Journal of Polish CIMEEAC
Faculty of Ocean Engineering & Ship Technology
GDAŃSK UNIVERSITY OF TECHNOLOGY

THE PROBLEM OF THE CALCULATION OF THE FREQUENCY OF DIAGNOSTIC EXAMINATIONS BASED ON DEVICE'S PROPER OPERATION TIME

Jacek Rudnicki

Politechnika Gdańska
Ul. Narutowicza 11/12, 80-233 Gdańsk, Polska
Tel.: +48 58 3472773, fax: +48 58 3472430
e-mail: jacekrud@pg.gda.pl

Abstract

The paper presents the proposal to apply the normal distribution to solve the problem of the frequency of diagnostic tests. Particular emphasis is placed on simplicity of the method. This method may be useful for the average user technical system. The method reduces the number of assumptions to a minimum. The results do not raise of serious doubts but they require verification of course.

Keywords : *reliability, diagnostics, Gaussian random variable*

1. Introduction

During the exploitation of many mechanical devices (including the devices of engine room), many times has it been proven that the duration of a proper operation of these objects is not an unequivocal measure of their wear [3], [4], [5], [6].

Insomuch as, one can assume that the hypothesis *"the process of exploitation of a considered technical object, which condition at whichever moment t_n ($n = 0, 1, ..., m$; $t_0 < t_1 < ... < t_m$) depends on the condition directly preceding and does not depend stochastically on conditions occurring before and their duration"* is true. This hypothesis explains the fact observed in real life, that is forecasting the duration of proper operation of devices accurately enough for practical purposes, knowing only their current technical condition and material-energetic resources.

Ipso facto, permanent control of the technical condition of eg. ship's energetic system, should be a part of any exploitation controlling system for security, economy and ergonomy purposes [3].

For the situation to be possible, it is crucial to fulfill some essential conditions, including:

- monitoring (continuous or periodic) of diagnostic parameters, enabling the record and creating a database of analyzed device;
- creation of a diagnostic model appropriate for kind and amount of registered diagnostic parameters;
- creation of exploitation process model that considers the results of processed diagnostic analysis, ie. diagnosis and perchance prognosis;
- creation of decision-making model.

Schematically, the information flow in exploitation controlling system considering the aforementioned factors, can be presented as [7]:

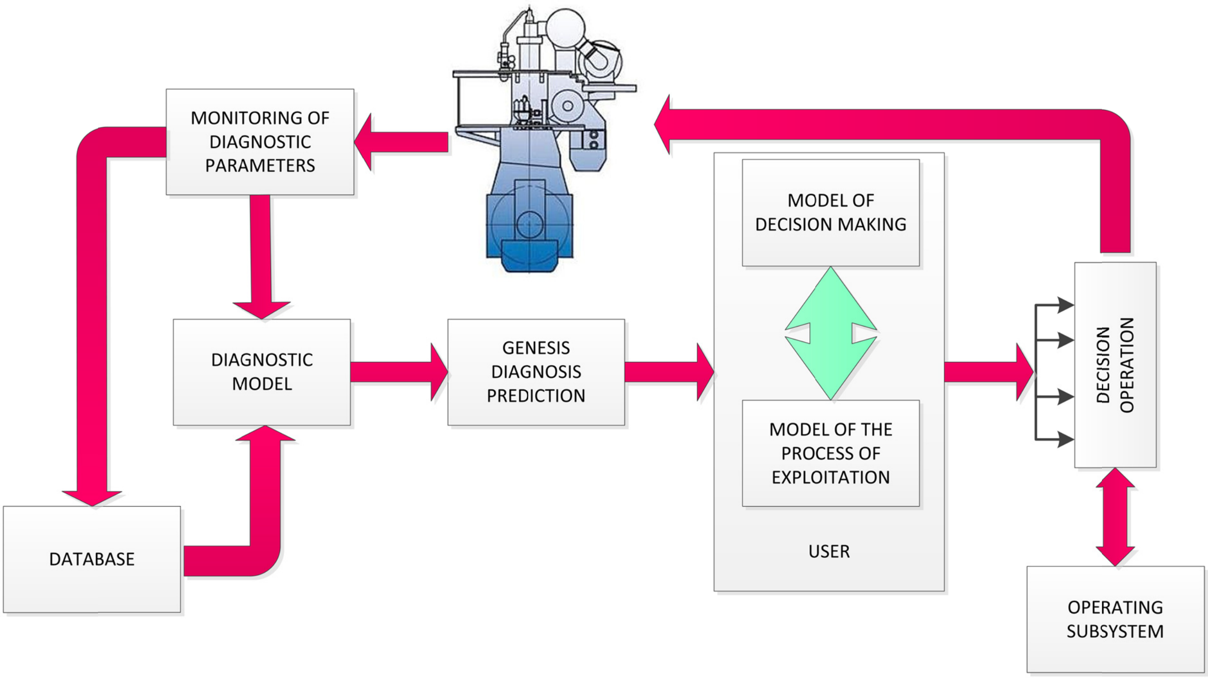

Fig.1 The information flow in exploitation controlling system

The information flow presented in the fig. 1 in the exploitation system enables the implementation of sub-goals which combined enable the implementation of the main goal, ie. the implementation of exploitation strategy according to technical condition.

The validity of this process causes, in general, no doubts, however, it creates high costs due to implementation and maintenance of a stationary diagnosing system. Financial constraints in this area prevent this scheme from being implemented, ie. permanent control of technical condition.

Implementation of elements of exploitation strategy using technical condition is however possible, causing significantly lower costs – using mobile, all-purpose diagnostic systems. Such systems and their qualified operators, can provide appropriate information that help making rational exploitation decisions, based on current technical condition of analyzed object, eg. the ship's main propulsion engine.

The forementioned situation limits, but does not eliminate additional financial exploitation costs. Therefore, the problem is the calculation of the rational frequency of the examinations that in the light of random character of damages does not depend directly on exploitation duration.

One of solutions of this problem can be using the distribution of random variables describing the time of device's proper operation time.

2. Theoretical distributions of random variables as reliability models of technical objects

The description of reliability properties is possible using theoretical distributions of random variables. In such cases the object is considered as a system of known structure. The proper operation time of particular sub-systems is described by random variables with density distribution depending on dominant, physical character of the damages.

Amongst many models of this type, two special are gamma and normal (Gaussian) distributions.

– **gamma distribution**

Density function of a random variable is denoted by [2]:

$$f(t) = \frac{b^g}{\Gamma(g)} t^{g-1} \exp(-bt) \tag{1}$$

where:

b – scale parameter,
g – shape parameter

Gamma distribution is useful in description of the proper operation time of devices when [1], [2]:

- objects in the inital moment ($t_0 = 0$) are homogeneous, which means the differences between crucial, selected parameters are irrelevant;
- mean wear velocity is constant;
- wear velocity is subject only to random fluctuations.

The property of this model is that it concerns the damages caused by accumulating damaging stimuluses (as a result of wear), assuming linearity of wear process. As an example of such wear model and the resulting damages may serve surface wear, such as abrasive wear, i.e. rolling and plain bearings, gears and volume wear involving rupturing of pieces as a result of accumulating strains and e.t.c.

- normal distribution (Gaussian)

Density function of a random variable is denoted by [2]:

$$f(t) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2}\frac{(t-\mu)^2}{\sigma^2}\right] \tag{2}$$

where:

μ – expected value,
σ – standard deviation.

Gaussian distribution is useful in description of the proper operation time of devices when similar to gamma distribution conditions are met. The fact that random variable of normal distribution can take negative values (while the proper operation time – only positive values) can be a limitation. However, if the random factor:

$$\frac{\sigma}{\mu} < \frac{1}{3}\ , \tag{3}$$

the probability of the random variable T taking negative values is negligibly small, which allows using this distribution [2], [8].

If the condition (3) is not met a normal distribution N(μ, σ) of random variable T cut in point t=0 having density function [2] is to be used:

$$f(t) = \frac{c}{\sigma \cdot \sqrt{2 \cdot \pi}} \exp\left[-\frac{(t-\mu)^2}{2 \cdot \sigma^2}\right] \tag{4}$$

where:

$$c = \frac{1}{1 - \Phi\left(-\frac{\mu}{\sigma}\right)}$$

It is important that the normal distribution cab be applied to describe reliability instead of the gamma distribution, assuming that shape parameters of this distribution are large enough (g > 12) [2]

3. Applying the normal distribution to determine diagnostic tests frequency

Because of universality and commonness of the normal distribution it can be used to determine diagnostic tests frequency, under certain conditions. The conditions are:

1. Based on 3σ rule it can be concluded with the probability of 99,73% that the proper operation time of the device will be included in [μ - 3σ, μ + 3σ] interval.
2. By designating μ - 3σ = t_{min} and μ + 3σ = t_{max} with the probability of 99,73% it can be concluded that the proper operation time of the device will be included in [t_{min}, t_{max}] interval.
3. From practical point of view the probability of 99,73% can be taken as a probability of a certain event.
4. The probability of the proper operation time of the device in [t_{min}, t_{max}] interval can be denoted (assuming condition 4) by P(t = t_{zd}) = 1- F(t_{zd}), where F(t_{zd}) is cumulative distribution of N(μ, σ).
5. A μ estimation can be taken as:
    a. The arithmetic average $\hat{t}_{śr}$ of the empirical reliability test results, if available.
    b. If such results are unavailable, the period of time between repairs of designated construction node of the device can be determined based on technical manual of the device.
6. A σ estimation can be taken as:
    a. The standard deviation $\hat{\sigma}$ of the empirical reliability test results, if available.

b. If such results or any other information is unavailable – the maximal acceptable value of this parameter, which is $\frac{\mu}{3}$.

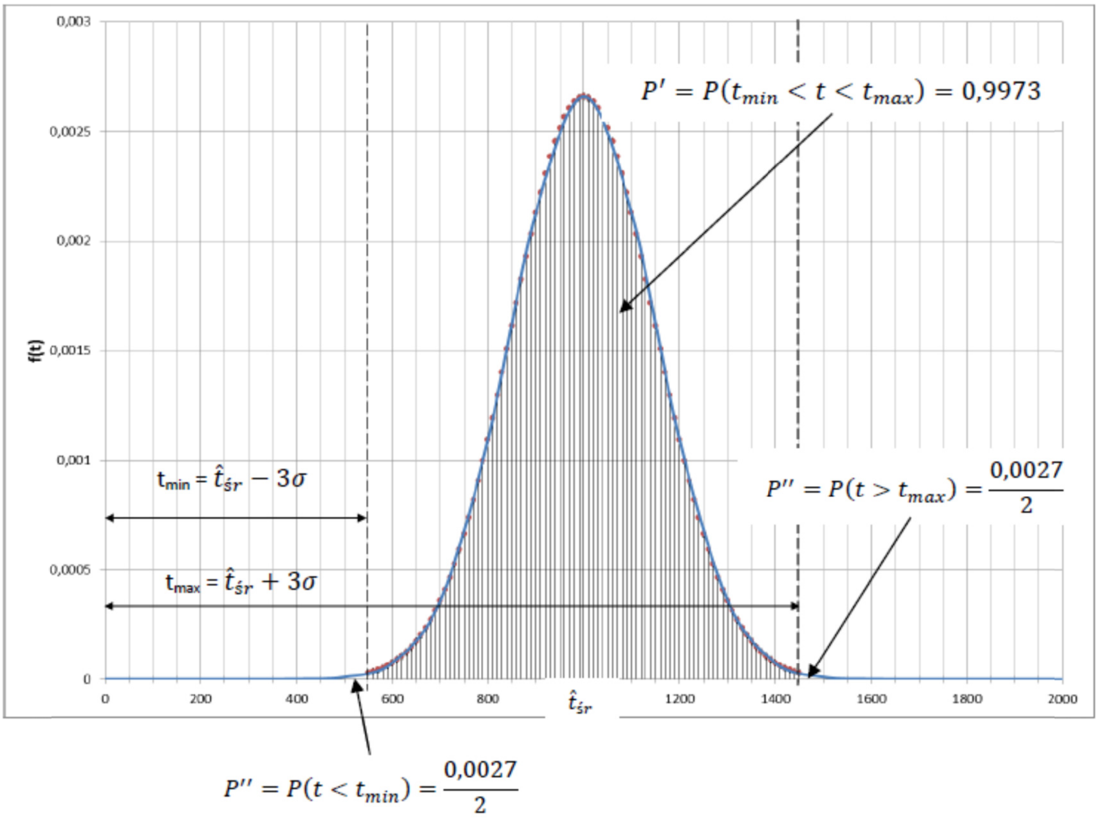

Fig. 2 The density function of the Gaussian distribution - 3σ rule

For a practical use of commonly known relationships shown in the fig. 2, with no need of using normal distribution density function tables, one can transform the graph shown in the fig. 2 into a pie chart shown in the fig. 3.

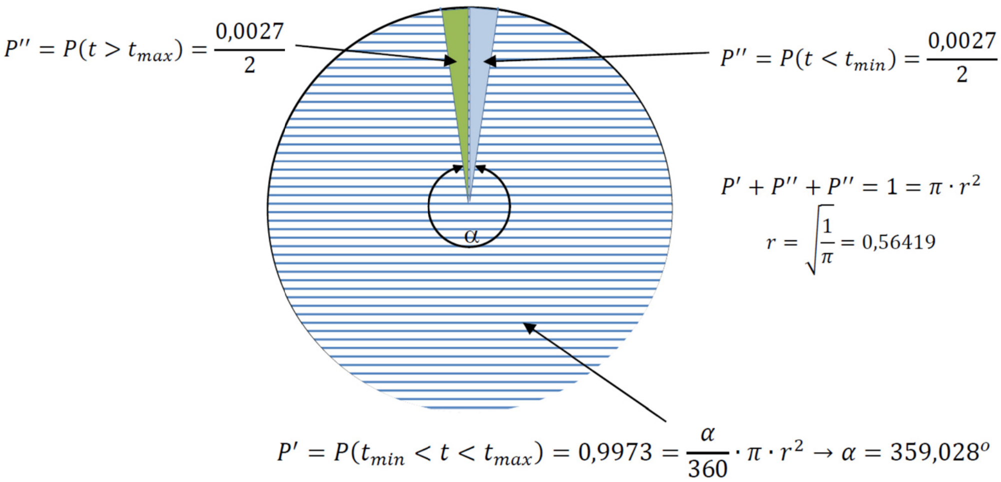

Fig. 3The principle of transformation of probability density Gaussian distribution

Using presented chart for estimation of time remaining to the beginning of diagnostic process proceeds in two stages shown below.

- Stage I

First of all, required value of probable proper operation time of analyzed object has to be established P_{zd} ($T \geq t_{zd}$), where:

T – random variable describing proper operation time;

t_{zd} – established time of object's proper operation.

Based on fig. 2 and 3, this probability is represented by equation below:

$$P_{zd}(T \geq t_{zd}) = 1 - [(P_{nzd}(T < t_{zd}) + P''(T < t_{min})] \tag{5}$$

assuming that probabilities P_{nzd} and P" are symbolically represented by areas of corresponding pie chart fragments shown in the fig. 4. In the same figure the angle and arc length corresponding to P_{nzd} - α_{zd}, l_{zd}.

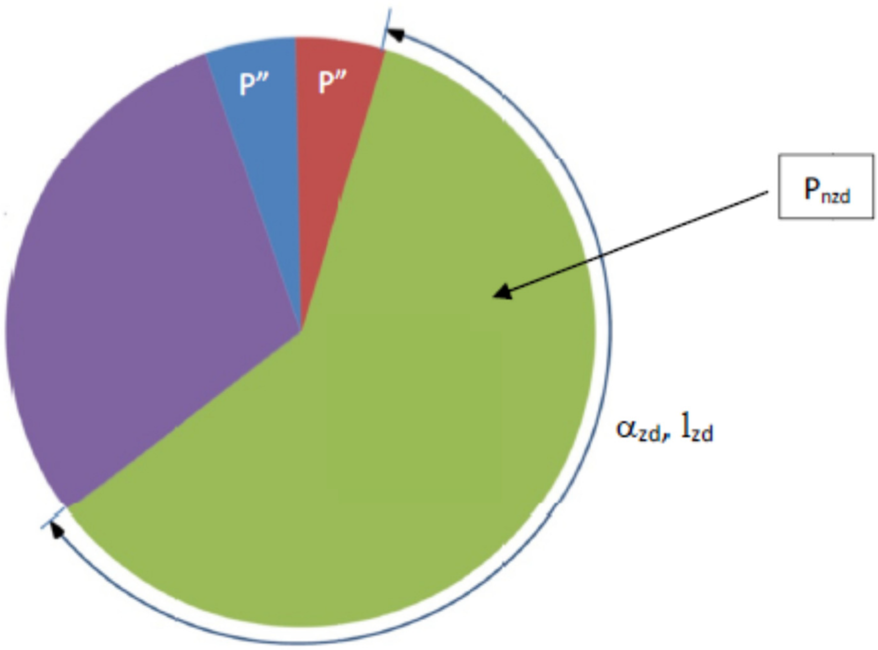

Fig. 4 Graphical illustration of the equation (5)

Equation (5) can be written:

$$\frac{\alpha_{zd}}{360} \cdot \pi \cdot r^2 + P" = 1 - P_{zd} \tag{6}$$

The relationship (6) allows to specify α_{zd} angle:

$$\alpha_{zd} = \frac{360 \cdot (1 - P_{zd} - P")}{\pi \cdot r^2} = \frac{360 \cdot (0{,}99865 - P_{zd})}{1} = 360 \cdot (0{,}99865 - P_{zd}), \tag{7}$$

and corresponding arc length l_{zd}:

$$l_{zd} = \frac{\alpha_{zd}}{360} \cdot 2 \cdot \pi \cdot r = \frac{\alpha_{zd}}{360} \cdot 3{,}5449 \tag{8}$$

- Stage II

The final procedure is rescaling the results acquired in stage I into real exploitation time values corresponding to accepted t_{min}, t_{max}, $\hat{t}_{śr}$ and σ values (fig. 2).

This operation can be presented in a way shown in the fig. 5.

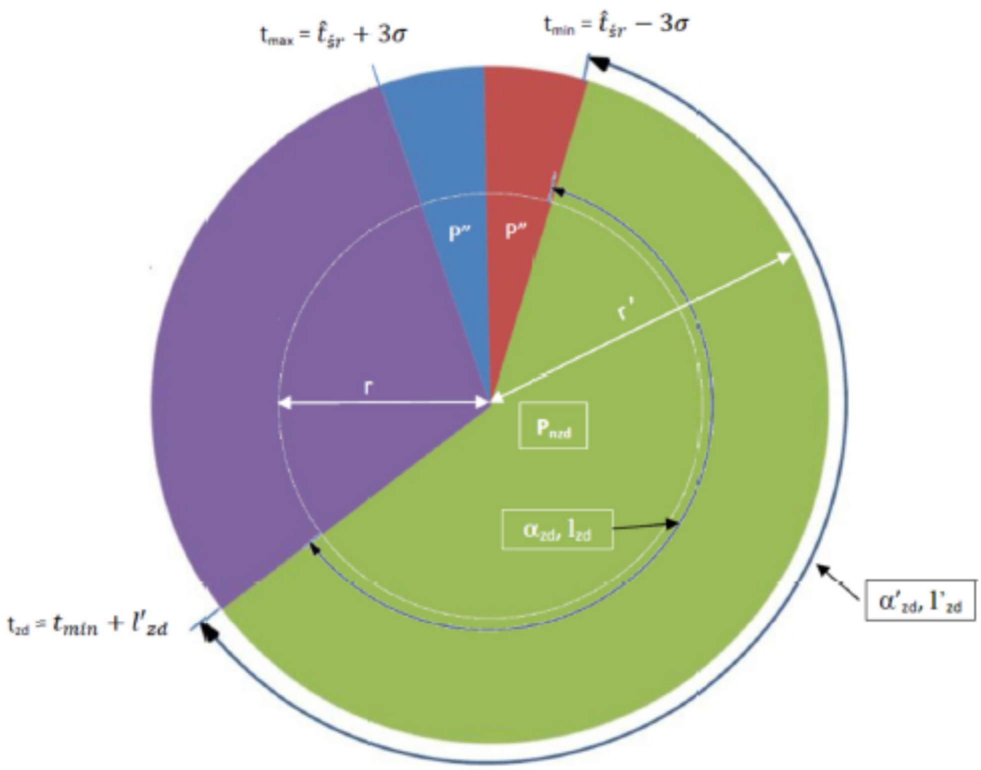

Fig. 5 Rescaling the results acquired in stage I into real exploitation time values

The angle α = 359,028° (fig. 3) corresponds to both l and l' arc length, wherein:

$$l = \frac{\alpha}{360} \cdot 2 \cdot \pi \cdot r - \text{fig. 3 i 4}$$
$$l' = \frac{\alpha}{360} \cdot 2 \cdot \pi \cdot r' - \text{fig. 5} \qquad (9)$$
$$l' = (\hat{t}_{śr} + 3\sigma) - (\hat{t}_{śr} - 3\sigma) = 6\sigma$$

Based on equation (9), radius r' can be calculated:

$$r' = r \cdot \frac{l'}{l} = \frac{360 \cdot 3 \cdot \sigma}{\alpha \cdot \pi}$$

Knowing the value of radius r' after taking into consideration the relationship $\alpha_{zd} = \alpha'_{zd}$ create a ratio:

$$\frac{l_{zd}}{l'_{zd}} = \frac{r}{r'} \rightarrow l'_{zd} = l_{zd} \cdot \frac{r'}{r} = \frac{360 \cdot (0{,}99865 - P_{zd})}{360} \cdot 3{,}5449 \cdot \frac{\frac{360 \cdot 3 \cdot \sigma}{\alpha \cdot \pi}}{0{,}56419} =$$
$$= (0{,}99865 - P_{zd}) \cdot 6{,}016 \cdot \sigma \qquad (10)$$

Finally, it can be written that time before which diagnostic examinations should be started with assumed probability of task completion P_{zd} and specified standard deviation is:

$$t_{zd} = t_{min} + l'_{zd} = (\hat{t}_{śr} - 3\sigma) + (0{,}99865 - P_{zd}) \cdot 6{,}016 \cdot \sigma \qquad (11)$$

4. Calculation example

Based on equation (11), a relatively simple solution can be presented for the problem which is the subject of the paper. The solution requires only three values, precisely:

- Required probability value of proper operation time of the device - $P_{zd;}$
- Mean time of proper operation time of the device mentioned - $\hat{\boldsymbol{t}}_{śr}$;
- Standard deviation of the random variable describing the proper operation time of the device - as it has been mentioned before, when there are no empirical data or any other information – maximal, acceptable value of this parameter can be assumed, that is $\frac{\mu}{3} \approx \frac{\hat{t}_{śr}}{3}$

It was assumed sample values:

- $\hat{t}_{śr} = 1000\ h$
- Standard deviation σ = 150 h

Results obtained have been presented in the fig. 6.

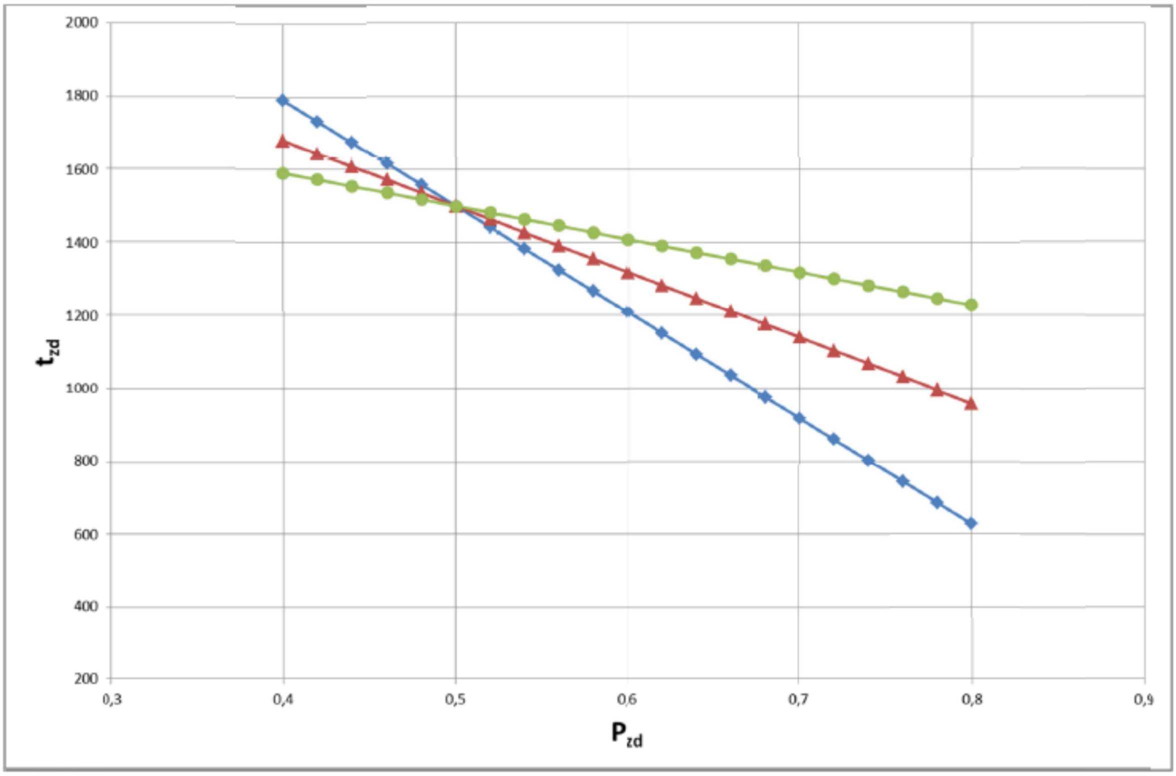

Fig. 6 The results obtained for $\hat{t}_{śr} = 1000\ h$ *and* $\sigma = 150\ h$

5. Summary

The presented method seems to be a proper supplement to the ways of description of reliability features of complex technical systems - for example: main ship propulsion engine, the crucial subsystem of ship power plant.

Credibility of the method should be verified during empirical research naturally.

The principal advantage of this method is its simplicity. An additional advantage is its versatility which makes that the method in question may be applied to reliability analysis of any device or energy subsystem where there are grounds for the application of the normal distribution.

6. Bibliography

[1] Benjamin J.R., Cornell C.A.: *Rachunek prawdopodobieństwa, statystyka matematyczna i teoria decyzji dla inżynierów*. WNT, Warszawa 1977.

[2] Girtler J.: *Modele matematyczne w badaniach niezawodności silników o zapłonie samoczynnym*. Konferencja Naukowo-Techniczna "Utrzymanie silników o zapłonie samoczynnym w ruchu" EXPLO-DIESEL'98. Instytut Technicznej Eksploatacji Siłowni Okrętowych WSM w Szczecinie, Szczecin 1998.

[3] Girtler J.: *Sterowanie procesem eksploatacji okrętowych silników spalinowych na podstawie diagnostycznego modelu decyzyjnego*. Zeszyty Naukowe AMW, nr 100A, Gdynia 1989.

[4] Girtler J.: *Możliwości zastosowania i przydatność procesów semimarkowskich jako modeli procesów eksploatacji maszyn*. Zagadnienia Eksploatacji Maszyn, z. 3/1996, s. 419-428.

[5] Okręglicki W., Łopuszyński B.: *Użytkowanie urządzeń mechanicznych*. WNT, Warszawa 1980.

[6] Rudnicki J.: *Model niezawodnościowo – funkcjonalny okrętowego, tłokowego silnika spalinowego*, 2000. Rozprawa doktorska (11.03.2000). Politechnika Szczecińska, Wydział Techniki Morskiej.

[7] Rudnicki J.: *Identyfikacja procesu eksploatacji okrętowych urządzeń i systemów energetycznych w aspekcie potencjalnych możliwości opracowania jakościowych i ilościowych modeli matematycznych tego procesu*. Praca wykonana w ramach projektu finansowanego przez MNiSW Nr N509 045 31/3500. Projekt badawczy pt.: „Kształtowanie bezpieczeństwa działania systemów energetycznych środków transportowych na przykładzie systemów okrętowych". Gdańsk, 2007.

[8] Żółtowski J.: *Wybrane zagadnienia z podstaw konstrukcji i niezawodności maszyn*. Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2004.