NUMERICAL INVESTIGATIONS OF THE NONLINEAR WAVES GENERATION IN A BUBBLE LAYER

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The purpose of the paper was to conduct numerical investigations of the nonlinear wave generation in a layer with uniformly distributed mono-size spherical bubbles. The mathematical model and examples of results of theoretical studies of this problem are presented. The mathematical model of the pressure propagation in the bubbly liquid layer is constructed using the linear non-dissipative wave and the Rayleigh-Plesset equations. The Commander and Prosperetti model is employed to compute the phase sound speed in the bubble layer. The spectra of transmitted and reflected waves are studied and the amplitudes of selected frequency harmonics of these waves examined.

INTRODUCTION

The nonlinear wave generation inside the bubble liquid layers with different physical properties is a very important problem. For example, it plays significant role in practice of parametric sonars production. The mathematical model of this problem consist of a set of two differential equations. The first of them, the linear non-dissipative wave equation, describes acoustic pressure changes in the bubble layer [1, 3]. The second of them, the Rayleigh-Plesset equation, allows us to predict radius changes of a bubble. It is worth mentioning that a correct choice of physical parameters is very important in the process of theoretical analysis. One of these parameters is the phase sound speed. In our work we use the model proposed by Commander and Prosperetti [2] to compute values of the phase sound speed.

In this paper we present mathematical model and examples of numerical investigations of the nonlinear wave propagation and generation efficiency in a bubble layer. Many different environmental and sounding signal parameters have influence on the nonlinear wave propagation in the bubble layer. Only a selection of them is examined in this paper. The changes of the transmitted and reflected waves are studied. Their magnitude spectra and the selected frequency harmonics for different values of the layer thickness, the volume fraction and bubble radius are analyzed.

1. MATHEMATICAL MODEL

We consider a liquid layer with single size uniformly distributed spherical bubbles located between $x=0$ and $x=L$. The media outside the layer are considered to be linear liquids. At the layer boundary $x=0$ and on the left of this boundary the acoustic field is the sum of the incident p_i and reflected p_r waves correspondingly. On the right of boundary $x=L$ only the transmitted wave p_t propagates. The density and sound speed inside the bubble layer are equal ρ_L and c_L correspondingly. These parameters outside this layer are equal ρ_0 and c_0 respectively. Because of small differences between the density of water at equilibrium state and in medium with bubbles we can put $\rho_L = \rho_0$.

The mathematical model of the acoustic pressure p propagated inside the layer is built on the basis of linear non-dissipative wave equation [1, 3]:

$$\frac{\partial^2 p}{\partial x^2}(x,t) - \frac{1}{c_L^2}\frac{\partial^2 p}{\partial t^2}(x,t) = -\rho_0 \frac{\partial^2 \beta}{\partial t^2}(x,t), \tag{1}$$

where β is the local fraction of volume occupied by the gas. Assuming a constant number N of air bubbles per unit volume, the volume fraction is given by

$$\beta(x,t) = \frac{4}{3}\pi R^3(x,t)N, \tag{2}$$

where R is the instantaneous radius of the bubbles which is modeled by means of the Rayleigh - Plesset equation:

$$R\frac{\partial^2 R}{\partial t^2} + \frac{3}{2}\left(\frac{\partial R}{\partial t}\right)^2 = \frac{1}{\rho_0}\left[p_g\left(\frac{R_0}{R}\right)^{3\gamma} + p_v - p_{stat} - \frac{2\sigma}{R} - p(x,t) - \rho_0\delta_t\omega R\frac{\partial R}{\partial t}\right], \tag{3}$$

where p_v is the gas and vapor pressure inside a bubble, p_{stat} is the ambient static pressure, $p_g = 2\sigma/R_0 + p_{stat} - p_v$, R_0 is the equilibrium bubble radius, ω is the angular frequency, γ is the polytropic exponent of gas, σ is the coefficient of surface tension, δ_t is the damping coefficient for the bubble.

To complete the formulation of our problem, the initial and boundary conditions are defined. The initial conditions are as follows:

$$\begin{aligned} &p(x,0) = 0, \frac{\partial p}{\partial t}(x,0) = 0, \quad \text{for } x \neq 0 \\ &R(x,0) = R_0, \frac{\partial R}{\partial t}(x,0) = 0. \end{aligned} \tag{4}$$

Boundary conditions are defined only for the pressure. Taking into account the continuity of the pressure and velocity at the layer boundaries $x=0$ and $x=L$ we obtain two boundary conditions:

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$$\frac{\partial p}{\partial t}(0,t) - c_0 \frac{\partial p}{\partial x}(0,t) = 2\frac{\partial p_i}{\partial t}(0,t),$$
$$\frac{\partial p}{\partial t}(L,t) + c_0 \frac{\partial p}{\partial x}(L,t) = 0. \tag{5}$$

Assuming that two different frequency harmonic waves are propagated, the incident wave at $x = 0$ is given by :

$$p_i(0,t) = P_A \sin(\omega_1 t) + P_A \sin(\omega_2 t). \tag{6}$$

A correct choice of physical parameters is very important in the process of numerical examination as they influence the correctness and accuracy of the results. The phase speed of acoustic waves c_L is calculated on the basis of the dispersion relation including the effective complex wave number κ in the gas-liquid mixture:

$$c_L = \frac{c_o}{\mathrm{Re}(\kappa / k)}. \tag{7}$$

where $k = \omega / c_0$ is the acoustic wave number. Considering a bubble population with the same equilibrium radius R_0 the square of the complex wave number is given by formula [2]:

$$\kappa^2 = k^2 + \frac{4\pi R_0 N}{\omega_0^2 / \omega^2 - 1 + i\delta_t}, \tag{8}$$

where ω_0 is the resonance angular frequency of a bubble. The resonance angular frequency ω_0 of a bubble with radius R_0 and the total damping coefficient δ_t which is the sum of three components: the viscous damping constant, the damping constant due to thermal effects and the acoustic radiation damping constant can be determined using formulas:

$$\omega_0^2 = \frac{p_0}{\rho_0 R_0^2}\left(\mathrm{Re}\,\Phi - \frac{2\sigma}{p_0 R_0}\right), \tag{9}$$

$$\delta_t = \frac{4\mu}{\rho_0 \omega R_0^2} + \frac{p_0}{\rho_0 R_0^2 \omega^2}\mathrm{Im}\,\Phi + \frac{\omega R_0}{c_0}, \tag{10}$$

with

$$\Phi = \frac{3\gamma}{1 - 3(\gamma - 1)\, i\, z[(i/z)^{1/2}\coth(i/z)^{1/2} - 1]}, \tag{11}$$

where $z = D/(\omega R_0^2)$ and D is the gas thermal diffusivity, μ is the coefficient of molecular viscosity of seawater. The quantity $p_0 = P_0 + 2\sigma / R_0$ is the undisturbed pressure in the bubble, where P_0 denotes the equilibrium pressure in the liquid.

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3. RESULTS OF NUMERICAL INVESTIGATIONS

Numerical calculations were carried out assuming that two harmonic waves with frequencies f_1=30 kHz, f_2=33 kHz and the same amplitudes P_A=100 kPa propagate in the bubble layer. We put the sound speeds c_0 =1450 m/s and density ρ_0=1000 kg/m^3. Figure 1 presents the magnitude spectrum of the transmitted and reflected waves normalized by pressure P_A calculated for volume fraction $\beta_0 = 10^{-6}$ and the thickness of the layer L=0.1 m.

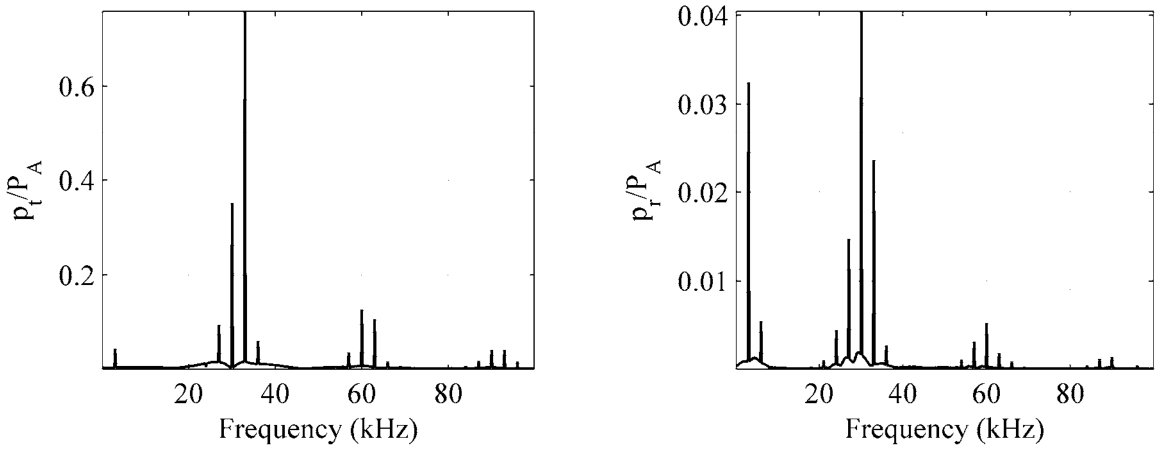

Fig.1. The magnitude spectrum of the transmitted (on the left) and reflected (on the right) waves: $\beta_0 = 10^{-6}$, $R_0 = 100\ \mu$m, L=0.1 m.

The changes of the f_i and $2f_i$ (i=1, 2) frequency waves were studied in this paper. Additionally the difference frequency waves were examined. Numerical calculations were carried out for different values of the volume fraction and the bubble layer thickness. The amplitude of different frequency harmonics of the transmitted wave normalized by pressure P_A as a function of volume fraction obtained for L=0.1 m is shown in Figure 2 (on the left). Similar results obtained for the reflected wave are given on the right of this figure.

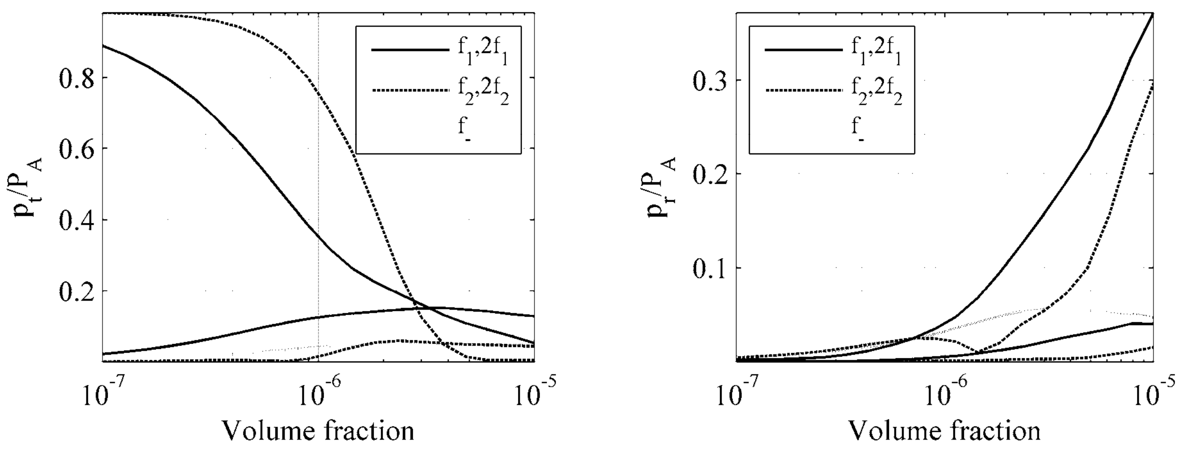

Fig.2. The amplitude of different frequency harmonics of the transmitted (on the left) and reflected (on the right) wave normalized by pressure P_A as a function of volume fraction: $R_0 = 100\ \mu$m, L=0.1 m.

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Figure 3 displays the amplitude of different frequency harmonics of transmitted and reflected waves normalized by pressure P_A as a function of layer thickness calculated for the volume fraction $\beta_0 = 10^{-6}$.

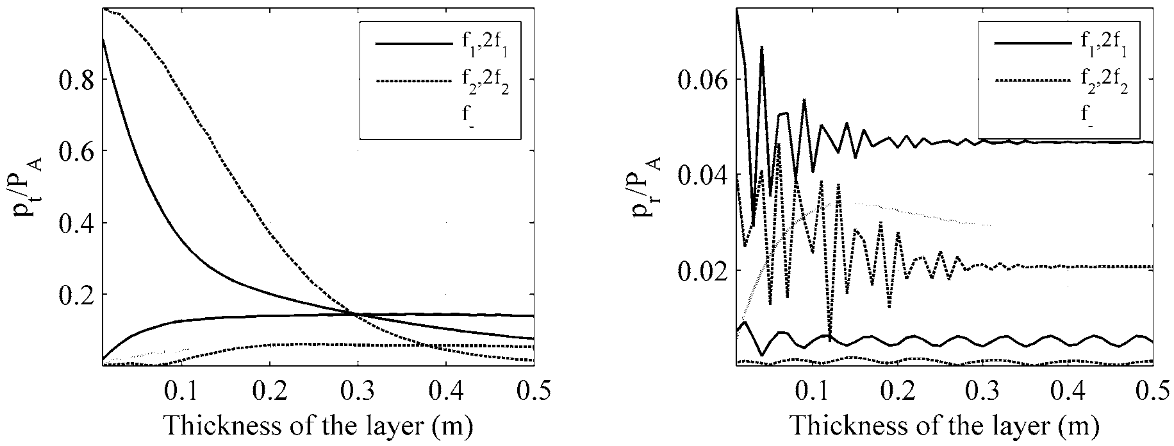

Fig.3. The amplitude of different frequency harmonics of the transmitted (on the left) and reflected (on the right) wave normalized by pressure P_A as a function of layer thickness: $\beta_0 = 10^{-6}$, $R_0 = 100\ \mu\text{m}$.

The results presented so far were obtained assuming that the bubble radius $R_0 = 100\ \mu\text{m}$. Figure 4 demonstrates amplitudes of different frequency harmonics of the transmitted and reflected wave normalized by pressure P_A as a function of the bubble layer thickness which were achieved for the bubble radius $R_0 = 40\ \mu\text{m}$ when $\beta_0 = 10^{-6}$.

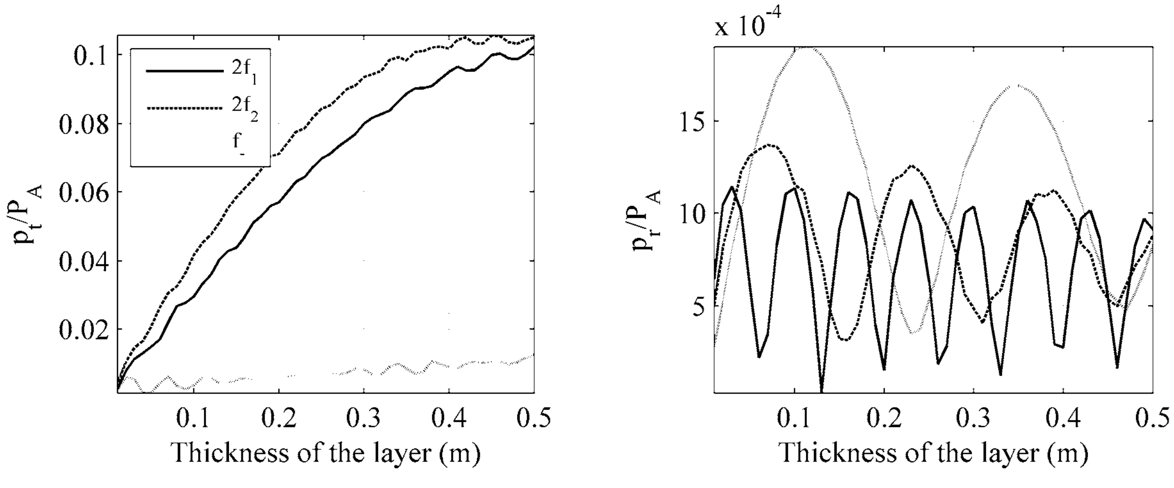

Fig.4. The amplitude of different frequency harmonics of the transmitted (on the left) and reflected (on the right) wave normalized by pressure P_A as a function of layer thickness: $\beta_0 = 10^{-6}$, $R_0 = 40\ \mu\text{m}$.

Last step of numerical investigations was analysis of wave generation efficiency. The amplitude of $2f_i$ (i=1, 2) frequency harmonic of the transmitted wave normalized by the amplitude of f_i frequency harmonic as a function of layer thickness for two different values

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of the volume fraction are shown in Figure 5. Figure 5a represents results obtained for $R_0 = 100\ \mu\text{m}$. Similar results obtained for $R_0 = 40\ \mu\text{m}$ are presented in Figure 5b.

a) b)

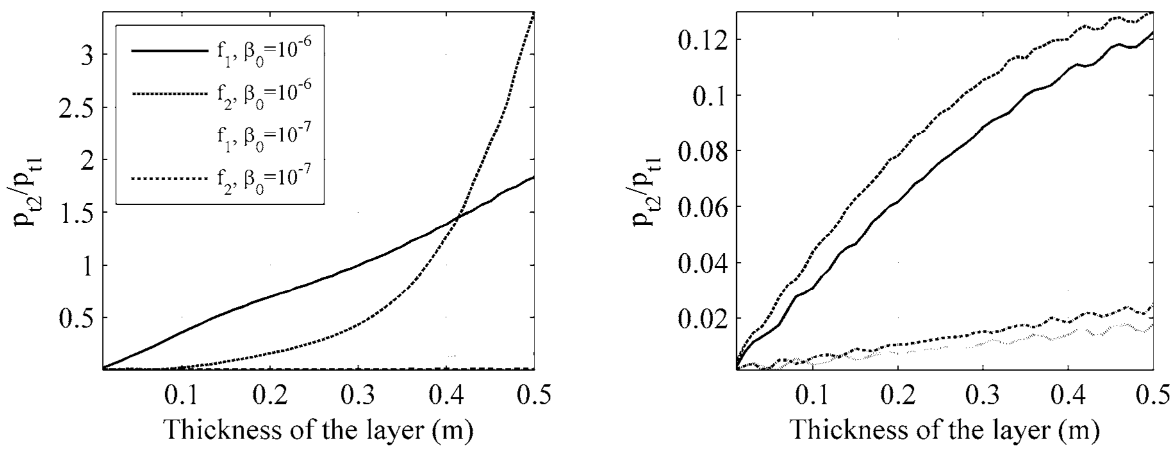

Fig.5. The amplitude of $2f_i$ (i=1, 2) frequency harmonic of the transmitted wave normalized by the amplitude of f_i frequency harmonic as a function of layer thickness for different values of the volume fraction: a) $R_0 = 100\ \mu\text{m}$, b) $R_0 = 40\ \mu\text{m}$.

4. CONCLUSIONS

The problem of nonlinear acoustic waves generation in one-dimensional bubbly liquid layer was considered and its mathematical model which is built on the basis of the non-dissipative wave equation and the Rayleigh-Plesset equation was presented. The finite-difference method was employed to solve the first of these equations while the second of them was solved using the classical forth order Runge-Kutta method. The results of numerical investigations were discussed.

The changes of the amplitudes of selected frequency harmonics of transmitted and reflected waves were analyzed. Numerical calculations were carried out for different values of the physical parameters. First of all the bubble thickness and the volume fraction were considered. Moreover calculations were carried out for the resonant bubbles and layers where the bubble's resonance frequency is far from the sounding signals. A detailed analysis shows that the volume fraction has significant influence on nonlinear wave propagation and, in consequence, the waves generation efficiency. It is worth mentioning that theoretical investigations became more difficult when value of this parameter increase. Additionally, the theoretical analysis is more complicated for the resonant bubbles then in the case of bubbles with different radii.

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