

# STUDY OF ARRAY OF MEMS INERTIAL MEASUREMENT UNITS UNDER QUASI-STATIONARY AND DYNAMIC CONDITIONS

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## ABSTRACT

*A measurement system includes all components in a chain of hardware and software that leads from a measured variable to processed data. In that context, the type and quality of the sensors or measuring devices are critical to any measurement system. MEMS/IMU sensors lag behind leading technologies in this respect, but the MEMS/IMU performance rapidly changes while is relatively inexpensive. For this reason, the paper proposes some investigations of currently available MEMS/IMUs, but in an array configuration. The article presents the results of research undertaken on this type of IMU sensor configuration under quasi-stationary and dynamic conditions and answers the question of whether the precision of current MEMS technologies for acceleration and angular velocity sensors is still improved using this kind of approach.*

**Keywords:** Gyro, Accelerometer, MEMS, Inertial Units, Sensor Array

## INTRODUCTION

There are numerous IMU application areas such as inertial positioning of spacecrafts, marine vehicles and drones, manned and unmanned aircraft, and autonomous vehicles. The ring laser gyro (RLG) is currently the industry standard for precision rotation measurement, though some researchers states that FOGs (fiber optic gyros) is not seen anymore as limited to medium grade, but on the contrary as the ultimate performance gyro that can surpass by at least one if not two orders of magnitude RLG technology [1]. Micro-Electromechanical Systems, Inertial Motion Unit (MEMS IMUs) performance is approaching FOG medium, tactical grade performance levels. FOGs still have an advantage on performance, but are much more costly than MEMS [2],[3]. Low-cost MEMS inertial sensors have emerged over the past decade and MEMS researchers have demonstrated a number

of microsensors for almost every possible sensing modality, including attitude, accelerations, pressure etc. [4],[5],[6]. The paper investigate an array configuration of currently available MEMS sensors to improve the final accuracy, so a topic that has to return from time to time due to changing technologies [7],[8],[9]. The issue of using simultaneous measurements from many identical devices is still topical due to dynamically changing technologies [10],[11],[12]. The article presents the use of this idea in the context of the LSM330DLC sensor array based on the MEMS technology, emphasizing however that a real improvement is possible when the measurements is carried out in quasi-stationary conditions, so in particular, this approach can be justified only, due to the random walk error, resulting from Brownian motion of the MEMS proof mass. The article answers the question if currently available, relatively low-cost IMU MEMS sensors can still provide better INS when studied in array configuration in quasi-stationary

and at the same time in dynamic conditions. Static or quasi-stationary measurements are considered sufficient in the context of low dynamics vehicles [13].

## MEMS IMU ERRORS AND CURRENT CAPABILITIES

MEMS technology is burdened with vibration, but mainly by static errors that are difficult to overcome i.e.: the instability of bias also called bias drift, typically expressed in [mg] or [m/s<sup>2</sup>] for accelerometers and for gyroscopes [deg/s] or [rad/s] [14]. This instability of bias, refers to the variation of the bias over time, assuming that other factors remain unchanged. This may be caused, for example, by the self-heating of the accelerometer/gyroscope itself and other components of the entire system, both mechanical and electrical. The instability of bias drift is the most important measure of the quality of the accelerometer/gyro and is defined as the lowest part on the Allan variance curve, as presented in Fig. 4 and Fig. 5. Thus, it represents the minimal bias stability that can be achieved for a given sensor, assuming that bias averaging takes place at the interval defined at the Allan variance minimum. This is one of the most important parameters for accuracy and overall performance in the context of unassisted inertial navigation. The scale factor is another inherent MEMS IMU error that multiplies the output of the sensor with respect to the true measurement, so this is a multiplicative error. The scale factor relates to an intended true input value, but can be positive or negative. Of course, the scale factor and bias, but not bias drift, can be eliminated by calibration. In a calibration procedure, the bias and scale factor are determined by comparing known parameters to measured output. Another important error, velocity or angular random walk (ARW) is a measure of accelerometer as well as gyro perturbation by some thermo-mechanical noise which fluctuates at a rate much greater than the sampling rate of the sensor. This is proportional to the square root of the integration time [15].

## MEMS IMU ARRAY SIMULTANEOUS MEASUREMENT HYPOTHESIS AND SIMULATION

To investigate the concept of simultaneous measurements from a MEMS IMU array, some simulations and analyses were carried out. The idea of course, stems from a measurement systems theory. Each IMU can be treated as measurement system comprised of an instrument or measuring device and an actual object parameter to be measured (Fig. 1). According to the theory of experimental design: performing measurements from multiple instruments should improve the precision and the accuracy. The assumption is that the response of an IMU instrument is the sum of three independent quantities:

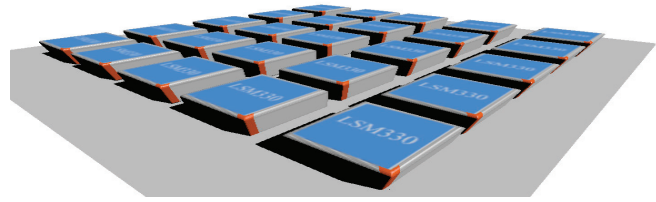


Fig. 1. Array of LSM330DLC MEMS IMUs. Tests carried out at an ambient temperature of 20°C.

the true value to measure  $\mu$ , a random measurement error  $X$  with mean 0 and variance  $\sigma^2$  ( $\sigma$  describes the instrument's imprecision) and a fixed error  $Y$  which can also be modelled as a random variable with mean 0, because these IMU/MEMSs are accurate, but because they do vary systematically from one to another, hence variance  $\tau^2$ . Although this model is rarely exactly right, it is usually a sufficiently good approximation that we can use it to find near-optimal combinations of measurements.

The results of repeated measurements by one IMU are independent, so each measurement can be treated as the sum  $Z_i = \mu + X_i + Y$  where  $i$  stands for the measurement, ranging from 1 through  $n$ , and  $Y$  is a property of the instrument itself, so it does not change from one measurement to another. We can compute the variance of the average of the measurements conceived of as an average of these random variables  $Z_i$  as:

$$\text{var}(\bar{Z}) = \frac{1}{n} \sigma^2 + \tau^2 \quad (1)$$

as  $n$  increases,  $\frac{\sigma^2}{n}$  decreases. Moreover, if we accept expectations in the sense of what an arbitrarily large number of measurements would produce on average,  $E(\bar{Z}) = \mu + Y$  shows that even the average is biased. The conclusion of this calculation is that averaging measurements from one instrument reduces the imprecision but has no effect on the accuracy.

On the other hand, if one supposes the measurements of all IMUs from Fig. 1 are independent, so supposing measurement by multiple IMUs, which is our case, then  $Z_i = \mu + X_i + Y_i$  where  $i$  is the indexes for the measurement and also for the instrument, therefore:

$$\text{var}(\bar{Z}) = \frac{1}{n} \sigma^2 + \frac{1}{n} \tau^2 \quad (2)$$

and (in the same sense as before, taking an arbitrarily large number of instruments),  $E(\bar{Z}) = \mu$ , as  $n$  increases, both  $\frac{\sigma^2}{n}$  and  $\frac{\tau^2}{n}$  decrease. Regardless, the expected value of the measurement is correct:  $\bar{Z}$  is more likely to be accurate in this case. Thus, averaging measurements from multiple instruments should improve the precision and accuracy.

The noise in the accelerometer or gyro is predominantly considered Gaussian white noise and therefore is a constant value across all frequencies [3]. Following that, numerical simulation was performed, using the Gaussian distribution which is most proper in the presence of ARW and bias in

run stability.

$$dev(\bar{Z}) = \frac{\sum_{i=1}^n (X_N)_{s=1}^{\infty}}{n} \quad (3)$$

where  $(X_N)_{s=1}^{\infty}$  is a random sequence where  $X \sim N_i(\mu_i, \sigma^2_i)$  is defined to be a Gaussian random variable, where of course  $\mu$  is the mean and  $\sigma^2$  is the variance of the Gaussian noise and  $n$  is the number of sensors,  $N$  is a size of the random vector. The result of the simulation is presented in Fig. 2, where the standard deviation error against the number of sensors in the MEMS array is mutually-dependant and is inversely proportional, as presented in Fig. 2.

Until  $n < 10$ , the gain or improvement is obvious, otherwise the approach can be questioned. However, depending on the available budget, increasing the number of IMUs in the array (as presented in Fig. 1) showed that an improvement of close to 10 times so of magnitude can be achieved.

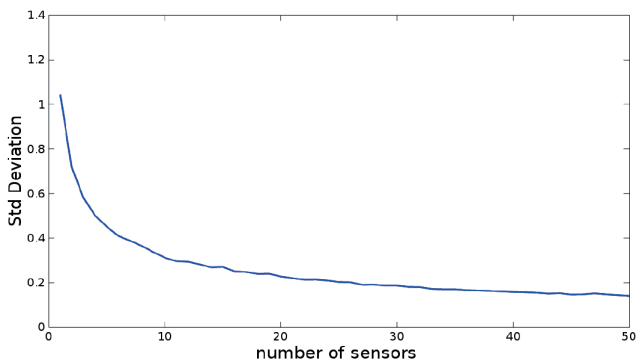


Fig. 2. Standard deviation error vs. number of sensors in MEMS IMU array for simulated data

## RESULTS OF INVESTIGATION OF REAL MEASUREMENTS FORM MEMS IMUS IN ARRAY CONFIGURATION USING ALLAN VARIANCE

The experimental data was collected from 24-hour simultaneous readings from an IMU LSM330 [17] sensor array at 50Hz rate and at 20°C temperature (Fig. 1). The environment temperature is of some importance for the overall quality of measurements. Raw accelerometer readings are expressed in [m/s], gyro readings are expressed in [deg/s], as presented in Table 1 [7].

Table 1. Format and example of collected records from MEMS IMU LSM330.

time	ax	ay	az	time	gx	gy	gz
[ms]	[m/s]	[m/s]	[m/s]	[ms]	[deg/s]	[deg/s]	[deg/s]
12536946977	0.239420	0.153229	9.864111	125369469785	0.088575	0.049480	0.026573

Two-Sample Variance Evaluation is a method of analyzing a sequence of data in the time domain, to measure the frequency stability of oscillators the variance also known as Allan variance. The method can also be used to determine the noise in a system as a function of the averaging time, and is currently a popular method for identifying and quantifying the different noise terms in inertial sensor data. The results from this method are related to four basic noise terms appropriate for inertial sensor data. These are quantization noise, angle random walk, bias instability, and rate random walk [17].

The investigation of MEMS IMU array using Allan deviation method is summarized in Table 2, where bias stability standard deviation error and normalized bias stability standard deviation error vs. number of sensor in the IMU array is presented.

Table 2. Allan Deviations of MEMS Accelerometer Array.

Number of IMUs in MEMS LSM330DLC array	Bias stability deviation error [m/s <sup>2</sup> ]	Normalized bias stability deviation error
1	4.459219e-04	1.000000
2	3.344659e-04	0.750055
3	2.889205e-04	0.647917
4	2.773783e-04	0.622033
5	2.495453e-04	0.559616
6	2.095125e-04	0.469841
7	1.828213e-04	0.409985
8	1.827415e-04	0.409806
9	1.711244e-04	0.383754
10	1.572241e-04	0.352582

The obtained results from Table 2 based on real data, are plotted over the simulated chart from Fig. 2 and presented in Fig. 3.

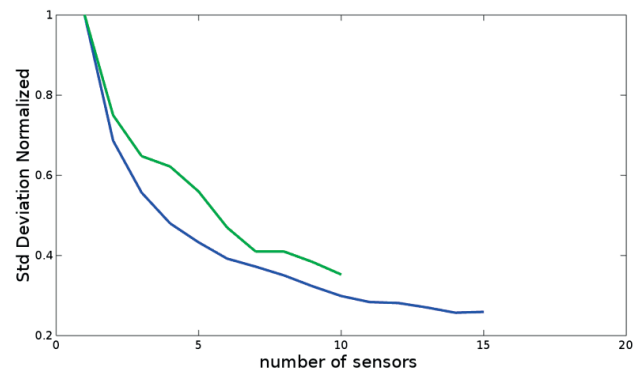


Fig. 3. Normalized standard deviation error vs. number of sensors in MEMS array for simulated data (blue) and real data from accelerometer (green)

Fig. 3 presents normalized standard deviation error vs. number of sensors in MEMS array for simulated data plotted in blue and the real data from the LSM330 array in green. Both curves are strongly related and rather strongly converge.

Table 3 presents some important detailed characteristics of raw measurements retrieved from the MEMS accelerometer in the context of some basic terms appropriate for INS. The in-run bias stability standard deviation error references the minima of the Allan deviation curve, as presented in Fig. 4, and it is over 3 times better for an array of n=10 MEMS accelerometer sensors. Fig. 4 presents the Allan variance for the data from the IMU array and for a single IMU in static conditions.

Table 3. Accelerometer Comparison.

Accelerometer Dynamic Accuracy	LSM330DLC	LSM330DLC array
bias stability deviation error [m/s <sup>2</sup> ]	4.4372e-04	1.57224e-04
variance	4.06215e-04	4.66665e-05
standard error [m/s <sup>2</sup> ]	0.020154	0.006831
bias [m/s <sup>2</sup> ]	0.050850	0.211349

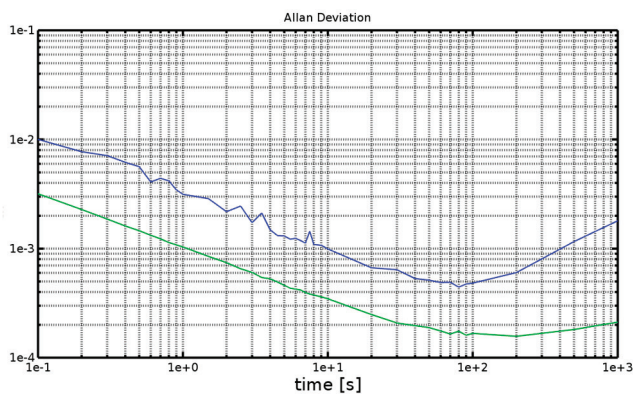


Fig. 4. Accelerometer Allan variance log-log plots of MEMS IMU array (green) and single MEMS IMU (blue)

In a similar manner, Table 4 and the following Fig. 5 present the results for the MEMS gyroscope. It turns out the performance is near 5 times better for an array of n=10 MEMS gyro sensors compared to a single MEMS gyro.

Table 4. Allan Deviations of MEMS Accelerometer Array.

Gyro Dynamic Accuracy	LSM330DLC	LSM330DLC array
bias stability deviation error [deg/s]	2.87124575e-04	6.3087195e-05
variance	1.34194979e-05	1.369146e-06
standard error [deg/s]	0.0036632	0.00117
bias [deg/s]	0.0512660	0.01552

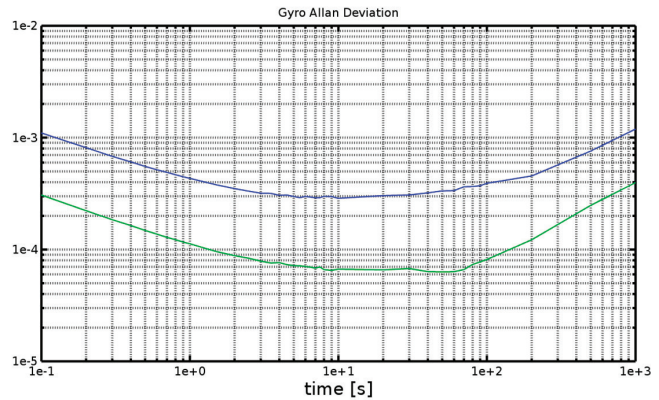


Fig. 5. Log-log plots of Allan deviation of gyro for MEMS IMU array (green) and single MEMS IMU (blue)

## IMU ARRAY UNDER DYNAMIC CONDITIONS

However, the quasi-stationary or stationary measurement conditions using the IMU sensor matrix indicate only one stationary aspect of the problem. In the quoted papers [7], [8], [9], [10] the fact of measurements under dynamic conditions for the IMU sensor array is omitted. Dynamic measurements, as mentioned in the introduction, emphasize the problem of system inertia and its nonlinearity and is especially important e.g. in the context of distance measurements using IMU. The comparative aspect of dynamic measurements for the sensor array is presented below. The Fig. 6 and Fig. 7 presents the step response of the IMU sensors array and an individual IMU sensor, i.e. the response to the step input for both cases. The step response for the accelerometer is presented in Fig. 6 for step input of the low and in Fig. 7 of higher dynamic or amplitude. The response of the IMU sensors array is presented in green, and the response of an individual IMU sensor is shown in blue.

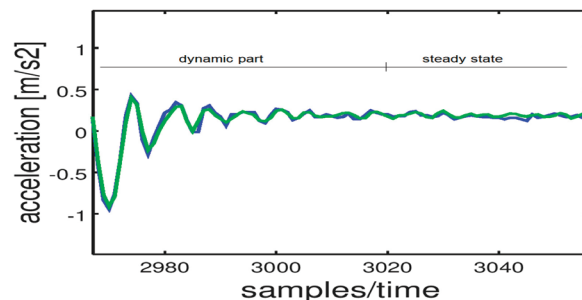


Fig. 6. Step response for MEMS IMU array (green) and single MEMS IMU (blue) with low step excitation



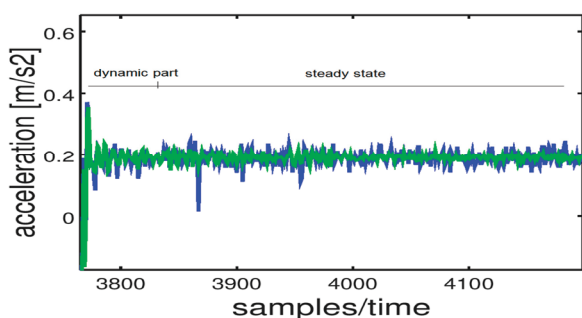


Fig. 7. Step response for MEMS IMU array (green) and single MEMS IMU (blue) with higher step excitation

The step response for IMU sensor is typical for the second order inertial system step response, so it possesses a steady and an undetermined part. In the transient part, as a consequence of the step input, the dynamic response of IMU dominates. The steady state part of the Figs is characteristic of quasi-stationary measurements, as presented the chapter 4. However, the most important observation is the lack of a clear difference for the dynamic part of the step response in both cases, so for the array of IMU sensors and an individual sensor. Additionally, depending on the dynamics of the step excitation, the time to settle or settling time differs, which is presented in Fig. 7. In this time the settling time is longer. Of course that fact has significant impact on further data processing, not only in the context of distance evaluation, but also for an angle or tilt estimation [13].

## CONCLUSIONS

High quality sensors are a matter of fundamental importance in the inertial navigation context, which is almost obvious. MEMS accelerometer/gyro sensors currently present quite good quality only for short ranges in the context of INS processing. Based on the conducted tests results as well as their analysis, we can conclude that by using currently available on the market MEMS IMUs sensor array, the precision of the entire INS system can be significantly improved but only under quasi-stationary conditions. The carried out studies comply with the presented theory and prove the performance is near 4.5 times better for the LSM330 gyro and over 3 times better for LSM330 accelerometer if they are used in the MEMS array configuration. Hereby, the experiment comply with the theory of experimental design that performing measurements from multiple IMU MEMS sensors significantly improves the precision and accuracy of the final acceleration and angle rate measurement. The presented theory and simulations prove that an almost tenfold improvement is possible, beyond this limit the improvement can be hard to observe. The experimental design also confirms, that a single MEMS IMU measurement model response is the compound of the sum of the three above mentioned independent quantities,

namely: the actual value, a random measurement error and a fixed error, characteristic of a particular instrument. However, this approach is not so effective under dynamic conditions. The matrix of the IMU sensor does not provide the expected improvement in this case and it is not possible to improve or significantly improve the accuracy of measurements under dynamic conditions using this approach.

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