

## **SUBMERGED OBJECT MODELING AND STORING IN A GLOBAL REFERENCE CONTEXT USING MULTIREOLUTION SPLINE TECHNIQUE**

**JERZY DEMKOWICZ**

Gdańsk University of Technology  
ul. G. Narutowicza 11/12  
80-233 Gdańsk, Poland  
demjot@eti.pg.gda.pl

*Contemporary records from multibeam sonars or even elevations from 3D shuttle radar topography missions feature high resolution. On the other hand, bathymetric models of a different resolution from different sensors are available as well, beginning from very high resolution MBS records and low resolution records coming from regular scattered measurements. Approximating and eventually visualizing the high volume scattered 3D raster data of different resolutions results in some difficulties related to the extent of computer processing power. The paper presents some advantages of using multiresolution splines combined with the Hilbert curve approach in the context. The proposed approach consists of two stages: firstly, data pertaining to different resolutions are interpolated using the spline technique and finally the knots and control points are saved using the Hilbert curve. Such an approach particularly facilitates high volume spatial data for the level of details (LoD) visualization technique.*

### **INTRODUCTION**

The topographic or bathymetric data features a wide range of vertical resolutions. In general, unlike terrestrial elevation data, undersea records tend not to lie in regular grids and also tend to be much denser near land-masses. Bathymetric records from multibeam sonar (MBS) possess resolutions of decimeters or an even higher resolution, whereas ocean bathymetry provides us with a 1 km resolution. The applications of high resolutions to many areas may obviously appear to be redundant or even ambiguous [3], but can simultaneously be useful in approximation approaches. The paper presents an approximation approach using hierarchical spline techniques. This approach permits a flexible and appropriate resolution for different scales in the visualization process as well as for 3D global elevation data imaging.

### 1. SPLINE FUNCTIONS

Spline functions can be expressed as a linear combination:

$$s(x) = \sum_{k \in Z} c(k)\varphi^n(x - k) \tag{1}$$

where  $c(k)$  is a control point, and  $\varphi^n$  a base function of  $n$  degrees. The linear combination is responsible for the spline function's smoothness. The spline functions of integer knots can be interpreted as functions of a different resolution within the context of multiscale representation [1]. The base function equation for  $n = 0$  yields  $\varphi^0(x/m)$ , and is 1 for  $x \in [0, m]$  and 0 otherwise:

$$\varphi^0(x/m) = \sum_{k=0}^{m-1} \varphi^0(x - k) = \sum_{k \in C} h_m^0(k)\varphi^0(x - k) \tag{2}$$

where  $h_m^0(k)$  represents the filter of Z transform  $H_m^0(z) = \sum_{k=0}^{m-1} z^{-k}$  therefore a discrete impulse of length  $m$ . The  $(n+1)$  convolution of the function reads:

$$H_m^n(z) = \frac{1}{m^n} (H_m^0(z))^{n+1} = \frac{1}{m^n} \left( \sum_{m=0}^{m-1} z^{-k} \right)^{n+1} \tag{3}$$

which represents  $n+1$ -th convolution of the discrete pulse and can be implemented by extremely rapid algorithms e.g. FIR filters. The coefficients of the filter both yield and resemble the Pascal triangle.

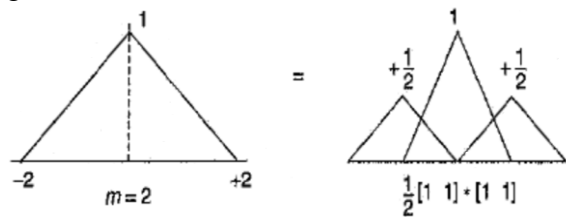


Fig. 1. Figures and photo should be in the text.

This situation is shown in Fig. 1 for a spline function of order 1, as a so-called spline function pyramid. Eventually, the spline function representation for  $n$  order is represented as:

$$\varphi^n(x/m) = \sum_{k \in Z} h_m^n(k)\varphi^n(x - k) \tag{4}$$

and can be interpreted as a spline function hierarchy.

### 2. MULTIREOLUTION SPLINE REPRESENTATION

The multiresolution spline approach can be analyzed using the sampling Nyquist theorem, which was introduced in Chapter 2 of the paper. The chapter underlines an important aspect of spline functions analysis, namely the possibility of their FIR filter implementation. However, in the context of the paper and the 3D data representation, a geometric approach seems to be of a more convenient and general nature.

Let  $V^k$  represent the control point space of resolution  $k$ . There is a transformation which transforms one space into the other:

$$R^{[k+1]} = SV^{[k]} \quad (5)$$

where  $S$  depicts the transformation. The multiresolution spline model of 3D records from MBS of different resolution is shown in Fig.4. The figure presents low and high resolution control point representations as a regular grid in different colors. Supposing  $\vec{O}$  represents a control point translation in a newer space of a higher resolution, the translation in this space reads as follows:

$$V^{[k+1]} = R^{[k+1]} \oplus \vec{O}^{[k+1]} \quad (6)$$

where  $\oplus$  operator can be interpreted as transposition  $\vec{O}$ . The result of the operation was shown in Fig. 4 as the high density yellow grid overlaying the base low resolution control points grid.

The hierarchical spline representation may be applied to different kinds of data, e.g. from the bathymetric data of electronic chart (low resolution) to the bathymetric data from multibeam sonar (high density resolution), and so on.

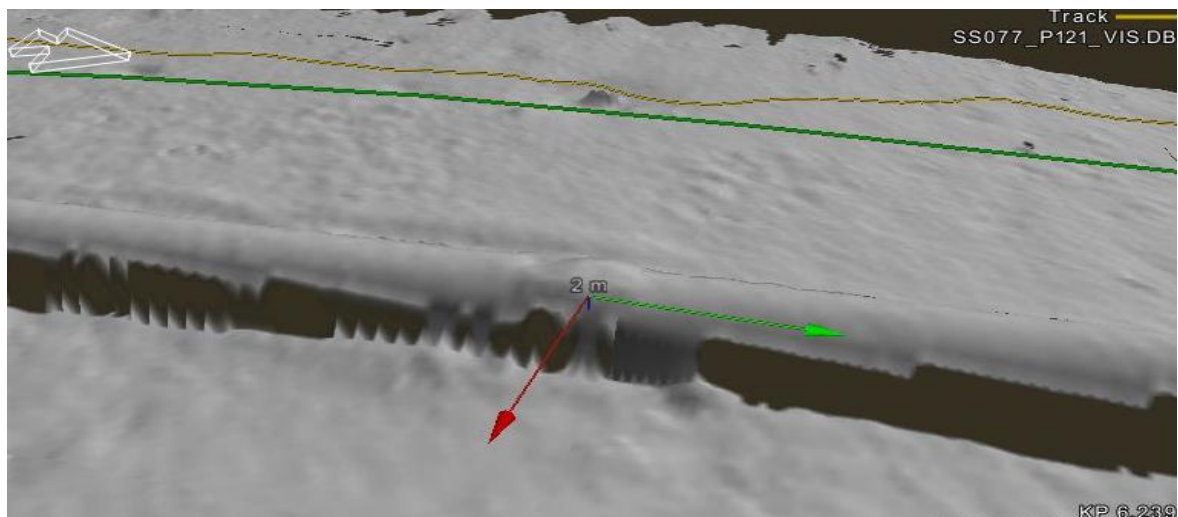


Fig. 2. Multibeam 3D data records of a submerged pipeline [11]



Fig. 3. Submerged pipeline observed from a ROV camcorder [11]

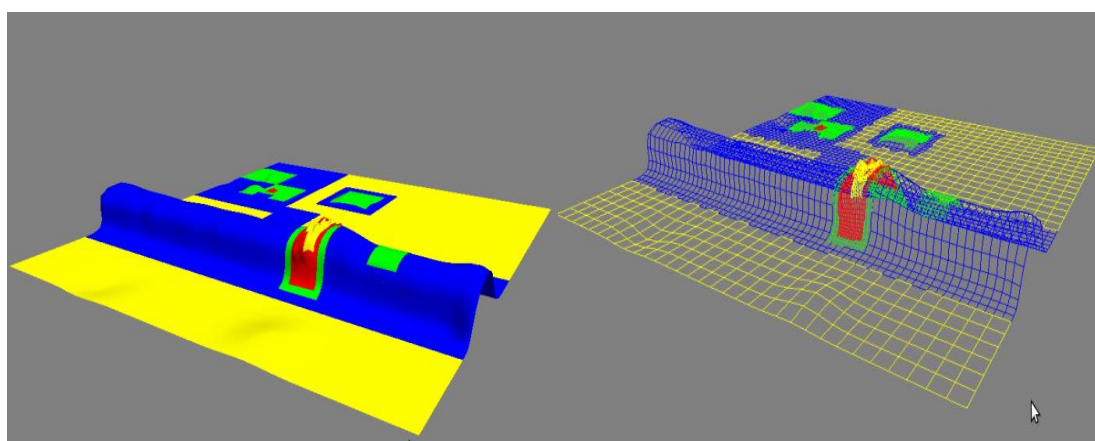


Fig. 4. Multi-resolution 3D data representation using hierarchical spline function techniques

The crucial question arising at this point refers to the approximation error, as the error could be used by the automatic multiresolution spline grid generation process. The error may be calculated by the following mean square error:

$$ER = \sqrt{\frac{\sum_{i=0}^n (f(x_i) - s(x_i))^2}{n - m}} \quad (7)$$

where  $f(x_i)$  represents the bathymetric measurement at point  $x$  and  $s(x_i)$  is the spline approximation at point  $x$ . The approximation error will depend on the sampling frequency (so the Nyquist theorem) and consequently this is the reason why the hierarchical splines are so useful, because the hierarchical splines can approximate the data, all the while taking into account their local density and locality. Therefore the local spline approximation error can be of use in the context of vertical spline knots or grid resolution estimation.

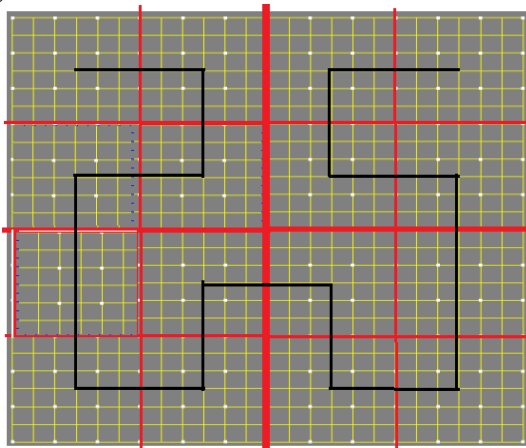
### 3. HILBERT CURVE APPLICATION IN THE MUTLIRESOLUTION DATA STORING AND DISPLAYING PROCESS

Hilbert Curves are space-filling curves, because they are one dimensional lines that regardlessly fill all available space in a fixed area, therefore representing part of the class of one-dimensional fractals. The process of quad-tree spatial indexing based on the Hilbert curve (Fig. 5) provides us with an efficient indexing method and if the ordered sequence of the spatial x, y coordinates required to draw a Hilbert curve is expressed in binary form, then the number of queries can be reduced to a minimum. Beginning at the root level (Fig. 5), the enumeration of the points is as follow: we choose a direction and a starting point, and proceed around the four quadrants, numbering them from 0 to 3. Then the sub-quadrants are enumerated, whilst maintaining the overall adjacency property. Each of the sub-quadrants' curves is a transformation of the original. This applies recursively to sub-sub quadrants, and so forth. The curve we use for a given quadrant is determined by the curve we used for the square in which it is located [10]. The conversion method between the x, y coordinates and the Hilbert curve positions uses the Hilbert map [10]:

**HILBERT\_MAP = {'a':**

**{(0, 0): (0, 'd'), (0, 1): (1, 'a'), (1, 0): (3, 'b'), (1, 1): (2, 'a')}, 'b': {(0, 0): (2, 'b'), (0, 1): (1, 'b'), (1, 0): (3, 'a'), (1, 1): (0, 'c')}, 'c': {(0, 0): (2, 'c'), (0, 1): (3, 'd'), (1, 0): (1, 'c'), (1, 1): (0, 'b')}, 'd': {(0, 0): (0, 'a'), (0, 1): (3, 'c'), (1, 0): (1, 'd'), (1, 1): (2, 'd')}}}**

a)



b)

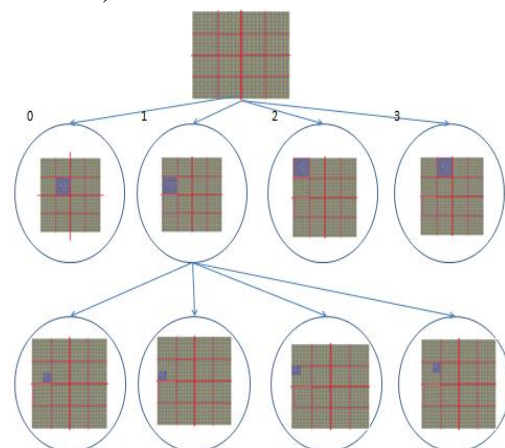


Fig. 5. Hilbert curve applied to sub-quadrants (a) and the quad-tree indexing process b) The quad-tree indexing process and meta code depicts the procedure [10]:

**POINT2HILBERT (x, y, zoom=16):**

**CURRENT\_SQUARE = 'a'**

**POSITION = 0**

**FOR i IN RANGE(order - 1, -1, -1):**

**POSITION <<= 2**

**QUAD\_X = 1 if x & (1 << i) else 0**

**QUAD\_Y = 1 if y & (1 << i) else 0**

**QUAD\_POSITION, CURRENT\_SQUARE = HILBERT\_MAP[CURRENT\_SQUARE][(quad\_x, quad\_y)]**

**POSITION |= QUAD\_POSITION**

**RETURN POSITION**

#### 4. CONCLUSIONS

The problem of efficient 3D spatial data representation is still open. There are two main reasons for this, namely data redundancy and excessive amounts of data. Interpolating, approximating, merging and eventually displaying scattered 3D raster data of high volumes and different resolutions leads to some difficulties as regards computer processing power. The proposed approach consists of two stages: firstly, all acquired high resolution data is interpolated with high density uniform spline interpolation and then data of a different resolution is stored using the Hilbert curve. The latter method, presented as a relatively straightforward concept is only well-suited for point indexing, which may be a drawback in other regards. At the same time, any redundancy and ambiguity of the records is not a drawback in the context of spline approximation, but rather as an advantage, and, in fact, is indispensable [3]. This flexible approach (Fig. 4) allows for spline multiresolution technique usage in the areas in which it is required only, namely areas of high resolution horizontal records.

#### REFERENCES

- [1] F.W. Wiley et al., "Hierarchical Spline Approximation", CiteSeer.IST, 2003.
- [2] J. Demkowicz, M. Moszyński, A. Stepnowski, "Application of splines and wavelets along with TIN decimation to 3D imaging of seafloor from multibeam sonar data". Proceedings of the Sixth European Conference on Underwater Acoustics. ECUA 2002. Gdańsk, 24-27 June 2002.
- [3] N.K. Govil, "Frontiers in Interpolation and Approximation", Chapman&Hall/CRC, 2006.
- [4] M. Unser, "Splines. A Perfect Fit for Signal and Image Processing", IEEE Signal Processing Magazine, Nov. 1999.
- [5] R. H. Bartels, J. C. Beatty, and B. Barsky. An Introduction to Splines for Use in Computer Graphics and Geometric Modeling. Morgan Kaufmann, 1979.
- [6] A. Certain, J. Popović, T. DeRose, T. Duchamp, D. Salesin, and W. Stuetzle. Interactive multiresolution surface viewing. Computer Graphics (Proceedings of SIGGRAPH 96), 30:97–115, August 1996.
- [7] A. Finkelstein and D. H. Salesin. Multiresolution curves. Computer Graphics (Proceedings of SIGGRAPH 94), 28:261–268, July 1994. [9] D. R. Forshey and R. H. Bartels. Hierarchical B-spline refinement. Computer Graphics (Proceedings of SIGGRAPH 88), 22(4):205–212, August 1988.
- [8] D. R. Forshey and R. H. Bartels. Surface fitting with hierarchical splines. ACM Transactions of Graphics, 14(2):134–161, April 1995.
- [9] M. Lounsbery, T. D. DeRose, and J. Warren. Multiresolution analysis for surfaces of arbitrary topological type. ACM Transactions on Graphics, 16(1):34–73, 1997.
- [10] N. Johnson, Algorithm of the Week: Spatial Indexing with Quad-trees and Hilbert Curves.
- [11] Data - courtesy TD, from offshore surveys on North Sea.