Reliability Analysis of Sea Cliff Slope Stability by Point Estimate Method

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Reliability Analysis of Sea Cliff Slope Stability by Point Estimate Method

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Abstract. The paper presents a reliability analysis of a sea cliff slope. The cliff, located in Jastrzębia Góra, is characterised by a complicated geological structure. Although currently stable, it is in fact close to the limit state. The objective of this paper is to present the point estimate method (PEM) of determining the mean value and standard deviation of the safety factor of the slope. Assuming its normal distribution, these statistical parameters make it possible to determine the probability of failure p_f and the reliability index β. In order to reduce the number of random variables, a sensitivity analysis was performed. The results were verified by the Monte Carlo simulation method. The main advantage of this approach is to minimize the number of sample calculations required to obtain estimators of the parameters investigated.

1. Introduction

Slope stability analysis abounds in various sources of uncertainties. They are associated mainly with inherent randomness of the soil, its parameters and model uncertainties. Thus the results of deterministic studies can be recognized to have limited value. The awareness of the possibility of slope failure is essential, but it is not sufficient in safety analysis. An engineer needs a quantitative measure of uncertainties. This measure may be obtained by probabilistic methods of slope analysis, which are often appropriate and desirable. A sea cliff in Jastrzębia Góra is an excellent example of a slope characterized by significant randomness resulting not only from its complicated geological structure, but also from its insufficient recognition. Moreover, even though the cliff is currently stable, the value of the safety factor F is only slightly larger than unity, which means that the probability of slope failure is high.

Interest in the reliability approach to slope stability analysis began over 50 years ago. The number of papers on this subject has increased considerably since the year 1975, when a second International Congress ICASP was held in Aachen. The dominant methods at that time were mainly the methods of correction factors, the First Order Second Moment method (FOSM) and the Monte Carlo method (MCM). The majority of probabilistic or reliability methods make use of traditional slope stability analysis techniques, that is, LEM. To avoid the main drawback of the FOSM, the First Order Reliability Method (FORM) has been widely used. A summary of research on probabilistic analysis of slope stability is given in the monograph [1]. The approach to probabilistic slope analysis changed significantly with the use of the finite element method (FEM) in geotechnical problems. Different probabilistic methods related to the FEM have been proposed, such as the perturbation method, spectral stochastic finite element method (SSFEM) and others. Along with the development of computers and software, the Monte Carlo Simulation (MCS) has become dominant. This method is a conceptually simple tool for analysing the reliability of slopes ([2], [11]). It is usually called the random finite-element method (RFEM) and it fully accounts for spatial correlation and averaging ([4], [5], [6]).

Unfortunately, in the case of complicated slopes, the RFEM requires extensive computational efforts. Over the past several decades, the point estimate method (PEM) proposed by Rosenblueth [7] has found an increasing use in many engineering fields. This method is capable of efficient estimation of the mean value, standard deviation and probabilistic moments of higher order of any random function. The method is straightforward and does not require perfection in the probability theory. The PEM uses a series of point estimates, that is, point-by-point evaluations of the response function at selected values of the input of random variables, to compute the moments of the response variable. The method applies appropriate weights to each point estimate of the response variables to compute moments under investigation. It can be readily applied to analyse response functions which cannot be formed explicitly, by closed-form expressions; another application of PEM is possible for the results of existing deterministic programs. In practice, the Rosenblueth point estimate method can be applied when the number of random variables is up to 5 or 6. However, Rosenblueth in [7] also proposed a modified technique for reducing the number of calculation points in the case of uncorrelated variables and in cases of neglecting the skewness. Many authors have applied this method for a probabilistic analysis of slope design (e.g. [8], [9], [10, [11], etc.).

In this paper, an attempt was made to apply the reliability approach to the slope stability analysis of a sea cliff in Jastrzębia Góra. A modified point estimate method was used, and the results were verified by the Monte Carlo Simulation technique. Deterministic slope stability calculations were performed by the FLAC software based on the two-dimensional explicit finite difference (FD) method.

2. Jastrzębia Góra – description of the cliff

The cliff at Jastrzębia Góra is a section of the southern Baltic coast of Poland. Its height reaches about 30 m, and it steeply descends to the beach. The morphology is a typical young glacial form created by inland glacier activity at the time of the last North-Polish continental glaciation of the Pomeranian phase. Since 2002, several landslides have developed here, causing the collapse of a pedestrian path and a building situated at the top of the cliff (Figure 1).

Figure 1. General view of the landslide on the cliff

The geological structure of the cliff was recognized in many investigations (e.g. [12], [13]). In the geological structure, two formations can be distinguished. The first one, up to the altitude of 15 m above sea level, is composed of relatively strong impermeable soils – clays and loams (Figure 2). Above this formation, a complex of sand-loam soils occurs [14]. Groundwater, in the form of a continuous table, is located relatively high in the cliff massif and appears on its face in the form of seepage springs.

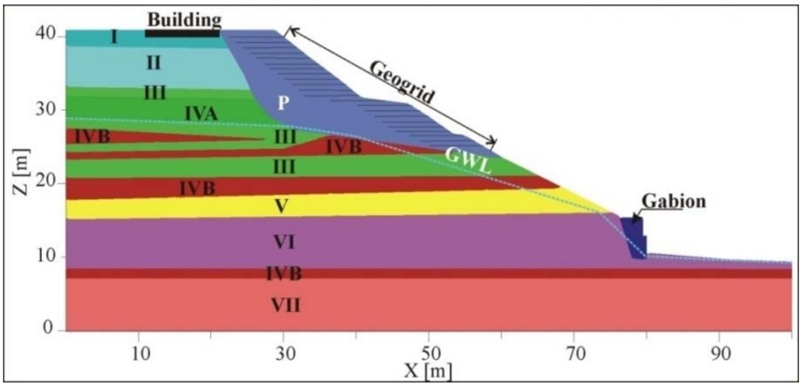

Figure 2. Analysed cross-section of the cliff

On the basis of geological recognition and the results of a wide range of laboratory tests performed on undisturbed samples, physical and strength parameters of particular soils composing the cliff layers were determined [13]. The characteristic values of these parameters, as well as the assumed standard deviations of strength parameters (σ_c and σ_ϕ), are given in Table 1.

Table 1. Geotechnical parameters of the layers in the calculation models

Layer number	Type of soil	$\rho^{(n)}$ [t/m^3]	$c^{(n)}$ [kPa]	σ_c [kPa]	$\phi^{(n)}$ [°]	σ_ϕ [°]
I	Silty loam	2.05	40.0	12.0	15.0	1.5
II	Sandy loam	1.80	15.0	4.5	29.0	2.9
III	Clay	2.05	45.0	13.5	10.0	1.0
IVA	Fine sand	1.80	0.0	0.0	33.0	3.3
IVB	Fine sand	1.80	0.0	0.0	35.0	3.5
V	Silty loam	2.05	58.0	17.4	19.0	1.9
VI	Loamy sand	2.18	21.8	6.54	27.0	2.7
VII	Clay	2.15	50.0	15.0	17.0	1.7
-	Geogrid	1.9	100	30	36	3.6

The landslides and cliff erosion are the results of two processes. They are caused by the so-called "land factors," connected with the geological structure of the massif, its significant inclination and water presence, as well as the height difference between the cliff crown and the sea level [12], [13]. The landslide processes are also an effect of difficult hydrogeological conditions, namely, the temporarily rising ground water table. The layers are slightly inclined seawards, and this configuration causes a slow creeping movement, especially of the cohesive soils in the lower part of the slope. A detailed investigation of the last incident proved that the landslide had been initiated by a slide of the sand-loam complex over the lower-lying impermeable clays. Water played an important role in this process.

After the last landslide (Figure 1), a programme of remedial works was undertaken. The colluvium was replaced with gravel reinforced by a geogrid (Figure 2). This support created a kind of "shelf" between the building and the cliff crown. Despite this support, signs of a new landslide have recently appeared on the cliff surface, and a new stability analysis has therefore been performed.

3. Deterministic stability analysis of the slope

The stability calculations of the safety factor (F) were performed by the two-dimensional explicit Finite Difference (FD) method included in the FLAC software. In contrast to traditional “limit state” analysis, this program provides a full solution of the coupled stress/displacement, equilibrium and constitutive equations. It was assumed that the mass model obeys the Mohr-Coulomb failure criterion. Calculations were based on the so-called "strength reduction technique" [15]. A series of simulations were performed using trial values of the F_{trial} factor to reduce the cohesion c and the friction angle ϕ. The calculations of the safety factor revealed that it was equal to 1.06055 and the slip zone (expressed by SSI - maximum shear strain increments, shown in Figure 3) encompassed the whole slope. It is seen that the building situated at the crest of the cliff is endangered by the slip, as it is inside the critical zone.

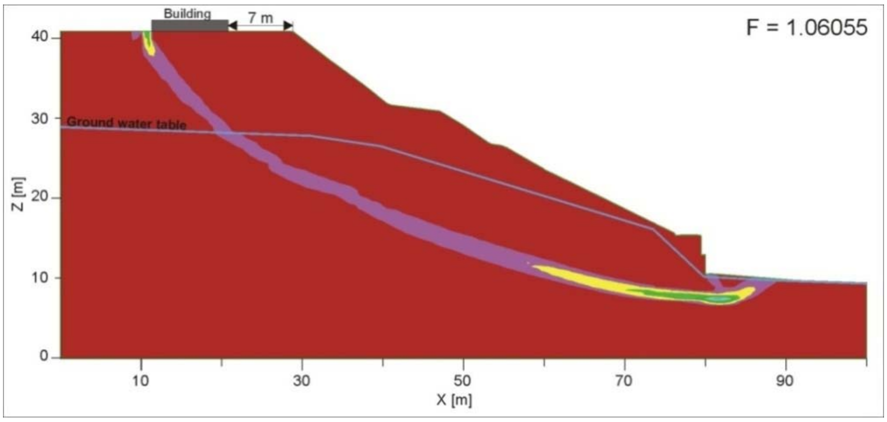

Figure 3. Location of the potential slip zone

4. Reliability analysis

According to Figure 2, the slope massif is divided into seven layers. It was assumed that only strength parameters, that is, the internal friction angle ϕ and cohesion c, were considered as random variables having a normal distribution. It was also assumed that the mean values of these parameters were equal to their deterministic values given in Table 1 and the coefficients of variations were equal to v_c=0.3, v_ϕ=0.1 [9]. Multiplying the coefficient of variation by the mean value, one receives the standard deviation of the random variable (Table 1). Layers IVa and b are cohesionless, but it is assumed that the geogrid is also a random variable, so eventually there are fourteen random variables in the case considered.

4.1. Sensitivity analysis

First, some trial computations were made to show the sensitivity of the cliff slope to changes in individual random parameters. Such an analysis was performed to reduce the number of variables that should be considered in the task, while maintaining the quality of the slope approximation calculations. For each individual random variable, three computational cases were considered: one in which the average value of the parameter was used (x_i=m_i) and two cases in which the parameter changed by the standard deviation value (x_i=m_i±σ_i). The remaining random parameters were set at the level of their individual average values. This approach is consistent with the PEM methodology, and the results of these samples were adopted in further analysis for calculating the slope reliability by this method. It was shown that only 7 out of 14 variables have a significant impact on the slope response. They are cohesion VII (3.13%), internal friction angle IVb (3.13%), cohesion III (2.76%), cohesion VI (2.58%), internal friction angle VI (2.58%), cohesion V (2.03%) and internal friction angle VII (1.11%). Other variables have an effect on the response at a level of less than 1%.

4.2. Point Estimate Methods

The point estimate method (PEM) can estimate efficiently the mean value, standard deviation and probabilistic moments of higher order of any random function. This method essentially involves the use of a Gaussian quadrature to determine probabilistic moments of a random function. The procedure is based on numerical integration. The method is straightforward and does not require knowledge of the probability theory. The PEM uses a series of point estimates, that is, point-by-point evaluations of the response function at selected values of the input random variables, to compute the moments of the response variable. The method prescribes appropriate weights to each point estimate of the response variables to compute moments under investigation. It can be readily applied to analyze response functions which cannot be formed explicitly, by closed-form expressions. Another application of PEM is possible for the results of existing deterministic programs. The standard PEM is inefficient when a large number of random variables are considered. In this case, n^2 samples should be used. Rosenblueth [7] also proposed a modified approach – a technique for reducing the number of calculation points to $2n+1$ in the case of uncorrelated variables and in cases of neglecting the skewness. It is assumed that the following deterministic function is known:

$$Y = f\left(x_1,\, x_2,\, ...,x_n\right) \tag{1}$$

The values of this function can be obtained by any deterministic finite element (in the considered case – discrete element) program. It should be noted that the distributions of input random variables are not necessary; only their first and second moments should be given. First, calculations are performed for an ideal model of all random variables described by their mean values:

$$y_0 = f\left(m_{x_1},\, m_{x_2},\, ...,m_{x_n}\right) \tag{2}$$

Next, two values shifted from the mean values by $\pm\sigma_{x_i}$ are calculated for each random variable

$$y_i^{-/+} = f\left(m_{x_1},\, m_{x_2},\, ...,m_{x_i} \mp \sigma_{x_i},\, ...,m_{x_n}\right) \tag{3}$$

On this basis the following parameters are defined:

$$\bar{y}_j = \frac{y_i^+ + y_i^-}{2} \tag{4}$$

$$v_{y_j} = \frac{y_i^+ - y_i^-}{y_i^+ + y_i^-} \tag{5}$$

Finally, the mean value and the coefficient of variation are determined:

$$\bar{Y} = y_0 \prod_{i=1}^{n} \left(\frac{\bar{y}_i}{y_0} \right) \tag{6}$$

$$v_Y = \sqrt{\left\{ \prod_{i=1}^{n} \left(1 + v_{y_i}^2 \right) \right\} - 1} \tag{7}$$

The mean value and standard deviation of the safety factor for the considered cliff slope obtained by PEM are presented in Table 2. These statistical parameters made it possible to determine the probability of failure p_f and the reliability index β, assuming that the safety factor has a normal distribution.

4.3. Monte Carlo Simulation

The Monte Carlo simulation (MCS) is a powerful statistical analysis tool. It is the most popular method to assess the reliability of structures with uncertain geometric and/or material parameters. Moreover, it has long been considered as the most accurate of all methods requiring knowledge of the probability distributions of the response function described by parameters having random uncertainties. Probabilistic analysis is done by generating random variables in a computer model (material strength, load, etc.) for previously assumed probability distributions. This generation is repeated many times, and the number of times a specific condition occurs is counted. Some conclusions on the model outputs are drawn on the basis of statistical analysis. The key problem of this method is to determine the number of simulations needed to obtain satisfactory probability accuracy [16]. It depends on the phenomenon studied and the complexity of the problem. In spite of the classical Monte Carlo simulation method, there are techniques that significantly reduce the number of simulations (e.g. stratified sampling, Latin hypercube sampling, Russian roulette).

The MCS was applied to verify the results obtained. In this case, a total of 100 sets of fourteen random variables were generated. The results of this method are also presented in Table 2. The outcomes of convergence, for both the average value and the standard deviation of the safety factor, are presented in Figure 4.

Table 2. Results of computational reliability analysis

Parameter	PEM	MCS
Average Value of F	1.01967	1.00071
Standard deviation of F	0.06956	0.08358
Probability of failure p_f	0.406	0.436
Reliability index β	0.238	0.162

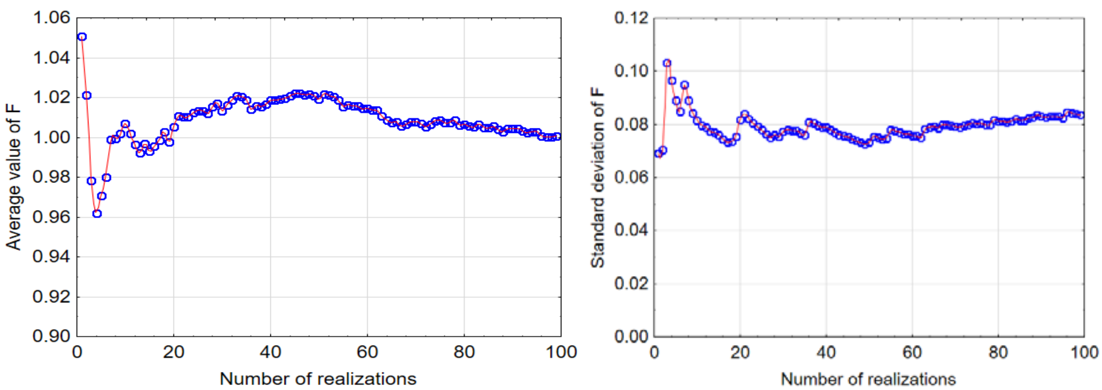

Figure 4. Convergence analysis of the average value and standard deviation of the safety factor

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Comparison of particular values from Table 2 for the PEM and MCS methods reveals some discrepancies. However, they are not significant, considering that the calculations by each method were carried out for a different number of random variables and that some difference resulted from differences between the two methods themselves.

5. Conclusions

The PEM has proved to be an efficient tool for probabilistic geotechnical engineering applications. In comparison with the MCS, a considerably smaller number of deterministic realizations is required here. In this specific case, slope stability calculations require tedious work, so the number of samples determines the efficiency of the method applied. The PEM seems to be an appropriate tool for such an analysis. It should be noted that an important advantage of the point estimate method is the possibility to neglect probability density functions of particular random variables in reliability analysis. Thus, the method is suitable for analysing a variety of engineering problems.

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