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Numerical and quantitative analysis of HIV/AIDS model with modified Atangana-Baleanu in Caputo sense derivative

Muhammad Farman [a,b], Saba Jamil [a], Muhammad Bilal Riaz [c,d,*], Muhammad Azeem [e], Muhammad Umer Saleem [f]

[a] Institute of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Pakistan
[b] Faculty of Arts and Sciences, Department of Mathematics, Near East University, Turkey
[c] Faculty of Applied Physics and Mathematics, Gdansk University of Technology, 80-233 Gdansk, Poland
[d] Department of Mathematics, University of Management and Technology, 54770 Lahore, Pakistan
[e] Department of Mathematics and Statistics, University of Lahore, Lahore, Pakistan
[f] Department of Mathematics, University of Education, Lahore, Pakistan

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Abstract Fractional calculus plays an important role in the development of control strategies, the study of the dynamical transmission of diseases, and some other real-life problems nowadays. The time-fractional HIV/AIDS model is examined using a novel method in this paper. Based on the Atangana-concept Baleanu's of a derivative in the Caputo sense, the current modified fractional derivative operator uses singular and non-local kernels. This new modified fractional operator is given a numerical approximation and applied to the HIV/AIDS model. In the presence of this novel operator, we present some significant analysis for the HIV/AIDS epidemic model. The uniqueness and stability criteria of the model have been demonstrated using the Picard successive approximation approach and Banach's fixed point theory. The Laplace Adomian decomposition method (LADM) was used to obtain the numerical solution for the modified Atangana-Baleanu Caputo derivative model. The convergence analysis is verified for the proposed scheme. Finally, numerical results and simulations are derived with the proposed scheme for HIV/AIDS model. On the dynamics of HIV/AIDS transmission, the effects of many biological parameters are examined.

1. Introduction

To establish public health strategies, mathematical models may be a useful tool [1,2]. Although it is unlikely that a mathematical model will be able to provide long-term predictions

* Corresponding author.
E-mail address: bilalsehole@gmail.com (M.B. Riaz).

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about the number of AIDS cases that are correct, one such model, based on interactions that contribute to disease transmission, could eventually allow researchers to answer many relevant questions [3]. The dynamics of HIV/AIDS transmission as a result have given rise to several mathematical models in recent years see, for example [4,5] and references mentioned therein.

The human immunodeficiency virus (HIV), one of the most contagious and lethal viruses, kills millions of people each year [6]. As a part of a larger problem of co-infection between HIV/AIDS (acquired immunodeficiency syndrome) and tuberculosis, Silva and Torres proposed the deterministic SICA model in 2015 [7]. Later, it has been widely utilized to study HIV/AIDS in various contexts and configurations, utilizing stochasticity [8], and fractional-order derivatives [9], and modified to various HIV/AIDS epidemics, like those in Morocco and Cape Verde [10,11]. We recommend the reader to [12] for a survey on SICA models for HIV transmission that demonstrates how they offer a useful framework for interventions and tactics to stop the spread of the HIV/AIDS epidemic.

Because of the dynamics of HIV epidemics, researchers have consistently discussed HIV fractional order models [13,14]. The fractional order model, which uses fractional calculus and integrates and transects differentiation, can aid in understanding real-world problems more effectively than classical derivatives [15–20]. Riemann Liouville first proposed the concept of a fractional derivative based on the power law. Atangana et al. [21,22] suggest the new fractional derivative using the exponential kernel. Trigonometric and exponential function-related issues with non-singular kernel fractional derivatives [23–26] illustrate several relevant strategies for the models of an epidemic. A numerical method for solving the nonlinear fractional differential equation has recently been introduced [27,28]. Antiretroviral therapy is included in the fractional order model of HIV/AIDS that has been researched in [29]. The analysis of the HIV/AIDS model under ABC fractional order derivative is proposed in [30].

We may encounter some problems with the initialization for nonsingular kernels. As in [31] for all equations in the form:

$$\int_a^x H(x,t)z(t)dt = k(x), a \neq x \neq b, \tag{1}$$

if $H(a,a) \neq 0$, then $k(a) = 0$, where $H(x,t)$ and $k(x)$ are continuous. Due to this condition, the corresponding differential equations with nonsingular kernels experience some unusual constraints. Refer to [31] on page 3 for more information about this significant problem that has not yet been resolved. The aforementioned problem for the operator introduced in [15] was recently resolved by [32,33]. To address the aforementioned issues with nonsingular operators, in [34] authors modified the operator with a Mittag–Leffler kernel and established that the associated FDEs based on this new operator is simple to initialize and show that there are multiple fractional differential problems that the ABC derivative cannot solve but that can be solved by the MABC derivative. The MABC derivative has the integrable singularity at the origin. In [35] the authors presented a revolutionary finite difference-based numerical technique for the MABC derivative, which makes it simple to initialize the associated fractional differential equations. The MABC derivative with Mittag–Leffler kernels was used by researchers to create modified fractional difference operators [36].

This article is structured as follows: section two presents the fundamental definitions and findings for the fractional operator and the Laplace transform. Section three covers the formation of fractional order models, and proved lemmas for the proposed model. Section four provides an iterative method for solving the aforementioned model using the LADM, and also discusses stability conditions using the Banach fixed point theory and the Picard successive approximation method. In section five, discussion and numerical simulations for various fractional-order values are carried out and graphically presented. Convergence is covered in section six, and then in section seven, we provide our conclusion.

2. Basic concepts

In this part, some basic definitions and conclusions from fractional calculus are given.

Definition 1. [37].

Let $x \in H^1([0,T]), T > 0,$ and $v \in (0,1)$. The Riemann–Liouville time-fractional derivative of order $v$is defined as

$$^{RL}D_t^v x(t) = \frac{1}{\Gamma(1-v)}\frac{d}{dt}\int_0^t (t-\tau)^{-v}x(\tau)d\tau, \tag{2}$$

the Riemann–Liouville derivative's Laplace transform is given by

$$\mathcal{L}\{^{RL}D_t^v x(t)\} = s^v\mathcal{L}\{x(t)\} - D_t^{v-1}x(t)\big|_{t=0}.$$

Definition 2. [37].

Let $x \in H^1([0,T]), T > 0,$ and $v \in (0,1)$. The usual Caputo time-fractional derivative of order v is expressed as

$$^{C}D_t^v x(t) = \frac{1}{\Gamma(1-v)}\int_0^t (t-\tau)^{-v}x\prime(\tau)d\tau, 0 \leqslant v \leqslant 1. \tag{3}$$

the Caputo derivative Laplace transform is

$$\mathcal{L}\{^{C}D_t^v x(t)\} = s^v\mathcal{L}\{x(t)\} - s^{v-1}x(0).$$

Definition 3. [15].

Let $x \in H^1([0,T]), T > 0,$ and $v \in (0,1)$. The CF derivative of a function $x(t)$ is defined by

$$^{CF}_{0}D_t^v x(t) = \frac{M(v)}{1-v}\int_0^t \frac{d}{d\tau}x(\tau)\exp\left(-\frac{v(t-\tau)}{1-v}\right)d\tau. \tag{4}$$

Caputo-Fabrizio derivative's Laplace transform is

$$\mathcal{L}\{^{CF}D_t^v x(t)\} = \frac{s\mathcal{L}\{x(t)\} - x(0)}{(1-v)s+v}, \left|\frac{v/(1-v)}{s}\right| < 1.$$

Definition 4. [22].

Let $x \in H^1([0,T]), T > 0,$ and $v \in (0,1)$. The ABC fractional derivative of a function $x(t)$ is presented as

$$^{ABC}_{0}D_t^v x(t) = \frac{AB(v)}{1-v}\int_0^t \frac{d}{d\tau}x(\tau)E_v\left(-\frac{v(t-s)^v}{1-v}\right)d\tau, \tag{5}$$

where $E_v(y)$ is the Mittag–Leffler kernel function of order v which is presented as follows

$$E_v(y) = \sum_{k=0}^{\infty}\frac{y^k}{\Gamma(vk+1)}. \tag{6}$$

and $AB(v)$ is normalization function and $AB(0) = AB(1) = 1$. The Laplace transform is obtained by:

$$\mathcal{L}\{{}^{ABC}D_t^v x(t)\} = \frac{AB(v)}{(1-v)} \frac{s^v \mathcal{L}\{x(t)\} - s^{v-1}x(0)}{s^v + \frac{v}{1-v}}, \left|\frac{v/(1-v)}{s}\right| < 1.$$

Definition 5. [34].

Let $x \in L^1(0, T)$, the MABC derivative of order $0 < v < 1$, is defined by

$$ {}^{MABC}D_0^v x(t) = \frac{AB(v)}{1-v}\Big[x(t) - E_v(-\mu_v t^v)x(0) - \mu_v \int_0^t (t-\tau)^{v-1} E_{v,v}(-\mu_v (t-\tau)^v) x(\tau) d\tau\Big], \quad (7)$$

where $\mu_v = \frac{v}{1-v}$ and

$$AB(v) = 1 - v + \frac{v}{\Gamma(v)}. \quad (8)$$

The Laplace transform of MABC derivative is as follows:

$$\mathcal{L}\{{}^{MABC}D_t^v x(t); s\} = \frac{AB(v)}{(1-v)} \frac{s^v \mathcal{L}\{x(t); s\} - s^{v-1}x(0)}{s^v + \mu_v}, \left|\frac{\mu_v}{s^v}\right| < 1. \quad (9)$$

3. Fractional order HIV/AIDS model

In this section, we propose a memory-affected MABC model for HIV/AIDS. The HIV/AIDS epidemic model presented in [11] is a classical derivative to be taken into consideration. Four sub-compartments make up the HIV/AIDS epidemic model. Here, susceptible people are designated as $S(t)$, infectious HIV-positive individuals as $I(t)$, and HIV-positive people in the chronic stage of their disease who are taking ART and have a viral load that is still low as C, and people with HIV who have AIDS-related clinical symptoms as A. βrepresents the HIV contact rate, and $\Lambda$is a unit of measure for population acquisition of susceptible people. In comparison to HIV-infected people without signs of AIDS, people who have AIDS, they considered to be more contagious. This is taken into consideration by the modification parameter $\eta_A \geqslant 1$. However, $\eta_C \leqslant 1$ translates the partial recovery of the immunological system in HIV-infected people who are properly using ART. Everybody dies naturally at the constant rate μ. HIV-positive people without AIDS symptoms, HIV treatment for those displaying AIDS symptoms is done at a γ rate, and I transition to the category of people infected with HIV and receiving ART treatment C at a rate of ϕ. At a rate ω, people in class C move on to class I. Further, we presume that a person with HIV exhibiting AIDS signs Awho begins medication goes to HIV-positive person group I and only if the treatment is continued moves to the chronic class C. HIV-positive people who do not receive ART develop to AIDS class A at a rate of ρdespite not having any symptoms of the disease. Notably, AIDS-related deaths occur at a rate of $d$only in HIV-positive people who have AIDS symptoms A. The MABC system, which explains the earlier presumptions, is as follows:

$$\begin{aligned}
{}^{MABC}D_t^\alpha S &= \Lambda - \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \mu S(t),\\
{}^{MABC}D_t^\alpha I &= \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \xi_3 I(t) + \gamma A(t) + \omega C(t),\\
{}^{MABC}D_t^\alpha C &= \phi I(t) - \xi_2 C(t),\\
{}^{MABC}D_t^\alpha A &= \rho I(t) - \xi_1 A(t).
\end{aligned} \quad (10)$$

subject to initial conditions are

$$S(0) = S_0 = N_1 \geqslant 0, I(0) = I_0 = N_2 \geqslant 0, C(0) = C_0 = N_3 \geqslant 0, A(0) = A_0 = N4 \geqslant 0.$$

3.1. Analysis of the Proposed Model

Here, we demonstrate that there is a nonzero solution to the homogeneous fractional initial value problem. We'll employ the following formulas:

$$\mathcal{L}(E_\alpha(\hbar t^\alpha)) = \frac{s^{\alpha-1}}{s^\alpha - \hbar}, \left|\frac{\hbar}{s^\alpha}\right| < 1, \quad (11)$$

$$\mathcal{L}(t^{\alpha-1}E_{\alpha,\alpha}(\hbar t^\alpha)) = \frac{1}{s^\alpha - \hbar}, \left|\frac{\hbar}{s^\alpha}\right| < 1. \quad (12)$$

Lemma 1. [34,37] Consider the fractional initial value problem

$$\begin{aligned}
&{}^{MABC}D_0^\alpha S(t) = \bar{\lambda} S, t > 0, S(0) = S_0,\\
&{}^{MABC}D_0^\alpha I(t) = \bar{\lambda} I, t > 0, I(0) = I_0,\\
&{}^{MABC}D_0^\alpha C(t) = \bar{\lambda} C, t > 0, C(0) = C_0,\\
&{}^{MABC}D_0^\alpha A(t) = \bar{\lambda} A, t > 0, A(0) = A_0,
\end{aligned}$$

where $0 < \alpha < 1$.

(1) For $\bar{\lambda} = \frac{B(\alpha)}{1-\alpha}$, the solution is given by

$$S(t) = S_0 \begin{cases} -\frac{t^{-\alpha}}{\mu_\alpha \Gamma(1-\alpha)}, t \neq 0, \\ 1, t = 0. \end{cases}$$

$$I(t) = I_0 \begin{cases} -\frac{t^{-\alpha}}{\mu_\alpha \Gamma(1-\alpha)}, t \neq 0, \\ 1, t = 0. \end{cases}$$

$$C(t) = C_0 \begin{cases} -\frac{t^{-\alpha}}{\mu_\alpha \Gamma(1-\alpha)}, t \neq 0, \\ 1, t = 0. \end{cases}$$

$$A(t) = A_0 \begin{cases} -\frac{t^{-\alpha}}{\mu_\alpha \Gamma(1-\alpha)}, t \neq 0, \\ 1, t = 0. \end{cases}$$

(2) For $\bar{\lambda} \neq \frac{B(\alpha)}{1-\alpha}$, the solution is given by

$$S(t) = S_0 \begin{cases} \frac{E_\alpha\left(\mu_\alpha \frac{\dagger_\alpha}{1-\dagger_\alpha} t^\alpha\right)}{1-\dagger_\alpha}, t \neq 0 \\ 1, t = 0 \end{cases}$$

$$I(t) = I_0 \begin{cases} \frac{E_\alpha\left(\mu_\alpha \frac{\dagger_\alpha}{1-\dagger_\alpha} t^\alpha\right)}{1-\dagger_\alpha}, t \neq 0 \\ 1, t = 0 \end{cases}$$

$$C(t) = C_0 \begin{cases} \frac{E_\alpha\left(\mu_\alpha \frac{\dagger_\alpha}{1-\dagger_\alpha} t^\alpha\right)}{1-\dagger_\alpha}, t \neq 0 \\ 1, t = 0 \end{cases}$$

$$A(t) = A_0 \begin{cases} \frac{E_\alpha\left(\mu_\alpha \frac{\dagger_\alpha}{1-\dagger_\alpha} t^\alpha\right)}{1-\dagger_\alpha}, t \neq 0 \\ 1, t = 0 \end{cases}$$

where $\dagger_\alpha = \frac{\bar{\lambda}(1-\alpha)}{B(\alpha)}$.

Proof.

(1) Given that

$$\int_0^t (t-\kappa)^{\alpha-1} E_{\alpha,\alpha}(-\mu_\alpha (t-\kappa)^\alpha) \kappa^{-\alpha} d\kappa = \Gamma(1-\alpha) E_\alpha(-\mu_\alpha t^\alpha), \quad (13)$$

for $t>0$, we obtain

$$\begin{aligned}
{}^{MABC}D_0^\alpha S(t) &= \tfrac{B(\alpha)}{1-\alpha}\big(S(t)-E_\alpha(-\mu_\alpha t^\alpha)S_0 \\
&\quad -\mu_\alpha \int_0^t (t-\kappa)^{\alpha-1}E_{\alpha,\alpha}(-\mu_\alpha(t-\kappa)^\alpha) \\
&\quad \times\Big(-\tfrac{S_0}{\mu_\alpha\Gamma(1-\alpha)}\kappa^{-\alpha}\Big)d\kappa\big),\\
{}^{MABC}D_0^\alpha I(t) &= \tfrac{B(\alpha)}{1-\alpha}\big(I(t)-E_\alpha(-\mu_\alpha t^\alpha)I_0 \\
&\quad -\mu_\alpha \int_0^t (t-\kappa)^{\alpha-1}E_{\alpha,\alpha}(-\mu_\alpha(t-\kappa)^\alpha) \\
&\quad \times\Big(-\tfrac{I_0}{\mu_\alpha\Gamma(1-\alpha)}\kappa^{-\alpha}\Big)d\kappa\big),\\
{}^{MABC}D_0^\alpha C(t) &= \tfrac{B(\alpha)}{1-\alpha}\big(C(t)-E_\alpha(-\mu_\alpha t^\alpha)C_0 \\
&\quad -\mu_\alpha \int_0^t (t-\kappa)^{\alpha-1}E_{\alpha,\alpha}(-\mu_\alpha(t-\kappa)^\alpha) \\
&\quad \times\Big(-\tfrac{C_0}{\mu_\alpha\Gamma(1-\alpha)}\kappa^{-\alpha}\Big)d\kappa\big),\\
{}^{MABC}D_0^\alpha A(t) &= \tfrac{B(\alpha)}{1-\alpha}\big(A(t)-E_\alpha(-\mu_\alpha t^\alpha)A_0 \\
&\quad -\mu_\alpha \int_0^t (t-\kappa)^{\alpha-1}E_{\alpha,\alpha}(-\mu_\alpha(t-\kappa)^\alpha) \\
&\quad \times\Big(-\tfrac{A_0}{\mu_\alpha\Gamma(1-\alpha)}\kappa^{-\alpha}\Big)d\kappa\big).
\end{aligned}$$

$$\begin{cases}
=\frac{B(\alpha)}{1-\alpha}(S(t)-E_\alpha(-\mu_\alpha t^\alpha)S_0+E_\alpha(-\mu_\alpha t^\alpha)S_0),\\
=\frac{B(\alpha)}{1-\alpha}(I(t)-E_\alpha(-\mu_\alpha t^\alpha)I_0+E_\alpha(-\mu_\alpha t^\alpha)I_0),\\
=\frac{B(\alpha)}{1-\alpha}(C(t)-E_\alpha(-\mu_\alpha t^\alpha)C_0+E_\alpha(-\mu_\alpha t^\alpha)C_0),\\
=\frac{B(\alpha)}{1-\alpha}(A(t)-E_\alpha(-\mu_\alpha t^\alpha)A_0+E_\alpha(-\mu_\alpha t^\alpha)A_0).
\end{cases}$$

$$\begin{cases}
=\bar\lambda\, S(t),\\
=\bar\lambda\, I(t),\\
=\bar\lambda\, C(t),\\
=\bar\lambda\, A(t).
\end{cases}$$

which makes the proof complete.

(2) Using Eqs. (11) and (12) for $t>0$,

$$\begin{cases}
\mathcal{L}\{{}^{MABC}D_0^\alpha S;s\}=\frac{B(\alpha)}{1-\alpha}\frac{1}{s^\alpha+\mu_\alpha}\left(\frac{S_0 s^\alpha}{1-\dagger_\alpha}\times\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}}-S_0 s^{\alpha-1}\right),\\
\mathcal{L}\{{}^{MABC}D_0^\alpha I;s\}=\frac{B(\alpha)}{1-\alpha}\frac{1}{s^\alpha+\mu_\alpha}\left(\frac{I_0 s^\alpha}{1-\dagger_\alpha}\times\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}}-I_0 s^{\alpha-1}\right),\\
\mathcal{L}\{{}^{MABC}D_0^\alpha C;s\}=\frac{B(\alpha)}{1-\alpha}\frac{1}{s^\alpha+\mu_\alpha}\left(\frac{C_0 s^\alpha}{1-\dagger_\alpha}\times\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}}-C_0 s^{\alpha-1}\right),\\
\mathcal{L}\{{}^{MABC}D_0^\alpha A;s\}=\frac{B(\alpha)}{1-\alpha}\frac{1}{s^\alpha+\mu_\alpha}\left(\frac{A_0 s^\alpha}{1-\dagger_\alpha}\times\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}}-A_0 s^{\alpha-1}\right).
\end{cases}$$

$$\begin{cases}
=\frac{B(\alpha)}{1-\alpha}S_0\frac{\dagger_\alpha}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}},\\
=\frac{B(\alpha)}{1-\alpha}I_0\frac{\dagger_\alpha}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}},\\
=\frac{B(\alpha)}{1-\alpha}C_0\frac{\dagger_\alpha}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}},\\
=\frac{B(\alpha)}{1-\alpha}A_0\frac{\dagger_\alpha}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}}.
\end{cases}$$

$$\begin{cases}
=\bar\lambda\,\frac{S_0}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}},\\
=\bar\lambda\,\frac{I_0}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}},\\
=\bar\lambda\,\frac{C_0}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}},\\
=\bar\lambda\,\frac{A_0}{1-\dagger_\alpha}\frac{s^{\alpha-1}}{s^\alpha-\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}}.
\end{cases}$$

$$\begin{cases}
=\bar\lambda\,\frac{S_0}{1-\dagger_\alpha}\mathcal{L}\Big(E_\alpha\Big(\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}t^\alpha\Big)\Big),\\
=\bar\lambda\,\frac{S_0}{1-\dagger_\alpha}\mathcal{L}\Big(E_\alpha\Big(\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}t^\alpha\Big)\Big),\\
=\bar\lambda\,\frac{S_0}{1-\dagger_\alpha}\mathcal{L}\Big(E_\alpha\Big(\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}t^\alpha\Big)\Big),\\
=\bar\lambda\,\frac{S_0}{1-\dagger_\alpha}\mathcal{L}\Big(E_\alpha\Big(\mu_\alpha\frac{\dagger_\alpha}{1-\dagger_\alpha}t^\alpha\Big)\Big).
\end{cases}$$

it completes the proof.

Lemma 2. [34,37] Consider the FDE

$$\begin{cases}
{}^{MABC}D_0^\alpha S(t)+\bar\lambda\,S=J_1(t),\ t>0,\ S(0)=S_0,\\
{}^{MABC}D_0^\alpha I(t)+\bar\lambda\,I=J_2(t),\ t>0,\ I(0)=I_0,\\
{}^{MABC}D_0^\alpha C(t)+\bar\lambda\,C=J_3(t),\ t>0,\ C(0)=C_0,\\
{}^{MABC}D_0^\alpha A(t)+\bar\lambda\,S=J_4(t),\ t>0,\ A(0)=A_0.
\end{cases}$$

For $0<\alpha<1$, and $\bar\lambda\neq-\frac{B(\alpha)}{1-\alpha}$, the solution of the above fractional initial value problem is given by

$$S(t)=\begin{cases}\hat S,\ t\neq0,\\ S_0,\ t=0,\end{cases}\quad I(t)=\begin{cases}\hat I,\ t\neq0,\\ I_0,\ t=0,\end{cases}\quad C(t)=\begin{cases}\hat C,\ t\neq0,\\ C_0,\ t=0,\end{cases}\quad A(t)=\begin{cases}\hat A,\ t\neq0,\\ A_0,\ t=0.\end{cases}\tag{14}$$

where

$$\begin{cases}
\hat S=S_0\frac{B(\alpha)}{\ell_\alpha}E_\alpha\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)+\frac{1-\alpha}{\ell_\alpha}J_1(t)+\frac{1-\alpha}{\ell_\alpha}\Big(\mu_\alpha-\frac{\bar\lambda\alpha}{\ell_\alpha}\Big)\Big(t^{\alpha-1}E_{\alpha,\alpha}\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)\Big)J_1,\\
\hat I=I_0\frac{B(\alpha)}{\ell_\alpha}E_\alpha\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)+\frac{1-\alpha}{\ell_\alpha}J_2(t)+\frac{1-\alpha}{\ell_\alpha}\Big(\mu_\alpha-\frac{\bar\lambda\alpha}{\ell_\alpha}\Big)\Big(t^{\alpha-1}E_{\alpha,\alpha}\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)\Big)J_2,\\
\hat C=C_0\frac{B(\alpha)}{\ell_\alpha}E_\alpha\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)+\frac{1-\alpha}{\ell_\alpha}J_3(t)+\frac{1-\alpha}{\ell_\alpha}\Big(\mu_\alpha-\frac{\bar\lambda\alpha}{\ell_\alpha}\Big)\Big(t^{\alpha-1}E_{\alpha,\alpha}\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)\Big)J_3,\\
\hat A=A_0\frac{B(\alpha)}{\ell_\alpha}E_\alpha\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)+\frac{1-\alpha}{\ell_\alpha}J_4(t)+\frac{1-\alpha}{\ell_\alpha}\Big(\mu_\alpha-\frac{\bar\lambda\alpha}{\ell_\alpha}\Big)\Big(t^{\alpha-1}E_{\alpha,\alpha}\Big(-\frac{\bar\lambda\alpha}{\ell_\alpha}t^\alpha\Big)\Big)J_4.
\end{cases}$$

and $\ell_\alpha=B(\alpha)+\bar\lambda(1-\alpha)$.

Proof.

Using Eqs. (11) and (12) one can easily verify that

$$\begin{cases}
\mathcal{L}(\hat S;\ s)=\frac{S_0B(\alpha)s^{\alpha-1}+(1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_1;\ s)}{\ell_\alpha s^\alpha+\bar\lambda\alpha},\\
\mathcal{L}(\hat I;\ s)=\frac{I_0B(\alpha)s^{\alpha-1}+(1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_2;\ s)}{\ell_\alpha s^\alpha+\bar\lambda\alpha},\\
\mathcal{L}(\hat C;\ s)=\frac{C_0B(\alpha)s^{\alpha-1}+(1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_3;\ s)}{\ell_\alpha s^\alpha+\bar\lambda\alpha},\\
\mathcal{L}(\hat A;\ s)=\frac{A_0B(\alpha)s^{\alpha-1}+(1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_4;\ s)}{\ell_\alpha s^\alpha+\bar\lambda\alpha}.
\end{cases}\tag{15}$$

By Eq. (9) we have

$$\begin{cases}
\mathcal{L}({}^{MABC}D_0^\alpha S+\bar\lambda\,S;\ s)=\frac{B(\alpha)}{1-\alpha}\frac{s^\alpha\mathcal{L}(\hat S;s)-s^{\alpha-1}S_0}{s^\alpha+\mu_\alpha}+\bar\lambda\,\mathcal{L}(\hat S;\ s),\\
\mathcal{L}({}^{MABC}D_0^\alpha I+\bar\lambda\,I;\ s)=\frac{B(\alpha)}{1-\alpha}\frac{s^\alpha\mathcal{L}(\hat I;s)-s^{\alpha-1}I_0}{s^\alpha+\mu_\alpha}+\bar\lambda\,\mathcal{L}(\hat I;\ s),\\
\mathcal{L}({}^{MABC}D_0^\alpha C+\bar\lambda\,C;\ s)=\frac{B(\alpha)}{1-\alpha}\frac{s^\alpha\mathcal{L}(\hat C;s)-s^{\alpha-1}C_0}{s^\alpha+\mu_\alpha}+\bar\lambda\,\mathcal{L}(\hat C;\ s),\\
\mathcal{L}({}^{MABC}D_0^\alpha A+\bar\lambda\,A;\ s)=\frac{B(\alpha)}{1-\alpha}\frac{s^\alpha\mathcal{L}(\hat A;s)-s^{\alpha-1}A_0}{s^\alpha+\mu_\alpha}+\bar\lambda\,\mathcal{L}(\hat A;\ s).
\end{cases}\tag{16}$$

Direct computations will result in

$$\begin{cases} \mathcal{L}({}^{MABC}D_0^\alpha S + \bar{\lambda} S; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}\left(\left(\ell_\alpha s^\alpha + \bar{\lambda}\alpha\right)\mathcal{L}(\hat{S}; s) - B(\alpha)s^{\alpha-1}S_0\right), \\ \mathcal{L}({}^{MABC}D_0^\alpha I + \bar{\lambda} I; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}\left(\left(\ell_\alpha s^\alpha + \bar{\lambda}\alpha\right)\mathcal{L}(\hat{I}; s) - B(\alpha)s^{\alpha-1}I_0\right), \\ \mathcal{L}({}^{MABC}D_0^\alpha C + \bar{\lambda} A; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}\left(\left(\ell_\alpha s^\alpha + \bar{\lambda}\alpha\right)\mathcal{L}(\hat{C}; s) - B(\alpha)s^{\alpha-1}C_0\right), \\ \mathcal{L}({}^{MABC}D_0^\alpha A + \bar{\lambda} C; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}\left(\left(\ell_\alpha s^\alpha + \bar{\lambda}\alpha\right)\mathcal{L}(\hat{A}; s) - B(\alpha)s^{\alpha-1}A_0\right). \end{cases} \tag{17}$$

Eq. (15) is substituted in Eq. (17) to obtain the following result:

$$\begin{cases} \mathcal{L}({}^{MABC}D_0^\alpha S + \bar{\lambda} S; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}(B(\alpha)s^{\alpha-1}S_0 + (1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_1; s) - B(\alpha)s^{\alpha-1}S_0), \\ \mathcal{L}({}^{MABC}D_0^\alpha I + \bar{\lambda} I; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}(B(\alpha)s^{\alpha-1}I_0 + (1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_2; s) - B(\alpha)s^{\alpha-1}I_0), \\ \mathcal{L}({}^{MABC}D_0^\alpha C + \bar{\lambda} C; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}(B(\alpha)s^{\alpha-1}C_0 + (1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_3; s) - B(\alpha)s^{\alpha-1}C_0), \\ \mathcal{L}({}^{MABC}D_0^\alpha A + \bar{\lambda} A; s) = \frac{1}{(1-\alpha)(s^\alpha+\mu_\alpha)}(B(\alpha)s^{\alpha-1}A_0 + (1-\alpha)(s^\alpha+\mu_\alpha)\mathcal{L}(J_4; s) - B(\alpha)s^{\alpha-1}A_0). \end{cases}$$

$$\begin{cases} = \mathcal{L}(J_1; s), \\ = \mathcal{L}(J_2; s), \\ = \mathcal{L}(J_3; s), \\ = \mathcal{L}(J_4; s), \end{cases}$$

and the proof is complete.

Remark

If $J_i \in C[0, T]$ where $i = 1, 2, 3, 4$, then

$$\begin{cases} \hat{S} = \frac{1}{\ell_\alpha}(S_0 B(\alpha) + (1-\alpha)J_1(0)), \\ \hat{I} = \frac{1}{\ell_\alpha}(I_0 B(\alpha) + (1-\alpha)J_2(0)), \\ \hat{C} = \frac{1}{\ell_\alpha}(C_0 B(\alpha) + (1-\alpha)J_3(0)), \\ \hat{A} = \frac{1}{\ell_\alpha}(A_0 B(\alpha) + (1-\alpha)J_4(0)). \end{cases}$$

Adding more conditions

$$\begin{cases} \bar{\lambda} S_0 = J_1(0), \\ \bar{\lambda} I_0 = J_2(0), \\ \bar{\lambda} C_0 = J_3(0), \\ \bar{\lambda} A_0 = J_4(0), \end{cases}$$

then $\hat{S} = S_0, \hat{I} = I_0, \hat{C} = C_0$, and $\hat{A} = A_0$, the solution given in Eq. (14) is continuous. The above given condition is required in order to ensure that a solution exists.

4. Iterative Scheme and Stability Analysis

We get the following system by applying the Laplace transform to both sides of model (10):

$$\begin{aligned} &\mathcal{L}\{{}^{MABC}D_t^\alpha S\} = \mathcal{L}\{\Lambda - \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \mu S(t)\}, \\ &\mathcal{L}\{{}^{MABC}D_t^\alpha I\} = \mathcal{L}\{\beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \xi_3 I(t) + \gamma A(t) + \omega C(t)\}, \\ &\mathcal{L}\{{}^{MABC}D_t^\alpha C\} = \mathcal{L}\{\phi I(t) - \xi_2 C(t)\}, \\ &\mathcal{L}\{{}^{MABC}D_t^\alpha A\} = \mathcal{L}\{\rho I(t) - \xi_1 A(t)\}, \end{aligned} \tag{18}$$

or

$$\begin{aligned} &\tfrac{B(\alpha)}{1-\alpha} \times \tfrac{s^\alpha \mathcal{L}\{S(t)\} - S(0)s^{\alpha-1}}{s^\alpha + v_\alpha} = \mathcal{L}\{\Lambda - \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \mu S(t)\}, \\ &\tfrac{B(\alpha)}{1-\alpha} \times \tfrac{s^\alpha \mathcal{L}\{I(t)\} - I(0)s^{\alpha-1}}{s^\alpha + v_\alpha} = \mathcal{L}\{\beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \xi_3 I(t) + \gamma A(t) + \omega C(t)\}, \\ &\tfrac{B(\alpha)}{1-\alpha} \times \tfrac{s^\alpha \mathcal{L}\{C(t)\} - C(0)s^{\alpha-1}}{s^\alpha + v_\alpha} = \mathcal{L}\{\phi I(t) - \xi_2 C(t)\}, \\ &\tfrac{B(\alpha)}{1-\alpha} \times \tfrac{s^\alpha \mathcal{L}\{A(t)\} - A(0)s^{\alpha-1}}{s^\alpha + v_\alpha} = \mathcal{L}\{\rho I(t) - \xi_1 A(t)\}. \end{aligned} \tag{19}$$

With the initial conditions, we get

$$\begin{aligned} &\mathcal{L}\{S(t)\} = \tfrac{S_0}{s} + \left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda - \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \mu S(t)\}\right], \\ &\mathcal{L}\{I(t)\} = \tfrac{I_0}{s} + \left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \xi_3 I(t) + \gamma A(t) + \omega C(t)\}\right], \\ &\mathcal{L}\{C(t)\} = \tfrac{C_0}{s} + \left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I(t) - \xi_2 C(t)\}\right], \\ &\mathcal{L}\{A(t)\} = \tfrac{A_0}{s} + \left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I(t) - \xi_1 A(t)\}\right]. \end{aligned} \tag{20}$$

Consider that the solutions $S(t), I(t)$, $C(t)$, and $A(t)$in the form of infinite series are presented by

$$S(t)=\sum_{q=0}^{\infty}S_q, I(t)=\sum_{q=0}^{\infty}I_q, C(t)=\sum_{q=0}^{\infty}C_q, A(t)=\sum_{q=0}^{\infty}A_q, \quad (21)$$

we resolve nonlinear terms as follows:

$$S(t)I(t)=\sum_{q=0}^{\infty}G_q, S(t)C(t)=\sum_{q=0}^{\infty}H_q, S(t)A(t)=\sum_{q=0}^{\infty}L_q, \quad (22)$$

where G_q, H_q, and L_q are further decomposed as follows:

$$\begin{aligned} G_q &= \frac{1}{\Gamma(q+1)}\frac{d^q}{d\lambda^q}\left[\sum_{j=0}^{q}\lambda^j S_j(t)\sum_{j=0}^{q}\lambda^j I_j(t)\right]\Bigg|_{\lambda=0} \\ H_q &= \frac{1}{\Gamma(q+1)}\frac{d^q}{d\lambda^q}\left[\sum_{j=0}^{q}\lambda^j S_j(t)\sum_{j=0}^{q}\lambda^j C_j(t)\right]\Bigg|_{\lambda=0} \\ L_q &= \frac{1}{\Gamma(q+1)}\frac{d^q}{d\lambda^q}\left[\sum_{j=0}^{q}\lambda^j S_j(t)\sum_{j=0}^{q}\lambda^j A_j(t)\right]\Bigg|_{\lambda=0} \end{aligned} \quad (23)$$

Substituting (21) and (22) into (20), we obtain

$$\begin{aligned} \mathcal{L}\left\{\sum_{q=0}^{\infty}S_q\right\} &= \frac{S_0}{s}+\left[\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\left\{\Lambda-\beta(\sum_{q=0}^{\infty}G_q+\eta_C\sum_{q=0}^{\infty}H_q +\eta_A\sum_{q=0}^{\infty}L_q)-\mu\sum_{q=0}^{\infty}S_q\right\}\right], \\ \mathcal{L}\left\{\sum_{q=0}^{\infty}I_q\right\} &= \frac{I_0}{s}+\left[\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\left\{\beta(\sum_{q=0}^{\infty}G_q+\eta_C\sum_{q=0}^{\infty}H_q +\eta_A\sum_{q=0}^{\infty}L_q)-\xi_3\sum_{q=0}^{\infty}I_q+\gamma\sum_{q=0}^{\infty}A_q+\omega\sum_{q=0}^{\infty}C_q\right\}\right], \\ \mathcal{L}\left\{\sum_{q=0}^{\infty}C_q\right\} &= \frac{C_0}{s}+\left[\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\left\{\phi\sum_{q=0}^{\infty}I_q-\xi_2\sum_{q=0}^{\infty}C_q\right\}\right], \\ \mathcal{L}\left\{\sum_{q=0}^{\infty}A_q\right\} &= \frac{A_0}{s}+\left[\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\left\{\rho\sum_{q=0}^{\infty}I_q-\xi_1\sum_{q=0}^{\infty}A_q\right\}\right]. \end{aligned} \quad (24)$$

The following iterative procedure is produced by matching the two sides of (24):

$$\begin{aligned} &\mathcal{L}\{S_0\}=\frac{N_1}{s}, \\ &L\{S_1\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(G_0+\eta_C H_0+\eta_A L_0)-\mu S_0\}, \\ &\mathcal{L}\{S_2\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(G_1+\eta_C H_1+\eta_A L_1)-\mu S_1\}, \\ &. \\ &. \\ &. \\ &\mathcal{L}\{S_{q+1}\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(G_q+\eta_C H_q+\eta_A L_q)-\mu S_q\},\ q\geqslant 1, \end{aligned} \quad (25)$$

$$\begin{aligned} &\mathcal{L}\{I_0\}=\frac{N_2}{s}, \\ &\mathcal{L}\{I_1\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\beta(G_0+\eta_C H_0+\eta_A L_0)-\xi_3 I_0+\gamma A_0+\omega C_0\}, \\ &\mathcal{L}\{I_2\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\beta(G_1+\eta_C H_1+\eta_A L_1)-\xi_3 I_1+\gamma A_1+\omega C_1\}, \\ &. \\ &. \\ &. \\ &\mathcal{L}\{I_{q+1}\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\beta(G_q+\eta_C H_q+\eta_A L_q)-\xi_3 I_q+\gamma A_q+\omega C_q\},\ q\geqslant 1, \end{aligned} \quad (26)$$

$$\begin{aligned} &\mathcal{L}\{C_0\}=\frac{N_3}{s}, \\ &\mathcal{L}\{C_1\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I_0-\xi_2 C_0\}, \\ &\mathcal{L}\{C_2\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I_1-\xi_2 C_1\}, \\ &. \\ &. \\ &. \\ &\mathcal{L}\{C_{q+1}\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I_q-\xi_2 C_q\},\ q\geqslant 1 \end{aligned} \quad (27)$$

$$\begin{aligned} &\mathcal{L}\{A_0\}=\frac{N_4}{s}, \\ &\mathcal{L}\{A_1\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I_0-\xi_1 A_0\}, \\ &\mathcal{L}\{A_2\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I_1-\xi_1 A_1\}, \\ &. \\ &. \\ &. \\ &\mathcal{L}\{A_{q+1}\}=\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I_q-\xi_1 A_q\},\ q\geqslant 1. \end{aligned} \quad (28)$$

When we consider the first three terms and take the Laplace inverse of (25–28) we get

$$\begin{aligned} S_0 &= N_1, \\ S_1 &= \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}(\Lambda-\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\mu N_1), \\ S_2 &= \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{\Lambda}{B(\alpha)}-\left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \\ &\quad (\Lambda-\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\mu N_1) \\ &\quad \times(\beta(N_2+N_3+N_4)+\mu)-\beta N_1\left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \\ &\quad ((N_2+\eta_C N_3+\eta_A N_4)+\xi_3 N_2+\gamma N_4 \\ &\quad +\omega N_3+\phi N_2-\xi_2 N_3+\rho N_2-\xi_1 N_4), \\ I_0 &= N_2, \\ I_1 &= \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}(\beta N_1(N_2+\eta_C N_3+\eta_A N_4) \\ &\quad -\xi_3 N_2+\gamma N_4+\omega N_3), \\ I_2 &= \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2\beta(\Lambda-\beta N_1(N_2+\eta_C N_3+\eta_A N_4) \\ &\quad -\mu N_1)(N_2+N_3+N_4) \\ &\quad +\left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2(\beta N_1(N_2+\eta_C N_3+\eta_A N_4) \\ &\quad -\xi_3 N_2+\gamma N_4+\omega N_3)(\beta N_1-\xi_3) \\ &\quad +\left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2(\phi N_2-\xi_2 N_3)(\beta N_1+\omega) \\ &\quad +\left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2\times(\rho N_2-\xi_1 N_4)(\beta N_1+\gamma), \\ C_0 &= N_3, \\ C_1 &= \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}(\phi N_2-\xi_2 N_3), \\ C_2 &= \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2(\phi[\beta N_1(N_2+\eta_C N_3+\eta_A N_4) \\ &\quad -\xi_3 N_2+\gamma N_4+\omega N_3]-\xi_2(\phi N_2-\xi_2 N_3)), \\ A_0 &= N_4, \\ A_1 &= \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}(\rho N_2-\xi_1 N_4), \\ A_2 &= \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2(\rho[\beta N_1(N_2+\eta_C N_3+\eta_A N_4) \\ &\quad -\xi_3 N_2+\gamma N_4+\omega N_3]-\xi_1(\rho N_2-\xi_1 N_4)), \end{aligned} \quad (29)$$

and so on.

The reaming terms can be calculated in this manner. Finally, the required solutions can be expressed as follows:

$$S(t) = N_1 + \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)} \times(\Lambda-\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\mu N_1) + \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{\Lambda}{B(\alpha)} - \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \times(\Lambda-\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\mu N_1) \times(\beta(N_2+N_3+N_4)+\mu) - \beta N_1\left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \times((N_2+\eta_C N_3+\eta_A N_4)+\xi_3 N_2+\gamma N_4 +\omega N_3+\phi N_2-\xi_2 N_3+\rho N_2-\xi_1 N_4), \tag{30}$$

$$I(t) = N_2 + \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)} \times(\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\xi_3 N_2+\gamma N_4+\omega N_3) + \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2\beta(\Lambda-\beta N_1(N_2+\eta_C N_3+\eta_A N_4) -\mu N_1)(N_2+N_3+N_4) + \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \times(\beta N_1(N_2+\eta_C N_3+\eta_A N_4) -\xi_3 N_2+\gamma N_4+\omega N_3)(\beta N_1-\xi_3) + \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2(\phi N_2-\xi_2 N_3)(\beta N_1+\omega) + \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2\times(\rho N_2-\xi_1 N_4)(\beta N_1+\gamma), \tag{31}$$

$$C(t) = N_3 + \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}(\phi N_2-\xi_2 N_3) + \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \times(\phi[\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\xi_3 N_2+\gamma N_4+\omega N_3] -\xi_2(\phi N_2-\xi_2 N_3)), \tag{32}$$

$$A(t) = N_4 + \left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}(\rho N_2-\xi_1 N_4) + \left[\left(1-\alpha+\frac{t^\alpha}{\Gamma(\alpha)}\right)\frac{1}{B(\alpha)}\right]^2 \times(\rho[\beta N_1(N_2+\eta_C N_3+\eta_A N_4)-\xi_3 N_2+\gamma N_4+\omega N_3] -\xi_1(\rho N_2-\xi_1 N_4)). \tag{33}$$

Theorem 4.1.

Let (B, |.|) be Banach space and $K: B \to B$ be a map satisfying:

$$\|K_x - K_y\| \leqslant \Pi\|X-K_x\| + \pi\|x-y\|,$$

for all $x, y \in$ B, where $0 \leqslant \Pi, 0 \leqslant \pi < 1$. Then, $K$is Picard K-stable.

Theorem 4.2.

Let K is a self map defined as below:

$$\begin{aligned}
K[S_q(t)] &= S_{q+1}(t) = S_n(0) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(I_q+\eta_C C_q+\eta_A A_q)S_q-\mu S_q\}\right],\\
K[I_q(t)] &= I_{q+1}(t) = I_n(0) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\beta(I_q+\eta_C C_q+\eta_A A_q)S_q -\xi_3 I_q+\gamma A_q+\omega C_q\}\right],\\
K[C_q(t)] &= C_{q+1}(t) = C_n(0) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I_q-\xi_2 C_q\}\right],\\
K[A_q(t)] &= A_{q+1}(t) = A_n(0) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I_q-\xi_1 A_q\}\right],
\end{aligned} \tag{34}$$

Then, the iteration is K-stable in $L^1(x,y)$ if the following statements are achieved:

$$\begin{aligned}
&(1-\beta(N_1+N_2)w_1(\vartheta)-\beta\eta_C(N_1+N_3)w_2(\vartheta) -\beta\eta_A(N_1+N_4)w_3(\vartheta)-\mu w_4(\vartheta)) < 1,\\
&(1+\beta(N_1+N_2)w_1(\vartheta)+\beta\eta_C(N_1+N_3)w_2(\vartheta) +\beta\eta_A(N_1+N_4)w_3(\vartheta)-\xi_3 w_5(\vartheta)+\gamma w_6(\vartheta)+\omega w_7(\vartheta)) < 1,\\
&(1+\phi w_8(\vartheta)-\xi_2 w_9(\vartheta)) < 1,\\
&(1+\rho w_{10}(\vartheta)-\xi_1 w_{11}(\vartheta)) < 1.
\end{aligned} \tag{35}$$

Proof.

To show that T has a fixed point, we evaluated the following for $(q,p) \in N \times N$:

$$\begin{aligned}
K[S_q(t)] - K[S_p(t)] &= S_q(t) - S_p(t) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(I_q+\eta_C C_q+\eta_A A_q)S_q-\mu S_q\}\right] - \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(I_p+\eta_C C_p+\eta_A A_p)S_p-\mu S_p\}\right],\\
K[I_q(t)] - K[I_p(t)] &= I_q(t) - I_p(t) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}L\{\beta(I_q+\eta_C C_q+\eta_A A_q)S_q-\xi_3 I_q+\gamma A_q+\omega C_q\}\right] - \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\beta(I_p+\eta_C C_p+\eta_A A_p)S_p-\xi_3 I_p+\gamma A_p+\omega C_p\}\right],\\
K[C_q(t)] - K[C_p(t)] &= C_q(t) - C_p(t) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I_q-\xi_2 C_q\}\right] - \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\phi I_p-\xi_2 C_p\}\right],\\
K[A_q(t)] - K[A_p(t)] &= A_q(t) - A_p(t) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I_q-\xi_1 A_q\}\right] - \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\rho I_p-\xi_1 A_p\}\right].
\end{aligned} \tag{36}$$

By calculating the norm of both sides of the first Equation of (36) we obtain

$$\|K(S_q(t)) - K(S_p(t))\| = \left\|S_q(t) - S_p(t) + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(I_q+\eta_C C_q+\eta_A A_q)S_q-\mu S_q\}\right] - \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\{\Lambda-\beta(I_p+\eta_C C_p+\eta_A A_p)S_p-\mu S_p\}\right]\right\|, \tag{37}$$

using triangular inequality and simplifying (37), we get

$$\|K(S_q(t)) - K(S_p(t))\| \leqslant \|S_q(t) - S_p(t)\| + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}[\|-\beta S_p(I_q-I_p)\|+\|-\beta I_q(S_q-S_p)\| +\|-\beta S_p(C_q-C_p)\|+\|-\beta C_p(S_q-S_p)\|+\|-\beta S_p(A_q-A_p)\| +\|-\beta A_p(S_q-S_p)\|+\|-\mu(S_q-S_p)\|]\right]. \tag{38}$$

Given the relative influence of both solutions, we let

$$\|S_q(t) - S_p(t)\| \cong \|I_q(t) - I_p(t)\| \cong \|C_q(t) - C_p(t)\| \cong \|A_q(t) - A_p(t)\|. \quad (39)$$

If we replace this in (38), we obtain the relation shown below:

$$\|K(S_q(t)) - K(S_p(t))\| \leqslant \|S_q(t) - S_p(t)\| + \mathcal{L}^{-1}\left[\tfrac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}[\|-\beta S_p(S_q - S_p)\| + \|-\beta I_q(S_q - S_p)\| + \|-\beta S_p(S_q - S_p)\| + \|-\beta C_p(S_q - S_p)\| + \|-\beta S_p(S_q - S_p)\| + \|-\beta A_p(S_q - S_p)\| + \|-\mu(S_q - S_p)\|]\right]. \quad (40)$$

Also the convergent sequence S_q, I_q, C_q and A_q are bounded. Next, we can obtain four different positive constants, N_1, N_2, N_3 and N_4 for all t such that

$$\|S_p\| < N_1, \|I_q\| < N_2, \|C_q\| < N_3, \|A_p\| < N_4, (q,p) \in N \times N. \quad (41)$$

Further, considering Eqs. (40) and (41), we get

$$\|K(S_q(t)) - K(S_p(t))\| \leqslant (1 - \beta(N_1 + N_2)w_1(\vartheta) - \beta\eta_C(N_1 + N_3)w_2(\vartheta) - \beta\eta_A(N_1 + N_4)w_3(\vartheta) - \mu w_4(\vartheta))\|(S_q - S_p)\|, \quad (42)$$

where w_1, w_2, w_3 and w_4 are functions of $\mathcal{L}^{-1}\left\{\frac{(1-\alpha)(s^\alpha+v_\alpha)}{B(\alpha)s^\alpha}\mathcal{L}\right\}$. Similarly, we are able to obtain

$$\begin{aligned}
&\|K(I_q(t)) - K(I_p(t))\| \leqslant (1 + \beta(N_1 + N_2)w_1(\vartheta) + \beta\eta_C(N_1 + N_3)w_2(\vartheta) + \beta\eta_A(N_1 + N_4)w_3(\vartheta) - \xi_3 w_5(\vartheta) + \gamma w_6(\vartheta) + \omega w_7(\vartheta))\|(I_q - I_p)\|,\\
&\|K(C_q(t)) - K(C_p(t))\| \leqslant (1 + \phi w_8(\vartheta) - \xi_2 w_9(\vartheta))\|(C_q - C_p)\|,\\
&\|K(A_q(t)) - K(A_p(t))\| \leqslant (1 + \phi w_8(\vartheta) - \xi_2 w_9(\vartheta))\|(A_q - A_p)\|,
\end{aligned} \quad (43)$$

where

$$\begin{aligned}
&(1 - \beta(N_1 + N_2)w_1(\vartheta) - \beta\eta_C(N_1 + N_3)w_2(\vartheta) - \beta\eta_A(N_1 + N_4)w_3(\vartheta) - \mu w_4(\vartheta)) < 1,\\
&(1 + \beta(N_1 + N_2)w_1(\vartheta) + \beta\eta_C(N_1 + N_3)w_2(\vartheta) + \beta\eta_A(N_1 + N_4)w_3(\vartheta) - \xi_3 w_5(\vartheta) + \gamma w_6(\vartheta) + \omega w_7(\vartheta)) < 1,\\
&(1 + \phi w_8(\vartheta) - \xi_2 w_9(\vartheta)) < 1,\\
&(1 + \rho w_{10}(\vartheta) - \xi_1 w_{11}(\vartheta)) < 1.
\end{aligned}$$

Therefore, K has a fixed point. Considering Eqs. (42) and (43), we assume:

$$\pi = (0,0,0,0),$$

$$\Pi = \begin{cases} (1 - \beta(N_1 + N_2)w_1(\vartheta) - \beta\eta_C(N_1 + N_3)w_2(\vartheta) - \beta\eta_A(N_1 + N_4)w_3(\vartheta) - \mu w_4(\vartheta)),\\ \left(\begin{array}{l}1 + \beta(N_1 + N_2)w_1(\vartheta) + \beta\eta_C(N_1 + N_3)w_2(\vartheta) + \beta\eta_A(N_1 + N_4)w_3(\vartheta) - \xi_3 w_5(\vartheta)\\ +\gamma w_6(\vartheta) + \omega w_7(\vartheta)\end{array}\right),\\ (1 + \phi w_8(\vartheta) - \xi_2 w_9(\vartheta)), (1 + \rho w_{10}(\vartheta) - \xi_1 w_{11}(\vartheta))1. \end{cases}$$

As a result theorem (4.1) conditions are satisfied. This completes the proof.

Theorem.4.3.

The iteration method produces a unique special solution to Eq. (10).

Proof.

Consider the subsequent Hilbert space $F : L^2(m,n) \times (0,T)$ it can be described as

$$f : (m,n) \times (0,T) \to R, \int\int gfdgdf < \infty.$$

Considering the following operator, we have

$$\pi(0,0,0,0), \Pi = \begin{cases} \Lambda - \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \mu S(t),\\ \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \xi_3 I(t) + \gamma A(t) + \omega C(t),\\ \phi I(t) - \xi_2 C(t),\\ \rho I(t) - \xi_1 A(t). \end{cases}$$

By using

$$P((S_{11} - S_{12}, I_{21} - I_{22}, C_{31} - C_{32}, A_{41} - A_{42}), (E_1, E_2, E_3, E_4)).$$

We have

$$\begin{aligned}
&\{\Lambda - \beta((I_{21} - I_{22}) + \eta_C(C_{31} - C_{32}) + \eta_A(A_{41} - A_{42}))(S_{11} - S_{12}) - \mu(S_{11} - S_{12})\}\\
&\leqslant \Lambda\|E_1\| + \beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\|\|E_1\| + \beta\|C_{31} - C_{32}\|\|S_{11} - S_{12}\|\|E_1\|\\
&+\beta\|A_{41} - A_{42}\|\|S_{11} - S_{12}\|\|E_1\| + \mu\|S_{11} - S_{12}\|\|E_2\|,\\
&\times\left\{\begin{array}{l}\beta((I_{21} - I_{22}) + \eta_C(C_{31} - C_{32}) + \eta_A(A_{41} - A_{42}))\\ (S_{11} - S_{12}) - \xi_3(I_{21} - I_{22})\\ +\gamma(A_{41} - A_{42}) + \omega(C_{31} - C_{32})\end{array}\right\}\\
&\leqslant \beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\|\|E_2\| + \beta\|C_{31} - C_{32}\|\|S_{11} - S_{12}\|\|E_2\| + \beta\|A_{41} - A_{42}\|\|S_{11} - S_{12}\|\|E_2\|\\
&+\xi_3\|I_{21} - I_{22}\|\|E_2\| + \gamma\|A_{41} - A_{42}\|\|E_2\| + \omega\|C_{31} - C_{32}\|\|E_2\|, \{\phi(I_{21} - I_{22}) - \xi_2(C_{31} - C_{32})\}\\
&\leqslant \phi\|I_{21} - I_{22}\|\|E_3\| + \xi_2\|C_{31} - C_{32}\|\|E_3\|,\\
&\{\rho(I_{21} - I_{22}) - \xi_1(A_{41} - A_{42})\} \leqslant \rho\|I_{21} - I_{22}\|\|E_4\| + \xi_1\|A_{41} - A_{42}\|\|E_4\|.
\end{aligned}$$

For convergence solution, we have

$$\|S - S_{11}\|, \|S - S_{12}\| \leqslant \frac{\zeta_{e_1}}{\delta},$$

$$\|I - I_{21}\|, \|I - I_{22}\| \leqslant \frac{\zeta_{e_2}}{\tau},$$

$$\|C - C_{31}\|, \|C - C_{32}\| \leqslant \frac{\zeta_{e_3}}{\upsilon},$$

and

$$\|A - A_{41}\|, \|A - A_{42}\| \leqslant \frac{\zeta_{e_4}}{\lambda},$$

where

$$\delta = 4(\Lambda + \beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\| + \beta\|C_{31} - C_{32}\|\|S_{11} - S_{12}\| + \beta\|A_{41} - A_{42}\|\|S_{11} - S_{12}\| + \mu\|S_{11} - S_{12}\|)\|E_1\|,$$
$$\tau = 4(\beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\| + \beta\|C_{31} - C_{32}\|\|S_{11} - S_{12}\| + \beta\|A_{41} - A_{42}\|\|S_{11} - S_{12}\| + \xi_3\|I_{21} - I_{22}\| + \gamma\|A_{41} - A_{42}\| + \omega\|C_{31} - C_{32}\|)\|E_2\|,$$
$$\upsilon = 4(\phi\|I_{21} - I_{22}\| + \xi_2\|C_{31} - C_{32}\|)\|E_3\|,$$
$$\lambda = 4(\rho\|I_{21} - I_{22}\| + \xi_1\|A_{41} - A_{42}\|)\|E_4\|.$$

But it is obvious that

$$(\Lambda + \beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\| + \beta\|C_{31} - C_{32}\|\|S_{11} - S_{12}\| + \beta\|A_{41} - A_{42}\|\|S_{11} - S_{12}\| + \mu\|S_{11} - S_{12}\|) \neq 0,$$
$$(\beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\| + \beta\|C_{31} - C_{32}\|\|S_{11} - S_{12}\| + \beta\|A_{41} - A_{42}\|\|S_{11} - S_{12}\| + \xi_3\|I_{21} - I_{22}\| + \gamma\|A_{41} - A_{42}\| + \omega\|C_{31} - C_{32}\|) \neq 0,$$
$$(\phi\|I_{21} - I_{22}\| + \xi_2\|C_{31} - C_{32}\|) \neq 0,$$
$$(\rho\|I_{21} - I_{22}\| + \xi_1\|A_{41} - A_{42}\|) \neq 0,$$

where $\|E_1\|, \|E_2\|, \|E_3\|, \|E_4\| \neq 0$. Therefore, we have

$$\|S_{11} - S_{12}\| = 0, \|I_{21} - I_{22}\| = 0, \|C_{31} - C_{32}\| = 0, \|A_{41} - A_{42}\|.$$

Which yields that

$$S_{11} = S_{12}, I_{21} = I_{22}, C_{31} = C_{32}, A_{41} = A_{42}$$

We get the required result. Hence,its proved.

5. Numerical simulation and discussion

Here, we examine numerical simulations of the HIV/AIDS model under MABC. The fractional operator is applied using the advanced approach for HIV/AIDS. Integral order derivative just analyzes HIV/AIDS in one place, while fractional order analyze HIV/AIDS from initial point where an infected individual carries HIV/AIDS and starts spreading till end. It actually helps to analyze complete behavior of HIV/AIDS from start till end. For simulation, the initial conditions and parameter values are listed below [9,11]:

$$\Lambda = 2.1, \beta = 0.01, \mu = \tfrac{1}{69.54}, \eta_A = 1.3, \eta_C = 0.015,$$
$$\phi = 1, \gamma = 0.33, \omega = 0.09,$$
$$\rho = 0.1, d = 1, S(0) = 338923, I(0) = 61, C(0) = 0, A(0) = 0.$$

We obtain an approximation of the solution of the fractional HIV/AIDS model (10) in series form by using ILTM sequentially up to four terms. Using the MABC fractional derivative, the model's numerical results for different fractional values are generated under the steady-state point. The influence of variables on the dynamics of the fractional-order model can be observed by looking at the end-time value of the specified parameter in several numerical ways.

For fractional values of $\alpha = 0.75, 0.85, 0.95$, and 1, tables 1–4 show approximations of all classes of the model (10). To estimate approximations of the solutions for infectious illness mathematical models, it has been found that the Modified ABC fractional operator is quite accurate and efficient.

Table 1 Table of $S(t)$ at different values of α.

t	$\alpha = 0.75$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 1$
0	3.5980	3.5793	3.4794	3.3892
0.2	3.3248	3.4835	3.5840	3.5995
0.4	2.8683	3.0696	3.2825	3.3838
0.6	2.2879	2.4201	2.6185	2.7422
0.8	1.6065	1.5654	1.6096	1.6746
1	0.8377	0.5238	0.2668	0.1811

Table 2 Table of $I(t)$ at different values of α.

t	$\alpha = 0.75$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 1$
0	0.4415	0.1612	0.0176	0.0000
0.2	1.7138	1.0094	0.4745	0.2805
0.4	3.0595	2.2206	1.4569	1.1214
0.6	4.5582	3.7602	2.9305	2.5225
0.8	6.1972	5.5971	4.8761	4.4838
1	7.9645	7.7101	7.2807	7.0055

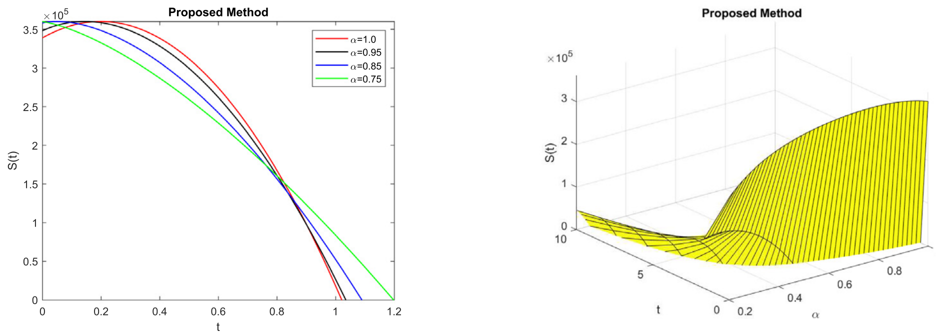
Table 3 Table of $C(t)$ at different values of α.

t	$\alpha = 0.75$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 1$
0	0.1293	0.0466	0.0052	0
0.2	0.3878	0.2455	0.1289	0.0828
0.4	0.6434	0.5183	0.3926	0.3309
0.6	0.9205	0.8600	0.7869	0.7444
0.8	1.2185	1.2644	1.3070	1.3232
1	1.5362	1.7272	1.9493	2.0673

Table 4 Table of $A(t)$ at different values of α.

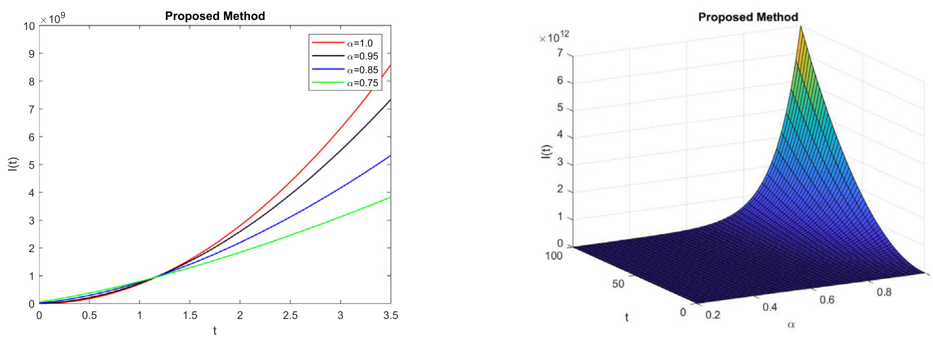
t	$\alpha = 0.75$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 1$
0	0.0961	0.0466	0.0049	0
0.2	0.3752	0.2969	0.1321	0.0828
0.4	0.6704	0.6545	0.4055	0.3310
0.6	0.9991	1.1089	0.8156	0.7446
0.8	1.3586	1.6512	1.3571	1.3235
1	1.7463	2.2749	2.0263	2.0679

Figs. (1) to (4) illustrate how all four compartments of the SICA model behave over time for various values of α. From Figs. 1–4, it is obvious that fractional order significantly affects the dynamic behavior of each component. We notice that when the derivative order α decreases from 1, the system's memory effect grows. As a result, the infection spreads slowly and the population's proportion of HIV- and AIDS-positive individuals rise over an extended time. The number of susceptible individuals progressively reduces and converges to zero, as seen in Fig. 1(a). The graph in Fig. 2(a) for infected individuals without a clinical AIDS symptom demonstrates that as the value α of decreases, the rate of rising likewise decreases. According to Figs. 3(a) and 4(a), the number of chronic individuals $C(t)$ and infected individuals with clinical AIDS symptoms $A(t)$ also rises for different values of α. Surface plots of



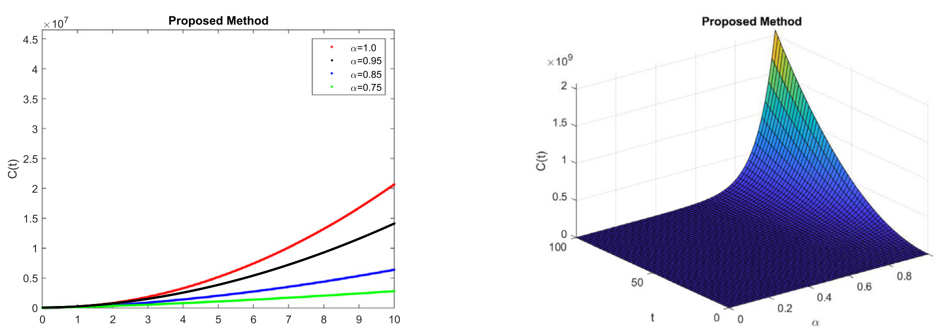
(a) (b)

Fig. 1 Two-dimensional and three-dimensional simulation of $S(t)$.



(a) (b)

Fig. 2 Two-dimensional and three-dimensional simulation of $I(t)$.



(a) (b)

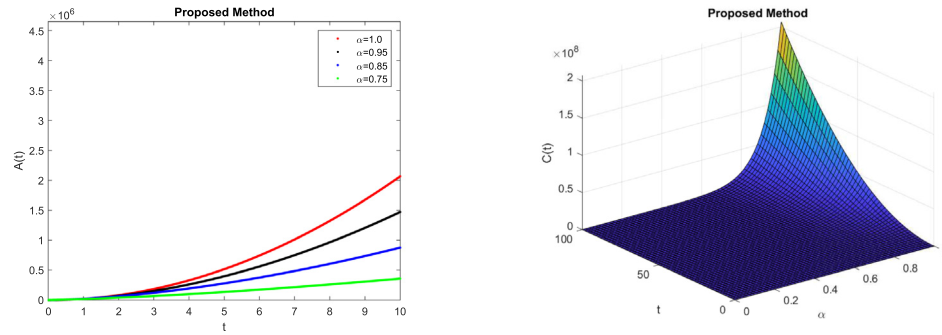
Fig. 3 Two-dimensional and three-dimensional simulation of $C(t)$.

all four compartments of the SICA model with respect to time t and α are shown in Figs. $1(b), 2(b), 3(b)$, and $4(b)$.

The graphical results show how effective LADM was in producing the desired result. It is also noticeable that by increasing the terms, the method's effectiveness can be raised significantly. To examine the impact of the fractional order model, observations have been made at various fractional values under the specified parameters. By reducing fractional values, solutions for all compartments reach the requisite accuracy and are more dependable. The simulations obviously demonstrate that we can obtain a better approximation to control the disease by employing fractional derivatives as compared to classical derivatives.

6. Convergence analysis

Solution (30–33) is a series that converges uniformly to the exact solution. We employ strategies to determine whether series (30–33) are converging (see [38]). Using [38], we provide the following theorem for this method's convergence under sufficient conditions.



(a) (b)

Fig. 4 Two-dimensional and three-dimensional simulation of $A(t)$.

Theorem.6.1.

Let Υbe a Banach Space and $\Psi : \Upsilon \rightarrow \Upsilon$be a contractive nonlinear operator then there exit

$Z, Z\prime \in \Upsilon, \|\Upsilon(Z) - \Upsilon(Z\prime)\| \leqslant d\|Z - Z\prime\|, 0 < d < 1.$

By using the criteria of Banach contraction, Ψ has a unique point Z such that $\Psi Z = Z$, where $Z = (S, I, C, A)$. Adomian Decomposition Method can be used to write the series shown in (30–33) as follows:

$$Z_k = TZ_{k-1}, Z_{k-1} = \sum_{j=0}^{k-1} Z_j, k = 1, 2, 3, \cdots$$

assume that $Z_0 \in \beta_r(Z)$, where $\beta_r(Z) = Z\prime \in \Upsilon : \|Z\prime - Z\| < r$; then we have

$(1) Z_k \in \beta_r(Z)$

$(2) \lim_{k\rightarrow\infty} Z_k = Z$

Proof.

(1) We will prove it by mathematical induction for $k = 1$, we get

$\|Z_0 - Z\| = \|\Psi(Z_0) - \Psi(Z)\| \leqslant d\|Z_0 - Z\|.$

assume that the result will be true for $k - 1$,then

$\|Z_0 - Z\| \leqslant d^{k-1}\|Z_0 - Z\|.$

We obtain

$$\begin{aligned}\|Z_k - Z\| &= \|\Psi(Z_{k-1}) - \Psi(Z)\| \leqslant d\|Z_{k-1} - Z\| \\ &\leqslant d^k\|Z_0 - Z\|,\end{aligned}$$

$$\|Z_k - Z\| \leqslant d^k\|Z_0 - Z\| \leqslant d^k r \leqslant r,$$

$\Rightarrow \; Z_k \in \beta_r(Z).$

(2) Since $\|Z_k - Z\| \leqslant d^k\|Z_0 - Z\|$ and

$\lim_{k\rightarrow\infty} d^k = 0,$

therefore, we have the

$\lim_{k\rightarrow\infty} \|Z_k - Z\| = 0,$

$\Rightarrow \lim_{k\rightarrow\infty} Z_k = Z.$

7. Conclusion

In this paper, we propose a MABC fractional order model of the HIV/AIDS epidemic. The MABC-fractional derivative is an extension of the ABC derivative in a wider space. The kernel of the MABC derivative has an integrable singularity at the origin. A new set of solutions to the associated fractional differential equations are obtained when ABC is modified, and the fundamental function of space is made explicit. Numerous fractional differential equations that the ABC derivative cannot solve can be solved by the MABC derivative. For instance, numerous homogeneous FDE with the MABC derivative allows a nonzero solution, and many linear fractional equations enable solutions without imposing additional conditions. The integral operator related to the ABC derivative and the MABC derivative is the same.

To provide public health professionals with some useful control techniques to help eradicate this communicable disease from the population, we developed a fractional mathematical model. The current work has shown that a modified Atangana-Baleanu fractional derivative operator can be used to represent infectious diseases efficiently. The Banach fixed point theory has also been used to verify the steady solution stability conditions and existence. For the MABC fractional HIV/AIDS model, approximate solutions and a graphical presentation utilizing the iterative Laplace transform technique have been shown. It should be emphasized that the memory aspects in the MABC derivative explore the concealed dynamics of infection in mathematical models of viral diseases, which are not realizable with integer-order derivatives. In the end, mathematical software is used to support all the theoretical findings through graphical and tabular representation. When accurate calculations of transmission structures are given in real-time, this model becomes very reliable. From the perspective of modeling, the new suggested modification will provide some insight into issues with and without singularity at the origin. By utilizing the MABC derivative, we will be able to better characterize the dynamics of complicated processes. The scope of applicability for these operators will be significantly increased in this way.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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