural rolling period (5000 TEU container vessel, draft $T = 13.50$ m and metacentric height $GM = 0.5$ m) – none of the dedicated software needs to be used

ϕ_A [°]	10	20	30	40	50	60	
KN [m]	2.61	5.21	7.80	10.08	11.72	12.70	Contemporary standard calculations
GZ [m]	0.10	0.27	0.57	0.78	0.63	0.17	
$\sum GZ$	0.10	0.47	1.31	2.66	4.07	4.87	
S_{ϕ_A}	0.01	0.04	0.11	0.23	0.36	0.42	
ϕ_A [rad]	0.175	0.349	0.524	0.698	0.873	1.047	Additional calculations
GM_{eq_simple} [m]	0.61	0.71	0.94	1.03	0.82	0.46	
τ [s] (Eq. (5.1))	25.3	23.4	20.3	19.4	21.7	28.9	

$S_{\phi_A} = [\Delta\phi_A/(2\cdot 57.3)]\sum GZ$

The validation of the very simplified calculation scheme presented in Table 2 is shown in Fig. 11. The results are compared to evaluate the conformity which is plotted in the graph.

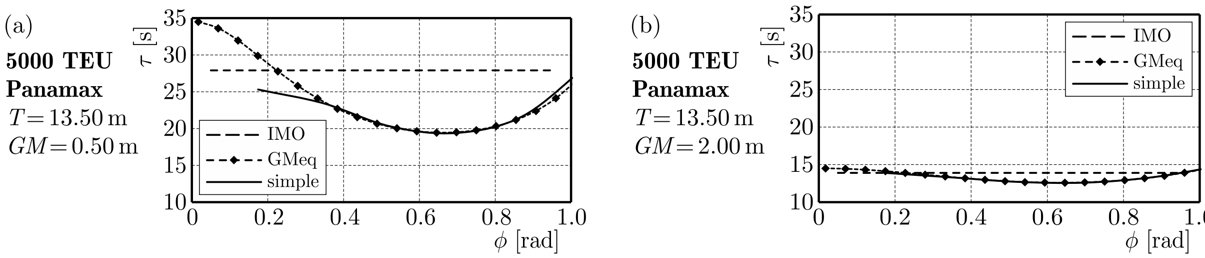

Fig. 11. Prediction of the rolling period vs. rolling amplitude for the 5000 TEU container vessel – validation of the simplified method applied according to formula (5.4) and with the use of calculation scheme presented in Table 2 (two sample loading conditions of the ship)

The graph shown in Fig. 11 reveals an astonishing conformity of the simplified method results as per the degree of the simplification used. The only range of significant discrepancies reaches from an angle of heel 0 up to about 15-20 degrees. This is due to the trapezoidal rule of numerical integration assuming a linear character of the integrated curve in every interval. However, this range of 0-15 degrees does not play an important role in terms of ship safety, especially safety against capsizing due to potential synchronous rolling motion. Basically, ships do not capsize at an angle of heel 15 or 20 degrees. The accuracy reached in the useful range of angles of heel allows one to propose the newly elaborated method instead of the IMO-recommended one.

6. Conclusions

In this paper, the problem related to the prediction of ship natural rolling period is outlined. It is revealed that rolling being a nonlinear phenomenon governed by the differential equation of ship motion can be described in terms of its period, by a relatively simple formula. Moreover, the accuracy of such an approach is outstanding as per the degree of used generalization. Both these features, e.g. simplicity and accuracy are essential in terms of practical application on-board ships. The proposed formula for the natural rolling period calculation can be implemented in fairly simple software being a decision support tool or it can even be used without any dedicated software in the course of straightforward calculation. Unlike the proposed formula, any numerical simulations of rolling cannot be performed on-board due to their complexity and high demands for crew education.

The elaborated method has been verified on the basis of the range of cases comprising seven different ships of dissimilar sizes and the number of their loading conditions covering draft and stability characteristics from the ballast condition up to the fully loaded state of a vessel. For each case and for the full range of possible rolling amplitudes, numerical simulations of rolling has been carried out. Positive verification of the new method is definitely a vital issue due to the importance of natural rolling period prediction. Generally, the greatest threat to the stability safety of a vessel is related to the resonance of rolling. The resonant motion could be predicted and avoided only in the case of quite precise prediction of the rolling period. Thus, the IMO-recommended formula ought to be rather modified and the newly proposed one could be introduced instead.

Although the proposed method has been widely verified, there is still room for further researches leading to tuning of the weight coefficients given in formula (5.1). The influence of some factors could be also assessed in the course of future researches, for instance the effects of damping, accuracy of the GZ curve approximation and variations of stability characteristics on waves.

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