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Spectral criterion of infinite fatigue life of beams under combined random loads

SUMMARY

This paper deals with fatigue assessment of metal beams under complex loading. Combined axial, bending and torsion loads are taken into account under assumption that the material complies with the Kelvin-Voigt's model and has a fatigue limit, and the stress components are stationary stochastic processes which are stationary correlated and differentiable in the mean-square sense.

By taking the power spectral densities of the stress components for known the criterion of infinite fatigue life of beams is formulated in the frequency domain. For this purpose the distortion-energy strength hypothesis and the theory of energy transformation systems are used.

INTRODUCTION

Most structures are complex systems generally made of (or modelled as composed of) members connected together in a particular way. The sub-division of a structure into the parts is a necessary stage for undertaking any stress analysis. It usually requires considerable simplification and idealization of the structure. In this paper it is assumed that the so obtained model includes beam elements subjected to random dynamic loads.

In general, complex loads can generate combined bending, torsion and tension-compression of a beam. Hence fatigue assessment of beams under such loading is considered. The aim of this paper is to elaborate a spectral fatigue criterion for beams under assumption that the loads represent stationary stochastic processes of zero mean values and known power spectral densities, as the multiaxial fatigue models exist almost exclusively in the time domain and the use of the frequency domain methods can lead to substantial computer time savings [1]. For this purpose the theory of energy transformation systems [2] and distortion-energy strength hypothesis [3] are employed.

APPLICATION OF THE DISTORTION--ENERGY STRENGTH HYPOTHESIS

In the general state of static stress in beams the strain energy of distortion per unit volume, ϕ, is given by :

$$\phi = \frac{1+\nu}{3E}\left[(\sigma_a + \sigma_b)^2 + 3\sigma_t^2\right] \tag{1}$$

where :

E- Young modulus
ν - Poisson's ratio
σ_a - normal stress due to axial force
σ_b - normal stress due to bending load
σ_t - shear stress due to torsion load.

According to the distortion-energy hypothesis the equivalent normal stress σ_e satisfies the equation :

$$\phi_e = \phi$$

where : ϕ_e - the strain energy of distortion per unit volume in the equivalent stress state,

that is :

$$\phi_e = \frac{1+\nu}{3E}\sigma_e^2 \tag{2}$$

Hence :

$$\sigma_e^2 = (\sigma_a + \sigma_b)^2 + 3\sigma_t^2 \tag{3}$$

Adaptation of the distortion-energy hypothesis to the stochastic stress is based on the assumption that (3) can be also utilised for the time-variable stress components [1,8].

$$\sigma_a = \sigma_a(t) \quad \sigma_b = \sigma_b(t) \quad \sigma_t = \sigma_t(t)$$

This hypothesis is valid for ductile materials. The energy dissipated by such materials under dynamic loads below the yield point can be calculated by means of the Kelvin-Voigt's model [3÷5].

For the sake of simplicity, the uniaxial Kelvin-Voigt's model :

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \tag{4}$$

(instead of the biaxial one [5]) is here applied to the stress components σ_a, σ_b and to the equivalent stress; and the analogous model :

$$\tau = G\gamma + \lambda\dot{\gamma} \tag{5}$$

is used for the stress component σ_t,

where :
- σ, ε - normal stress and strain, respectively
- τ, γ - shear stress and strain, respectively
- G - shear modulus
- η, λ - coefficients of internal viscous damping of the material.

Equations (1) through (5) yield :

$$\left(E\varepsilon_e + \eta\dot{\varepsilon}_e\right)^2 = \left(E\varepsilon_a + \eta\dot{\varepsilon}_a + E\varepsilon_b + \eta\dot{\varepsilon}_b\right)^2 + 3\left(G\varepsilon_t + \lambda\dot{\varepsilon}_t\right)^2 \tag{6}$$

where :
- ε_e - the equivalent normal strain
- ε_i - the strain associated with σ_i (i = a, b, t).

In uniaxial problems the following approximate expressions are commonly accepted :

$$\varepsilon(t) = \frac{1}{E}\sigma(t) \qquad \gamma(t) = \frac{1}{G}\tau(t) \tag{7}$$

so that (6) becomes :

$$\left(\sigma_e + \frac{\eta}{E}\dot{\sigma}_e\right)^2 = \left(\sigma_a + \frac{\eta}{E}\dot{\sigma}_a + \sigma_b + \frac{\eta}{E}\dot{\sigma}_b\right)^2 + 3\left(\sigma_t + \frac{\lambda}{G}\dot{\sigma}_t\right)^2 \tag{8}$$

The time domain relationship (8) is not convenient for evaluation of parameters of the equivalent stress from spectral data. Therefore (8) should be transformed into the frequency domain.

FORMULATION OF THE PROBLEM

It is assumed that the stress components are stationary (in the wide sense) stochastic processes which are stationary correlated and differentiable in the mean-square sense [6,7]. Accordingly, 2nd order time derivatives of the correlation functions :

$$K_{\sigma_i}(\tau) = \left\langle \sigma_i^*(t_1)\sigma_i(t_2)\right\rangle \qquad \tau = t_2 - t_1$$
$$K_{\sigma_i\sigma_k}(\tau) = \left\langle \sigma_i^*(t_1)\sigma_k(t_2)\right\rangle \qquad i,k = a,b,t \qquad i \neq k \tag{9}$$

exist, where :
- $\langle\cdot\rangle$ - denotes the expected value and
- $*$ - stands for the complex conjugate,

the following relationships hold :

$$K_{\dot{\sigma}_i\sigma_i}(\tau) = \left\langle \dot{\sigma}_i^*(t_1)\sigma_i(t_2)\right\rangle = -\frac{d}{d\tau}K_{\sigma_i}(\tau)$$
$$K_{\sigma_i\dot{\sigma}_i}(\tau) = \left\langle \sigma_i^*(t_1)\dot{\sigma}_i(t_2)\right\rangle = \frac{d}{d\tau}K_{\sigma_i}(\tau) \tag{10}$$
$$K_{\dot{\sigma}_i}(\tau) = \left\langle \dot{\sigma}_i^*(t_1)\dot{\sigma}_i(t_2)\right\rangle = -\frac{d^2}{d\tau^2}K_{\sigma_i}(\tau)$$

and :

$$K_{\dot{\sigma}_i\sigma_k}(\tau) = \left\langle \dot{\sigma}_i^*(t_1)\sigma_k(t_2)\right\rangle = -\frac{d}{d\tau}K_{\sigma_i\sigma_k}(\tau)$$
$$K_{\sigma_i\dot{\sigma}_k}(\tau) = \frac{d}{d\tau}K_{\sigma_i\sigma_k}(\tau) \qquad K_{\dot{\sigma}_i\dot{\sigma}_k}(\tau) = -\frac{d^2}{d\tau^2}K_{\sigma_i\sigma_k}(\tau) \tag{11}$$

For the equivalent stress σ_e the same assumptions are valid. However in this case, to facilitate the formulation of the fatigue criterion, σ_e is defined as a periodic (in the mean-square sense) process [8] :

$$\sigma_e(t) = a\sin\left(\omega_e t + \varphi\right) = a_1\exp\left(j\omega_e t\right) + a_{-1}\exp\left(-j\omega_e t\right) \tag{12}$$

where :
- a - random amplitude phase angle
- φ - random phase angle
- ω_e - constant circular frequency

and [7] :

$$\left\langle a_1\right\rangle = \left\langle a_{-1}\right\rangle = \left\langle a_1^* a_{-1}\right\rangle = \left\langle a_{-1}^* a_1\right\rangle = 0$$
$$a_1 = \frac{a}{2j}\exp\left(j\varphi\right) \qquad a_{-1} = a_1^* \tag{13}$$

In order to obtain the frequency domain formulation of (8) it is rewritten in terms of correlation functions of the processes involved therein as follows :

$$\left\langle\left[\sigma_e^*(t_1) + \frac{\eta}{E}\dot{\sigma}_e^*(t_1)\right]\left[\sigma_e(t_2) + \frac{\eta}{E}\dot{\sigma}_e(t_2)\right]\right\rangle =$$
$$= \left\langle\left[\sigma_a^*(t_1) + \frac{\eta}{E}\dot{\sigma}_a^*(t_1) + \sigma_b^*(t_1) + \frac{\eta}{E}\dot{\sigma}_b^*(t_1)\right]\cdot\right.$$
$$\left.\cdot\left[\sigma_a(t_2) + \frac{\eta}{E}\dot{\sigma}_a(t_2) + \sigma_b(t_2) + \frac{\eta}{E}\dot{\sigma}_b(t_2)\right]\right\rangle +$$
$$+3\left\langle\left[\sigma_t^*(t_1) + \frac{\lambda}{G}\dot{\sigma}_t^*(t_1)\right]\left[\sigma_t(t_2) + \frac{\lambda}{G}\dot{\sigma}_t(t_2)\right]\right\rangle \tag{14}$$

By using (9) through (13), the following is obtained from (14) :

$$\frac{1}{4}\left(1 + \frac{\eta^2}{E^2}\omega_e^2\right)\left\langle a^2\right\rangle\left[\exp(j\omega_e\tau) + \exp(-j\omega_e\tau)\right] =$$
$$= K_{\sigma_a}(\tau) - \frac{\eta^2}{E^2}\frac{d^2}{d\tau^2}K_{\sigma_a}(\tau) + K_{\sigma_b}(\tau) - \frac{\eta^2}{E^2}\frac{d^2}{d\tau^2}K_{\sigma_b}(\tau) +$$
$$+ K_{\sigma_a\sigma_b}(\tau) - \frac{\eta^2}{E^2}\frac{d^2}{d\tau^2}K_{\sigma_a\sigma_b}(\tau) +$$
$$+ K_{\sigma_b\sigma_a}(\tau) - \frac{\eta^2}{E^2}\frac{d^2}{d\tau^2}K_{\sigma_b\sigma_a}(\tau) + 3K_{\sigma_t}(\tau) - 3\frac{\lambda^2}{G^2}\frac{d^2}{d\tau^2}K_{\sigma_t}(\tau) \tag{15}$$

where $\left\langle a^2\right\rangle$ is the mean-square value of the equivalent stress amplitude. Fourier transformation of (15) yields as follows :

$$\frac{1}{4}\left(1+\frac{\eta^2}{E^2}\omega_e^2\right)\langle a^2\rangle[\delta(\omega-\omega_e)+\delta(\omega+\omega_e)]=$$

$$=\left(1+\frac{\eta^2}{E^2}\omega^2\right)\left[S_{\sigma_a}(\omega)+S_{\sigma_b}(\omega)+S_{\sigma_a\sigma_b}(\omega)+S_{\sigma_b\sigma_a}(\omega)\right]+$$

$$+3\left(1+\frac{\lambda^2}{G^2}\omega^2\right)S_{\sigma_t}(\omega) \qquad (16)$$

where :

- δ - Dirac's delta function
- $S_{\sigma_a}, S_{\sigma_b}, S_{\sigma_t}$ - power spectral densities of the processes σ_a, σ_b and σ_t, respectively
- $S_{\sigma_a\sigma_b}, S_{\sigma_b\sigma_a}$ - cross power spectral densities of the processes σ_a and σ_b, respectively, which satisfy the relation [7] :

$$S_{\sigma_b\sigma_a}(\omega)=S^*_{\sigma_a\sigma_b}(\omega) \qquad (17)$$

The quantities $\langle a^2\rangle$ and ω_e cannot be determined from (16) without additional assumptions. This problem is below solved by means of the theory of energy transformation systems.

APPLICATION OF THE THEORY OF ENERGY TRANSFORMATION SYSTEMS

According to the theory of energy transformation systems [2], a uniaxial stress and multiaxial stress can be regarded as equivalent in terms of fatigue lifetime of the material if during service life the internally and externally dissipated energies per unit volume in these states are respectively equal [8].

For stationary stochastic processes the frequency domain formulation of those conditions can be expressed as follows.

A one-dimensional stress process and vector stress process can be regarded as equivalent in terms of fatigue lifetime of the material if over the whole frequency range the internally and externally dissipated powers per unit volume in these stress states are respectively equal.

So, considering the internally dissipated powers, one gets from (16) :

$$\frac{\eta^2}{4E^2}\omega_e^2\langle a^2\rangle\int_{-\infty}^{\infty}[\delta(\omega-\omega_e)+\delta(\omega+\omega_e)]d\omega=$$

$$=\frac{\eta^2}{E^2}\left[\int_{-\infty}^{\infty}\omega^2S_{\sigma_a}(\omega)d\omega+\int_{-\infty}^{\infty}\omega^2S_{\sigma_b}(\omega)d\omega+\right. \qquad (18)$$

$$\left.+\int_{-\infty}^{\infty}\omega^2S_{\sigma_a\sigma_b}(\omega)d\omega+\int_{-\infty}^{\infty}\omega^2S_{\sigma_b\sigma_a}(\omega)d\omega\right]+3\frac{\lambda^2}{G^2}\int_{-\infty}^{\infty}\omega^2S_{\sigma_t}(\omega)d\omega$$

Equations (16) and (18) imply that :

$$\frac{1}{4}\langle a^2\rangle\int_{-\infty}^{\infty}[\delta(\omega-\omega_e)+\delta(\omega+\omega_e)]d\omega=$$

$$=\int_{-\infty}^{\infty}S_{\sigma_a}(\omega)d\omega+\int_{-\infty}^{\infty}S_{\sigma_a\sigma_b}(\omega)d\omega+ \qquad (19)$$

$$+\int_{-\infty}^{\infty}S_{\sigma_b\sigma_a}(\omega)d\omega+\int_{-\infty}^{\infty}S_{\sigma_b}(\omega)d\omega+3\int_{-\infty}^{\infty}S_{\sigma_t}(\omega)d\omega$$

Equations (17) through (19) give the following equivalence conditions between the considered stress and that equivalent :

$$\frac{1}{2}\omega_e^2\langle a^2\rangle=$$

$$=\int_{-\infty}^{\infty}\omega^2S_{\sigma_a}(\omega)d\omega+2\int_{-\infty}^{\infty}\omega^2\bar{S}_{\sigma_a\sigma_b}(\omega)d\omega+ \qquad (20)$$

$$+\int_{-\infty}^{\infty}\omega^2S_{\sigma_b}(\omega)d\omega+3\left(\frac{\lambda E}{\eta G}\right)^2\int_{-\infty}^{\infty}\omega^2S_{\sigma_t}(\omega)d\omega$$

$$\frac{1}{2}\langle a^2\rangle=\int_{-\infty}^{\infty}S_{\sigma_a}(\omega)d\omega+2\int_{-\infty}^{\infty}\bar{S}_{\sigma_a\sigma_b}(\omega)d\omega+$$

$$+\int_{-\infty}^{\infty}S_{\sigma_b}(\omega)d\omega+3\int_{-\infty}^{\infty}S_{\sigma_t}(\omega)d\omega \qquad (21)$$

where the bar over $S_{\sigma_a\sigma_b}$ denotes its real part, namely :

$$\bar{S}_{\sigma_a\sigma_b}(\omega)=\mathrm{Re}\left[S_{\sigma_a\sigma_b}(\omega)\right] \qquad (22)$$

Hence :

$$\langle a^2\rangle=2\left[\int_{-\infty}^{\infty}S_{\sigma_a}(\omega)d\omega+2\int_{-\infty}^{\infty}\bar{S}_{\sigma_a\sigma_b}(\omega)d\omega+\right. \qquad (23)$$

$$\left.+\int_{-\infty}^{\infty}S_{\sigma_b}(\omega)d\omega+3\int_{-\infty}^{\infty}S_{\sigma_t}(\omega)d\omega\right]$$

$$\omega_e=\left[\frac{\int_{-\infty}^{\infty}\omega^2S_{\sigma_a}(\omega)d\omega+2\int_{-\infty}^{\infty}\omega^2\bar{S}_{\sigma_a\sigma_b}(\omega)d\omega+\int_{-\infty}^{\infty}\omega^2S_{\sigma_b}(\omega)d\omega+3\left(\frac{\lambda E}{\eta G}\right)^2\int_{-\infty}^{\infty}\omega^2S_{\sigma_t}(\omega)d\omega}{\int_{-\infty}^{\infty}S_{\sigma_a}(\omega)d\omega+2\int_{-\infty}^{\infty}\bar{S}_{\sigma_a\sigma_b}(\omega)d\omega+\int_{-\infty}^{\infty}S_{\sigma_b}(\omega)d\omega+3\int_{-\infty}^{\infty}S_{\sigma_t}(\omega)d\omega}\right]^{1/2} \qquad (24)$$

It should be pointed out that in the case of a single stress σ_a or σ_b, (24) is reduced to the Rice's formula for the mean frequency of a stationary Gaussian process [7]. However, the assumption of normality of the stress processes was avoided in this paper.

CRITERION OF INFINITE FATIGUE LIFE

The problem to be faced in this section is the use of (23) in designing for infinite fatigue life if the material capacity is characterized by its fatigue limit F under fully reversed tension-compression. In this case the simplest criteria are as follows [9] :

$$\langle \mu \rangle \geq 0 \tag{25}$$

and

$$\langle \bar{\mu} \rangle \geq 0 \tag{26}$$

where μ and $\bar{\mu}$ are the relative fatigue safety margins defined as :

$$\mu = 1 - \frac{1}{f} \qquad f = \frac{F}{a} \tag{27}$$

$$\bar{\mu} = 1 - \frac{1}{f^2} \tag{28}$$

By regarding F as a deterministic quantity (25) and (27) give :

$$\langle a \rangle \leq F \tag{29}$$

whereas (26) and (28) lead to :

$$\langle a^2 \rangle \leq F^2 \tag{30}$$

Of course, the criterion (30) is more conservative than (29) because :

$$\langle a \rangle^2 = \langle a^2 \rangle - \mathrm{Var}(a) \tag{31}$$

where Var (a) is the variance of the equivalent stress amplitude.

Having determined the mean-square value of the equivalent stress amplitude one can obtain the criterion (30) in the following form :

$$\int_{-\infty}^{\infty} S_{\sigma_a}(\omega)d\omega + 2\int_{-\infty}^{\infty} \bar{S}_{\sigma_a\sigma_b}(\omega)d\omega + \int_{-\infty}^{\infty} S_{\sigma_b}(\omega)d\omega + +3\int_{-\infty}^{\infty} S_{\sigma_t}(\omega)d\omega \leq \frac{1}{2}F^2 \tag{32}$$

The problem of estimation of the relevant probability of infinite fatigue life requires further analysis.

In view of the scatter inherent in fatigue tests, it is advisable to assume F to be a random variable, and to replace (32) by :

$$\int_{-\infty}^{\infty} S_{\sigma_a}(\omega)d\omega + 2\int_{-\infty}^{\infty} \bar{S}_{\sigma_a\sigma_b}(\omega)d\omega + \int_{-\infty}^{\infty} S_{\sigma_b}(\omega)d\omega + +3\int_{-\infty}^{\infty} S_{\sigma_t}(\omega)d\omega < \frac{1}{2}\langle F^2 \rangle \tag{33}$$

where $\langle F^2 \rangle$ is the mean-square value of the fatigue limit F.

EXAMPLE

Task

Determine the criterion of infinite fatigue life based on (30) if beam is subjected to a random axial force and bending moment 'hich generate, at a given point, the stationary (in the wide sense) nd stationary correlated stress components :

$$\sigma_i(t) = \sum_{p=1}^{n} \left(A_{ip} \cos \omega_p t + B_{ip} \sin \omega_p t \right) \quad i = a, b \tag{34}$$

where A_{ip} and B_{ip} are random variables.

Solution

Applying Euler formulae :

$$\cos \omega_p t = \frac{1}{2}\left[\exp(j\omega_p t) + \exp(-j\omega_p t)\right]$$

$$\sin \omega_p t = \frac{1}{2j}\left[\exp(j\omega_p t) - \exp(-j\omega_p t)\right]$$

one gets :

$$\sigma_i(t) = \sum_{p=1}^{n}\left[C_{ip}\exp(j\omega_p t) + D_{ip}\exp(-j\omega_p t)\right] \tag{35}$$

where :

$$C_{ip} = \frac{1}{2}A_{ip} + \frac{1}{2j}B_{ip} \qquad D_{ip} = \frac{1}{2}A_{ip} - \frac{1}{2j}B_{ip}$$

Denoting $-\omega_p = \omega_{-p}$ equation (35) can be rewritten into the form :

$$\sigma_i(t) = \sum_{p=-n}^{n} H_{ip}\exp(j\omega_p t) \tag{36}$$

where :

$$H_{ip} = C_{ip} \quad \text{for } p = 1,2,...,n \text{ and}$$

$$H_{ip} = D_{ip} \quad \text{for } p = -1,-2,...,-n$$

The correlation functions of the stress components become :

$$K_{\sigma_i}(t_1,t_2) = \left\langle \sum_{p=-n}^{n} H_{ip}^{*}\exp(-j\omega_p t_1) \sum_{m=-n}^{n} H_{im}\exp(j\omega_m t_2) \right\rangle$$

$$K_{\sigma_a\sigma_b}(t_1,t_2) = \left\langle \sum_{p=-n}^{n} H_{ap}^{*}\exp(-j\omega_p t_1) \sum_{m=-n}^{n} H_{bm}\exp(j\omega_m t_2) \right\rangle \tag{37}$$

Hence :

$$K_{\sigma_i}(t_1,t_2) = \sum_{p=-n}^{n}\sum_{m=-n}^{n}\exp\left[j(\omega_m t_2 - \omega_p t_1)\right]\langle H_{ip}^{*}H_{im}\rangle$$

$$K_{\sigma_a\sigma_b}(t_1,t_2) = \sum_{p=-n}^{n}\sum_{m=-n}^{n}\exp\left[j(\omega_m t_2 - \omega_p t_1)\right]\langle H_{ap}^{*}H_{bm}\rangle \tag{38}$$

Stationarity in the wide sense implies that :

$$\left\langle H_{ip}^{*}H_{im}\right\rangle =\begin{cases}R_{ip}\text{ for } p=m\\ 0 \quad \text{for } p\neq m\end{cases}$$

$$\left\langle H_{ap}^{*}H_{bm}\right\rangle =\begin{cases}R_{abp}\text{ for } p=m\\ 0 \quad \text{for } p\neq m\end{cases}$$

where :

$$R_{ip}=\frac{1}{4}\left\langle A_{ip}^{2}+B_{ip}^{2}\right\rangle$$

$$R_{abp}=\frac{1}{4}\left[\left\langle A_{ap}A_{bp}+B_{ap}B_{bp}\right\rangle +\frac{1}{j}\left\langle A_{ap}B_{bp}-A_{bp}B_{ap}\right\rangle \right]$$

Thus :

$$K_{\sigma_i}(t_1,t_2)=K_{\sigma_i}(\tau)=\sum_{p=-n}^{n}R_{ip}\exp\left(j\omega_p\tau\right)$$
$$K_{\sigma_a\sigma_b}(t_1,t_2)=K_{\sigma_a\sigma_b}(\tau)=\sum_{p=-n}^{n}R_{abp}\exp\left(j\omega_p\tau\right) \tag{39}$$

Fourier transformation of (39) yields :

$$S_{\sigma_i}(\omega)=\sum_{p=-n}^{n}R_{ip}\delta\left(\omega-\omega_p\right)$$
$$S_{\sigma_a\sigma_b}(\omega)=\sum_{p=-n}^{n}R_{abp}\delta\left(\omega-\omega_p\right) \tag{40}$$

The mean-square value of the equivalent stress amplitude is found by substituting (40) into (23), which gives :

$$\left\langle a^2\right\rangle =2\left(\sum_{p=-n}^{n}R_{ap}+2\sum_{p=-n}^{n}\overline{R}_{abp}+\sum_{p=-n}^{n}R_{bp}\right) \tag{41}$$

where :

$$\overline{R}_{abp}=\frac{1}{4}\left\langle A_{ap}A_{bp}+B_{ap}B_{bp}\right\rangle$$

Hence :

$$\left\langle a^2\right\rangle =2\left(\frac{1}{4}\sum_{p=-n}^{n}\left\langle A_{ap}^{2}+B_{ap}^{2}\right\rangle +\right.$$
$$\left.+\frac{1}{2}\sum_{p=-n}^{n}\left\langle A_{ap}A_{bp}+B_{ap}B_{bp}\right\rangle +\frac{1}{4}\sum_{p=-n}^{n}\left\langle A_{bp}^{2}+B_{bp}^{2}\right\rangle \right)=$$
$$\sum_{p=1}^{n}\left\langle A_{ap}^{2}+B_{ap}^{2}\right\rangle +2\sum_{p=1}^{n}\left\langle A_{ap}A_{bp}+B_{ap}B_{bp}\right\rangle +\sum_{p=1}^{n}\left\langle A_{bp}^{2}+B_{bp}^{2}\right\rangle \tag{42}$$

So, the criterion (30) takes the form :

$$\sum_{p=1}^{n}\left\langle A_{ap}^{2}+B_{ap}^{2}\right\rangle +2\sum_{p=1}^{n}\left\langle A_{ap}A_{bp}+B_{ap}B_{bp}\right\rangle +\sum_{p=1}^{n}\left\langle A_{bp}^{2}+B_{bp}^{2}\right\rangle \leq F^2 \tag{43}$$

CONCLUSIONS

In this paper a criterion of infinite fatigue life of metal beams in the frequency domain was formulated. For this purpose the distortion-energy strength hypothesis and theory of energy transformation systems, were applied. Moreover, it was assumed that the material is ductile and has a fatigue limit, and that the stress components represent stationary (in the wide sense) and stationary correlated stochastic processes differentiable in the mean-square sense. By taking the power spectral densities of individual stress components for known an equivalent stress was defined.

Appraised by Marek Sperski, Assoc.Prof.,D.Sc.

NOMENCLATURE

a	-	equivalent stress amplitude
E	-	Young modulus
f	-	fatigue safety factor
F	-	fatigue limit under fully reversed tension-compression
G	-	shear modulus
j	-	imaginary unity
K_{σ_i}	-	auto-correlation function of the process σ (i = a,b,t)
$K_{\sigma_a\sigma_b}$	-	cross-correlation function of the processes σ_a and σ_b
S_{σ_i}	-	power spectral density of the process σ_i (i = a,b,t)
$S_{\sigma_a\sigma_b}$	-	cross power spectral density of the processes σ_a and σ_b
$\overline{S}_{\sigma_a\sigma_b}$	-	real part of $S_{\sigma_a\sigma_b}$
t	-	time
γ	-	shear strain
δ	-	Dirac's delta function
ε	-	normal strain
ε_e	-	equivalent strain
ε_i	-	i-th strain component (i = a,b,t)
η	-	coefficient of internal viscous damping of the material in tension-compression
λ	-	coefficient of internal viscous damping of the material in torsion
$\mu, \overline{\mu}$	-	relative margins of the fatigue safety
ν	-	Poisson's ratio
σ	-	normal stress
σ_e	-	equivalent stress
σ_i	-	i-th stress component (i = a,b,t)
τ	-	time interval, shear stress
ϕ, ϕ_e	-	strain energies of distortion per unit volume in the general static and equivalent stress states, respectively
ω	-	circular frequency
ω_e	-	equivalent circular frequency
$\langle\cdots\rangle$	-	expected value
$(\ )^*$	-	complex conjugate.

Indices

a,b,t - quantities associated with axial, bending and torsion loads, respectively.

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