Task Assignments in Logistics by Adaptive Multi-Criterion Evolutionary Algorithm with Elitist Selection

Jerzy Balicki

Gdańsk University of Technology,
Faculty of Electronics,
Telecommunications and
Informatics ul. Narutowicza 11/12,
81-230 Gdańsk, Poland
Email: balicki@eti.pg.gda.pl

Abstract— An evolutionary algorithm with elitist selection and an immunological procedure has been developed for Pareto task assignment optimization in logistics. A multi-criterion optimization problem has been formulated for finding a set of efficient alternatives. Some criteria have been applied for evaluation of solutions: bottleneck machine workload, a machine cost, and a system performance. Moreover, some numerical experiments have been performed and the machine constraints have been respected.

I. INTRODUCTION

Genetic algorithms, evolutionary algorithms and evolution strategies are the alternative evolutionary approaches to the other meta-heuristic multicriteria optimization methods such as simulated annealing, immunological systems [1, 6], tabu search [12], scatter search or Hopfield neural networks [4]. Evolutionary calculations process simultaneously a solution population, which permits finding a subset of P-optimal alternatives by one run as a replacement for several isolated runs of the other multiobjective optimization techniques [7]. From this reason evolution approaches are convenient, if we look for the subset of Pareto-optimal solutions [16, 17].

Experimental outcomes demonstrate that elitism can increase performance of multi-objective evolutionary algorithms radically [18]. Moreover, elitism avoids the damage of non-dominated alternatives, if they have been established. A concept of elitism for multi-criterion evolutionary algorithms is taken from evolution strategies regarding evolution strategy developed for combinatorial and multi-objective optimization problems [5]. The other evolution strategy with an archive for finding P-optimal solutions has been suggested in [13].

In this paper, a problem of task allocation in logistics has been verbalized as a combinatorial and multi-objective optimization question characterized by some partial criteria: a machine cost, a bottleneck machine workload, and the system performance. Moreover, three kinds of constraints have been considered. The first constraint sort is related with

the assumption that each task should be allocated to the machine, and the second one – with assumption one and only one machine should be allocated to a place of the task performing. Furthermore, the resource constraints are respected.

II. EVALUATION CRITERIA

We assume that some important logistic tasks are performed by some machines, automatically. A bottleneck machine is characterized by the heaviest logistic task load. A bottleneck workload is should be minimized as a critical factor that can balance a whole load among elements. The weight of the bottleneck machine is calculated, as below:

$$Z_{max}(x) = \max_{i \in 1, I} \left\{ \sum_{j=1}^{J} \sum_{v=1}^{V} t_{vj} x_{vi}^{task} x_{ij}^{p} + \sum_{v=1}^{V} \sum_{\substack{u=1 \\ u \neq v}}^{V} \sum_{i=1}^{I} \sum_{\substack{k=1 \\ k \neq i}}^{I} \tau_{ikvu} x_{vi}^{task} x_{uk}^{task} \right\}, \quad (1)$$

where

t_{vj} – a time of the overhead performing for the task number v by the machine sort number j,

τ_{ikvu} – a time of a resource transport between the task number v at the place w_i and the task number u at the place w_k.

$$x = [x_{11}^{task}, ..., x_{vi}^{task}, ..., x_{VI}^{task}, x_{11}^{p}, ..., x_{ij}^{p}, ..., x_{IJ}^{p}]^{T},$$

$$x_{ij}^{p} = \begin{cases} 1 \text{ if } p_j \text{ is assigned to the } w_i, \\ 0 \text{ in the other case.} \end{cases} i = \overline{1, I}, \; j = \overline{1, J},$$

$$x_{vi}^{task} = \begin{cases} 1 \text{ if task } T_v \text{ is assigned to } w_i, \\ 0 \text{ in the other case,} \end{cases} v = \overline{1, V}, i = \overline{1, I}.$$

The machine cost is determined regarding the formula, as below:

$$K(x) = \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_j x_{ij}^{\pi}, \quad (2)$$

where α_j is the machine cost for the sort number j.

The total machine performance is calculated, as follows:

$$\Theta(x) = \sum_{i=1}^{I} \sum_{j=1}^{J} \beta_j x_{ij}^p, \qquad (3)$$

where β_j is the logistic machine performance for its sort number j.

III. MUTIOBJECTIVE OPTIMIZATION PROBLEM

An optimal configuration of task in a logistic system that can be modeled as a task assignment may reduce the total cost of a set of tasks execution or the workload of bottleneck machine. It can decrease the cost of machines because of the machine sort selection, too. A total amount of system performance is another measure that can be maximized by task scattering and by the machine sort selection. An advantage of the system with an optimal task assignment may exceed 50% value of any criterion for a system with a task scattering designed without an optimization technique [2]. The bottleneck machine load is assessment criterion of the system configuration that causes the load balance and also minimizes a response time [3].

In that problem, the admissible solution satisfies three classes of constraints. Because each unit is allocated to one node, the logistic task allocation constraints are devised, as below:

$$\sum_{i=1}^{I} x_{vi}^{task} = 1, v = \overline{1,V}. \qquad (4)$$

We assume one and only one machine should be allocated at each node. It implies the machine allocation constraints, as follows:

$$\sum_{j=1}^{J} x_{ij}^p = 1, i = \overline{1,I}. \qquad (5)$$

Each machine provides some resource capacities to perform some assigned tasks. Let some resources $z_1,...,z_l,...,z_L$ be required in a logistic system. We introduce μ_{jl} to represent the lth resource capacity in the machine p_j. We assume the task number v holds η_{vl} units of z_l. The values μ_{jl} and η_{vl} are nonnegative and limited.

The resource capacity limit in any machine in the ith place cannot be exceeded, what can be written, as bellows:

$$\sum_{v=1}^{V} \eta_{vl} x_{vi}^{task} \leq \sum_{j=1}^{J} \mu_{jl} x_{ij}^p, \quad l = \overline{1,L}, \ i = \overline{1,I}. \qquad (6)$$

The multiobjective optimization problem may be established a triple $(\mathcal{X}, F, P)$ to find the Pareto representation of some optimal solutions, as follows [15]:

1) $\mathcal{X}$ - an admissible solution set

$$\mathcal{X} = \{x \in \mathcal{B}^{I(V+J)} | \sum_{j=1}^{J} x_{ij}^p = 1, i = \overline{1,I};$$

$$\sum_{i=1}^{I} x_{vi}^{task} = 1, v = \overline{1,V};$$

$$\sum_{v=1}^{V} \eta_{vl} x_{vi}^{task} \leq \sum_{j=1}^{J} \mu_{jl} x_{ij}^p, \quad l = \overline{1,L}, \ i = \overline{1,I}\}$$

$$\mathcal{B} = \{0, 1\}$$

2) f – a multi-objective optimization criterion

$$f : \mathcal{X} \rightarrow \mathcal{R}^3, \qquad (7)$$

where $f(x) = [K(x), \Theta(x), Z_{max}(x)]^T$, $x \in \mathcal{X}$

3) P - the relationship of optimization preferences [14]

IV. MUTIOBJECTIVE OPTIMIZATION ALGORITHMS

Some advanced evolutionary algorithms have been developed for several multi-objective optimization problems [8, 9, 11]. What is more, some of them have been tested for finding the set of Pareto-optimal task assignments [2, 3].

A ranking idea for non-dominated individuals was introduced to avoid the prejudice of the interior Pareto solutions [10]. Then, the first algorithm called NSGA with the ranking procedure has built on the ideas mentioned by Goldberg [15].

In a current population, some non-dominated individuals get a rank equal to 1. Then, the second level of non-dominated alternatives is assigned the rank 2. This assigning procedure is recurred until the population is preceded. It is worth to mention that all non-dominated individuals have the same reproduction fitness because of the equivalent rank.

Deb et al. have improved NSGA by introduction an elitist procedure [8]. In an evolutionary algorithm called NSGA-II, a selection of potential parents is based on a binary tournament. Both an offspring population and a parent population are combined to select a new parent population. If two chromosomes are characterized by the other ranks, it with smaller rank is preferred. If individuals have the same rank, there is preferred the logistic configuration of tasks in a less overcrowded region.

V. ADAPTIVE MULTI-CRITERION EVOLUTIONARY ALGORITHM WITH ELITIST SELECTION

Adaptive evolutionary algorithm with elitist selection called AMEA+ is noticed as an advanced optimization technique for multi-criterion logistic task assignment [3]. Figure 1 shows a diagram of AMEA+. The preliminary set of chromosomes is erected to satisfy constraints (4) and (5)

(Fig. 1, line 3). Generated individuals are constructed by integer coding, as below:

$$X = (X_1^{task},...,X_v^{task},...,X_V^{task}, X_1^{p},...,X_i^{p},...,X_I^{p}), \quad (8)$$

where $X_v^{task} = i$ if $x_{vi}^{task} = 1$ and $X_i^{p} = j$ if $x_{ij}^{p} = 1$. Besides, $1 \leq X_v^{task} \leq I$ and $1 \leq X_i^{p} \leq J$.

Fitness function is calculated for some admissible solutions (Fig. 1, line 4), as below:

$$F(x) = P_{max} - \lambda(x) + \lambda_{max} + 1, \quad (9)$$

where $\lambda(x)$ is the admissible solution rank, $1 \leq \lambda(x) \leq \lambda_{max}$.

---

1. BEGIN
2. $\varphi := 0$, enter ψ – the size of chromosome population, γ – the chromosome length, $p_m := (\psi \gamma)^{-1}$;
3. Create a preliminary set **Pop**(φ), **M**(φ):= **Pop**(φ);
4. Compute ranks $\lambda(x)$ and values of fitness
   $F(x), x \in \textbf{Pop}(\varphi)$
5. work:=TRUE
6. WHILE work DO
7.   BEGIN /* create a new population */
8. $\varphi := \varphi +1$, **Pop**$(\varphi) := \varnothing$
9.   Estimate probabilities $p_s(x), x \in \textbf{P}(\varphi-1)$
10.   FOR $\psi/2$ DO
11.   BEGIN /* reproduction cycle */
12.   2WT potential parent selection (**a,b**) from **P**$(\varphi$ -1**)**
13.   S-crossover of a pair (**a,b**) with the adaptive
    crossover rate $p_c := e^{-\varphi/T_{max}}$
14.   S-mutation of an offspring pair (**a',b'**) with p_m
15.   **P**(t):=**P**(t) $\cup$ (**a',b'**}
16.   END
17.   **P**(φ):= **M**(φ) $\cup$ **P**(φ)
18.   Compute ranks $\lambda(x)$ and values of fitness
    $F(x), x \in \textbf{P}(\varphi)$
19.   An elitist selection of ψ solutions with the largest $F(x)$ in **P(t)**; if more than ψ items have the same rank, use the crowd measure to select ψ solutions;
20.   IF $(\varphi \geq T_{max}$ OR **P**(φ) converges) THEN
    work:=FALSE
21. END
22. END

Fig. 1. A diagram of an adaptive multi-criteria genetic algorithm with elitist selection

The 2WT potential parent selection is the two-weight tournament because two times the roulette rule is made.

The chromosome crossover point is randomly selected and two offspring are formed regarding $p_c := e^{-\varphi/T_{max}}$. The first part of the parent **a** is concatenated with the second part of the parent **b**. Similarly, the first part of the parent **b** is concatenated with the second part of the parent **a**. Each pair of potential individuals is randomly chosen for crossover with the probability pc. If chromosomes are not taken for crossover, parents are transferred to a set of offspring. A crossover rate decreases during the progressing of evolution. So, the changes of a search area retire slowly.

The S-mutation is based on the integer random modification by another feasible value. For X_v^{task}, the set $\{1,...,I\}$ is considered, and for X_i^{p}, the set $\{1,...,J\}$ is adequate. A constant mutation rate is assigned.

A search space for the considered evolutionary algorithm consists of $I^{V}J^{I}$ elements. It can be proved that S-crossover and S-mutation give ability for obtaining each solution in the search space. So, we can expect to find non-dominated assignments after an exhausted search.

Some numerical experiments demonstrate that elitism may improve the quality of alternatives [13, 14, 18]. An improved elitist selection is carried out as follows. Let M(t) be an old population P(t-1) and P(t) be a new population created from M(t) by mating, crossover and mutation. Firstly, a sum of two populations M(t) $\cup$ P(t) is created. Secondly, ranks are calculated for an entire population M(t) $\cup$ P(t). Each non-dominated solution in the extended set is characterized by the fitness value regarding its rank. Finally, L solutions are qualified with the higher fitness f(x) to P(t).

Let T_{max} be a maximal number of new populations that is O(n), where n = max{I, V, J}, I is O(J), and V is O(J). Moreover, let a size L of population be O(n), too. Now, we can assess a complexity of the AMEA-II. Fitness (Fig. 1, line 18) is calculated $O(n^2)$ times what gives the complexity $O(n^6)$ for the AMEA-II applied to the multi-objective logistic problem.

VI. CONVERGENCE MEASURE

The convergence of the studied algorithm can be considered by measuring the quality of obtained logistic task assignments to the Pareto front. So, we introduce a closeness measure for the obtained efficient points to the given Pareto points $\{P_1, P_2,..., P_U\}$. They may be found by enumerative search for small instances. An evolutionary algorithm can find the set of sub-efficient points $\{A_1, A_2,..., A_{U'}\}$, where $U' \leq U$. If there is an outcome $A_u=(A_{u1}, P_{u2}, A_{u3})$ with the same cost of computers as the uth Pareto result $P_u=(P_{u1}, P_{u2}, P_{u3})$, then the distance between these points is equal to $\sqrt{(P_{u1} - A_{u1})^2 + (P_{u4} - A_{u4})^2}$. If A_u is missed by the AMEA+, then the distance $\sqrt{\sum_{\substack{i=1 \\ i \neq 2}}^{3} (P_{ui} - A_{ui}^{-})^2}$ is considered, where A_{u1}^{-} is the maximal load of the bottleneck machine, and A_{u3}^{-} is the minimal performance of machines, for the instance of the problem (7).

Some ξ initial populations are generated, and ξ values of the distances S_u^1 are calculated. A convergence criterion to the P-set is calculated, as below:

$$S = \frac{1}{\xi}\sum_{l=1}^{\xi}\sum_{u=1}^{U} S_u^1. \qquad (10)$$

The AMEA+ gives better results than AMEA without an elitist procedure. In the 200 iteration, S is 1.3% for the AMEA+ and 1.6% for the AMEA. Above algorithms have been run 30 times.

For the instance with 2 places, 10 tasks, and 5 machines types, 30 binary variables generate 1 073 741 824 solutions in the search space. Θ is a value from [200, 600], K is from [2, 10] [money unit], and Z_{max} is from [26; 75] [time unit].

A formula I ^{V}J ^{I} permits to calculate an upper bound of the number of an admissible set. Figure 2 displays the two criteria space of evaluations. The ideal point y^o and the anti-ideal point y^- can be used for finding a compromise solutions.
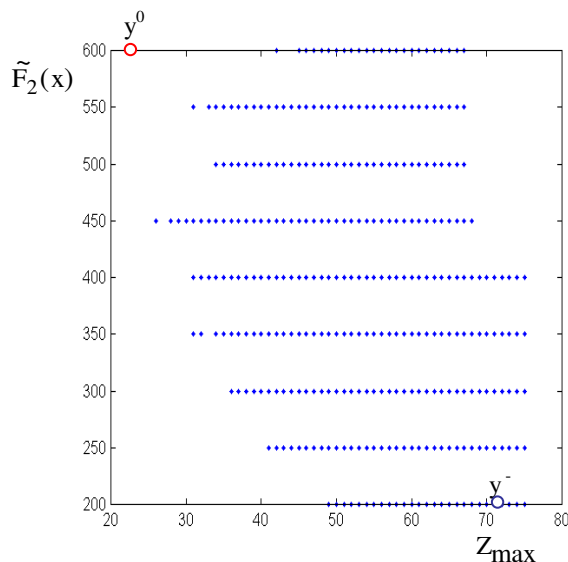

Fig. 2. Bi-criteria evaluation space

VII. CONCLUDING REMARKS

The AMEA+ is capable techniques for solving a multiobjective optimization problems focused on finding logistic task allocations that minimize the cost of machines, a workload of the bottleneck machine, and maximize performance of logistic system.

Every one of non-dominated solutions in the combined population is assigned a fitness based on the rank of solutions. Dominated solutions are assigned fitness worse than the worst fitness of any non-dominated solutions in a population. This assignment of fitness makes sure that the search is directed towards the non-dominated solutions, too.

Our future works will focus on finding the combination the multicriteria evolutionary algorithm with the immunological algorithm to include some ranking procedures to handle constraints and for improving the obtained Pareto-optimal task assignments.

References

1. Aragon V. S., Esquivel S. C.; Coello Coello C. A.: A modified version of a T-Cell Algorithm for constrained optimization problems. International Journal for Numerical Methods in Engineering, Vol. 84, Issue: 3, 2010, pp. 351-378.
2. Balicki J.: An Adaptive Quantum-based Multiobjective Evolutionary Algorithm for Efficient Task Assignment in Distributed Systems. Proceedings of The 13th WSEAS International Conference on Computers The 13th WSEAS Multiconference on Circuits, Systems, Communications and Computers, July22-26, 2009, Rodos Island, Greece, WSEAS Press, pp. 417-422
3. Balicki J.: Negative selection with ranking procedure in tabu-based multi-criterion evolutionary algorithm for task assignment. In: V.N. Alexandrov et al. (Eds.): Computational Science – ICCS 2006, Proceedings of the 6th International Conference on Computational Science, Reading, UK, May 28-31, 2006, Lecture Notes in Computer Science, vol. 3993, Springer-Verlag, Berlin Heidelberg New York, 2006, pp. 863-870
4. Balicki J., Kitowski Z., Stateczny A.: Extended Hopfield Model of Neural Networks for Combinatorial Multiobjective Optimization Problems. Proceedings of 1998 IEEE World Congress on Computational Intelligence, Anchorage, May 4-9, 1998, Vol. 2, pp. 1646-1651.
5. Binh T. T., Korn U., (1997) Multiobjective evolution strategy for constrained optimization problems. Proceedings of the 15th IMACS World Congress on Scientific Computation, Modelling and Applied Mathematics, Berlin, 357-362.
6. Canova A., Freschi F.: Multiobjective design optimization and Pareto front analysis of a radial eddy current coupler. International Journal of Applied Electromagnetics and Mechanics, Vol. 32, Issue: 4, 2010, pp. 219-236.
7. Coello Coello C. A., (1999) A comprehensive survey of evolutionary-based multiobjective optimization techniques, Knowledge and Information Systems. An International Journal, **1**, 269-308.
8. Deb K., Agrawal S., Pratap, A., Meyarivan T. (2000) A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. Technical Raport No. 200001, Indian Institute of Technology, Kanpur.
9. Freschi F., Repetto M.: VIS: An artificial immune network for multi-objective optimization. Engineering Optimization, Vol. 38 Issue: 8, 2006, pp. 975-996.
10. Goldberg D. E., (1989) Genetic algorithms in search, optimization, and machine learning, Addison-Wesley Publishing Company, Inc. Reading, Massachusetts.
11. Hiremath N. C., Sahu S., Tiwari M. K.: Designing a Multi Echelon Flexible Logistics Network using Co-evolutionary Immune-particle Swarm Optimization with Penetrated Hyper-mutation (COIPSO-PHM). Proc. of the 2nd International Conference on Mechanical and Aerospace Engineering, Bangkok, July 29-31, 2011, Book Series: Applied Mechanics and Materials, Vol. 110-116, 2012, pp. 3713-3719.
12. Jozefowska J., Waligora G., Weglarz J., (1998) Tabu search with the cancellation sequence method for a class of discrete - continuous scheduling problems. Fifth International Symposium on Methods and Models in Automation and Robotics, Miedzyzdroje, 1047 - 1052.
13. Knowles J., Corne D. W., (2000) Approximating the nondominated front using the Pareto archived evolution strategy, Evolutionary Computation, **8**, 2, 149-172.
14. Sinha A. K., Zhang W. J., Tiwari M. K.: Co-evolutionary immuno-particle swarm optimization with penetrated hyper-mutation for distributed inventory replenishment. Engineering Applications of Artificial Intelligence, Vol. 25, Issue: 8, 2012, pp. 1628-1643.
15. Srinivas N, Deb K., (1994) Multiobjective optimization using non-dominated sorting in genetic algorithms, Evolutionary Computation, **2**, 3, 221-248.

16. Ulungu E.L., Teghem J., (1994) Multi-objective combinatorial optimization problems: A survey, *Journal of Multi-Criteria Decision Analysis,* 3, 83-104.

17. Van Veldhuizen D. V., Lamont G. B., (2000) Multiobjective evolutionary algorithms: Analyzing the state-of-the-art, *Evolutionary Computation,* 8, 2, 125-147.

18. Zitzler E., Deb K., Thiele L., (2000) Comparison of multiobjective evolutionary algorithms: Empirical results, *Evolutionary Computation,* 8, 2, 173-195.