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VESSEL ENERGY REQUIREMENT PREDICTION FROM ACCELERATION STAGE TOWING TESTS ON SCALE MODELS

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ABSTRACT

One of the most crucial tasks for naval architects is computing the energy required to meet the ship's operational needs. When predicting a ship's energy requirements, a series of hull resistance tests on a scale model vessel is carried out in constant speed stages, while the acceleration stage measurements are ignored. Another important factor in seakeeping analysis is the ship's hydrodynamic added mass. The second law of dynamics states that all this valuable information, that is, the dependence of the hull resistance on the vessel's speed and the added mass, is accessible from just one acceleration stage towing test done up to the maximum speed. Therefore, the acceleration stage, often overlooked in traditional towing experiments, can be a valuable source of information. For this reason, this work aims to generalise Froude's scaling procedure to full-scale vessels for the accelerated stage towing tests.

Keywords: ships energy requirement, towing tank tests, acceleration stage, hydrodynamic added mass

INTRODUCTION

In 1870, W. Froude initiated an investigation into ship resistance with the use of vessel models. The resistance is the horizontal component of the force opposing the forward motion of a vessel's hull. Froude noted that the wave configurations around geometrically similar forms were similar if compared at speeds proportional to the square root of the model length. He propounded that the total resistance could be divided into skin friction resistance and residuary – mainly wave-making – resistance. The specific residuary resistance would remain constant at corresponding speeds between the model and the full-scale vessel. Next, estimates of frictional resistance from a series of measurements on planks of different lengths and with different surface finishes were derived [1]. The scaling procedure proposed by Froude was based on towing experiments at constant speed on the model vessel. Further, in 1874 Froude carried out full-scale tests on HMS Greyhound (100 ft), and the results showed substantial agreement with the model predictions [2]. Finally, in 1877 he gave a detailed explanation of wave-making resistance, supporting his scaling methodology [3]. Froude's ideas still dominate this subject.

Nowadays, when predicting ships' energy requirements, resistance tests on model vessels are still conducted. For constant speed, the resistance is determined by towing force measurements. In the next step, the resistance test results are scaled from the model to the full-scale ship. A modification of Froude's scaling method by splitting the residual resistance into the form resistance and the wave-making resistance, suggested by Hughes [4], and known as the form factor (1+k) approach, was later adopted by the International Towing Tank Conference (ITTC).

The scaling procedures, as mentioned above, refer to towing tests at constant speed. Another important aspect is the derivation of the hydrodynamic added mass of the ship, which may account for up to 30% of the ship's mass and therefore represents significant inertia for the accelerated motion. It follows from the laws of dynamics that all this information, i.e., the dependence of the resistance on the speed and the added mass, is accessible from the acceleration stage towing test done up to the maximum speed. The measuring apparatus in the 19th century did not allow Froude to conduct his research on the acceleration stage with the same level of precision as is possible now. Despite the development, great accuracy, and sampling rate of measurements, however, the author is unaware of any scaling procedures from the acceleration stage towing tests. Therefore, this paper derives a dynamical scaling proposition for the propulsion force needed to estimate the full-scale ship's energy consumption. A fully dynamic model can simulate any profile: constant speed, acceleration, deceleration, and gliding. Moreover, such an approach allows for optimisation of the required energy based on dynamical systems, which is especially desired for short-range vessels, where constant speed is not the major stage [5].

This paper is organised as follows. First, all the components of the ship's resistance in accelerated motion, with some historical background, are introduced. Next, the standard scaling procedure for the constant speed towing tank tests is described because most of the methodology used for the scaling from the accelerated motion tests is the same. Finally, a proposition for the towing tank tests in the accelerated stage and a scaling procedure for such tests are explained.

NEWTON'S SECOND LAW OF DYNAMICS FOR TOWING TESTS

To explain the concept of the proposed scaling procedure from the acceleration towing tank tests, let us start with Newton's second law of dynamics, which for any vessel takes the following form:

$$mv' = F_P(v, v') - R_T(v)\ . \qquad (1)$$

Here m stands for the total mass, which is the sum of the mass of the vessel m_v and the hydrodynamic added mass of the water m_{add}, i.e.,

$$m = m_v + m_{add}\ . \qquad (2)$$

In general, the added mass is a second-order tensor relating the fluid acceleration vector to the resulting force vector on the body. Only the surge added mass is taken into consideration in this work. Further, in formula (1), v' denotes the speed derivative over time, F_P is the propulsion force for the full-scale vessel, or the towing force in the case of the model vessel, and R_T is the total hull resistance force.

In the case of constant speed, i.e. $v' = 0$, the towing force is equal in magnitude to the total hull resistance, and the second law takes the following form:

$$F_P(v, 0) = R_T(v). \qquad (3)$$

Then, after rewriting (1), we get

$$F_P(v, v') = F_P(v, 0) + (m_v + m_{add})\ v'. \qquad (4)$$

Formula (4) shows that, from the acceleration stage towing tests, which give data $F_P(v, v')$, information on both the total hull resistance dependence on constant speed $F_P(v, 0)$ and the hydrodynamic added mass m_{add} are accessible.

THE ADDED MASS

In 1786 Du Buat found by experiment that the motion of spheres oscillating in water could only be described if an added mass was included in the equations of motion. In fluid mechanics, the added mass is defined as an extra fluid mass that accelerates with the body. It is the inertia added to a system because the accelerating body, to pass through, must move aside and then close in behind a specific volume of the surrounding fluid. The fluid thus possesses kinetic energy that it would lack if the immersed body were not in accelerated motion. The body has to impart this kinetic energy to the fluid by doing work on the fluid. Any corresponding equations of motion for the immersed body must take into account this loss of kinetic energy. This can be modelled in the equations of motion as some volume of fluid moving with the object although, in reality, the fluid will be accelerated to varying degrees. When the body moves at a constant speed, the corresponding motion of the fluid is steady; thus, the kinetic energy of the fluid is constant. It follows that for constant-speed motion, the added mass terms can be omitted in the equations of motion [6].

The added mass depends on the size and shape of the immersed body, the direction in which it moves through the fluid with respect to its axis, and the density and viscosity of the fluid. It can be described by a dimensionless coefficient which depends on the shape of the immersed body. The dimensionless added mass coefficient C_M is the added mass divided by the displaced fluid mass [7]; that is, divided by the fluid density ρ times the volume of the body under water V; therefore

$$m_{add} = C_M \rho V. \qquad (5)$$

The same principles apply to ships. In the marine sector, added mass is referred to as hydrodynamic added mass. The hydrodynamic added mass has also been investigated in the maritime area. Motora first conducted model testing for a ship called Mariner to predict the added mass [8]. Ghassemi and Yari proposed a numerical calculation of the marine propeller's added mass using the boundary element method [9]. Zeraatgar et al. investigated the surge added mass of planing hulls by model vessel experiments and by approximations with a quasi-analytical method [10]. The conclusion was that the surge added water mass could account for 10% of the total mass for the investigated planing hulls.

Essentially for ships, the added mass can reach even one-third of their mass, representing significant inertia in addition to the viscous and wave-making drag forces. Thus, the energy required to accelerate the added mass should also be considered when performing a seakeeping analysis.

When conducting a towing test in the acceleration stage, the surge added mass can be obtained from the equations of motion by extrapolating the towing force to zero speed $F_P(0, v')$, i.e.,

$$(m_v + m_{add})v' = F_P(0, v') - R_T(0). \tag{6}$$

Then the total hull resistance can be neglected, $R_T(0) = 0$, and it follows that

$$m_{add} = \frac{F_P(0, v')}{v'} - m_v. \tag{7}$$

THE TOTAL HULL RESISTANCE

Even in calm water, a ship experiences the water's resistance to its motion. This force is referred to as the total hull resistance R_T. This resistance force is needed to calculate the ship's effective power. Many factors combine to form the total resistance force acting on the hull. The physical factors affecting ship resistance are the friction and viscous effects of the water acting on the hull and the energy required to create and maintain the ship's characteristic bow and stern waves. Finally, a minor contribution is made by the resistance that the air provides to the ship's motion. This may be written in the following form:

$$R_T = R_V + R_W + R_A, \tag{8}$$

where R_T is the total hull resistance, R_V is the viscous friction resistance, R_W is the wave-making resistance, and R_A stands for the air resistance.

The total hull resistance R_T can also be formulated by means of the dimensionless total resistance coefficient C_T with the following equation:

$$R_T = \frac{1}{2} C_T \rho S v^2. \tag{9}$$

Here ρ is the water density, S is the wetted surface area of the underwater hull, and v is the speed of the vessel.

As the total hull resistance R_T is the sum of the viscous R_V and wave-making R_W resistance, when neglecting the air resistance, one can write an equation for the total dimensionless resistance coefficient in terms of the viscous and wave-making coefficients, such that

$$C_T = C_V + C_W, \tag{10}$$

where C_T is the coefficient of the total hull resistance, C_V is the coefficient of the viscous frictional resistance, and C_W is the wave-making resistance coefficient.

To quantify these dimensionless resistance coefficients, two numbers are used. The Reynolds number Re quantifies the influence of viscous forces on the fluid's motion. It indicates the ratio of inertial to viscous forces and, for the ship, is defined as a dimensionless ratio

$$\mathscr{Re} = \frac{\mathrm{v}\rho L}{\mu} = \frac{\mathrm{v}L}{\nu}, \tag{11}$$

where v is the vessel's speed, L is the length of the wetted surface, μ is the dynamic viscosity, and ν is the kinematic viscosity.

The Froude number Fr, in hydrology and fluid mechanics, is used to quantify the influence of gravity on a fluid's motion. It indicates the ratio of the inertia forces to the gravitational forces related to the mass of water displaced by a floating vessel. It is defined by a dimensionless ratio:

$$\mathscr{Fr} = \frac{\mathrm{v}}{\sqrt{gL}}. \tag{12}$$

Here g denotes the gravity acceleration. Then, the relationship between these two numbers can be written in the following form, which is practical for scaling purposes:

$$\mathscr{Re} = \frac{\rho}{\mu}\sqrt{g}\, L^{1.5}\, \mathscr{Fr}. \tag{13}$$

THE VISCOUS RESISTANCE

Although water has low viscosity, it produces a significant friction force opposing the ship's motion. The viscous resistance R_V is made up of the skin friction resistance and the viscous pressure resistance. Experimental data have shown that water friction can account for most of the hull's total resistance at low speeds and is still dominant for higher speeds [11]. The ship's hull shape influences the magnitude of the viscous pressure drag. Vessels with a lower length-to-beam ratio will have greater drag than those with a higher length-to-beam ratio.

The dimensionless viscous coefficient C_V, taking into account both the skin friction and the viscous pressure resistance, can be derived from the formula

$$C_V = (1 + k)C_F. \tag{14}$$

Here $(1+k)$ is the form factor, which depends on the hull form, and C_F is the skin friction coefficient based on the flat plate results. The form factor $(1+k)$ can be derived from low-speed tests when, at low Froude numbers Fr, the wave resistance coefficient C_W tends to zero and therefore $(1+k) = C_T/C_F$. The skin friction resistance coefficient C_F is assumed to be dependent on the Reynolds number Re and is recommended to be calculated through the ITTC-1957 skin friction line as

$$C_F(\mathscr{Re}) = \frac{0.075}{(\log_{10}\mathscr{Re} - 2)^2}. \tag{15}$$

The ITTC-1978 powering prediction procedure for deriving the viscous coefficient C_V recommends the use of formula (15), together with the form factor $(1+k)$. The same methodology for calculating the C_V coefficient can be used for the proposed scaling procedure from the acceleration stage towing tests.

THE WAVE-MAKING RESISTANCE

When a submerged vessel travels through a fluid, pressure variations are created around the body. Near a free surface, the pressure variations manifest themselves through changes in the fluid level, creating waves. Such a wave system is made up of transverse and divergent waves. With a body moving through a stationary fluid, the waves travel at the same speed as the body. It follows that the transverse wavelength depends on the ship's speed. The mathematical form of such a wave system is called the Kelvin wave after Lord Kelvin [12]. The first step in formulating an analytical expression for the wave resistance was taken by Michell in 1898 [13]. A review of Michell's wave resistance approach and its impact on ship hydrodynamics is given by Tuck [14]. Further, in 1909 wave resistance was investigated both theoretically and experimentally by Havelock [15] and elaborated in [16]. The findings are that the amplitudes of the waves directly depend on the ship's Froude number *Fr*. Thus the dimensionless coefficient for the wave-making resistance C_W is assumed to depend only on the Froude number. The wave resistance for low speeds is negligible, but for Froude numbers over 0.35, the wave resistance may exceed the viscous resistance for most vessels [11]. Setting equal Froude numbers for the model and full-scale ship, such that the wave resistance coefficients are equal, still dominates the subject of scaling procedures. This assumption will also be used in the proposed scaling procedure for the acceleration stage towing tests.

THE RESISTANCE BREAKDOWN

Within the subject of the resistance breakdown, it is worth emphasising the fundamental difference between the scaling methods proposed by Froude and Hughes. Froude assumed that all residuary resistance scales according to Froude's law, that is, for the same Froude number *Fr*. This is not physically correct because the viscous pressure drag included within the C_V dimensionless coefficient should scale according to Reynolds' law. Hughes assumes that the total viscous resistance, i.e., the friction and the form, scales according to Reynolds' law. This leads to the dimensionless resistance coefficient breakdown:

$$C_T(\mathscr{Re}, \mathscr{Fr}) = C_V(\mathscr{Re}) + C_W(\mathscr{Fr}). \quad (16)$$

This also needs to be adjusted, as the viscous resistance interferes with the wave-making resistance. The reason is that the boundary layer growth suppresses the stern wave; thus, the wave resistance can depend on *Re*. Moreover, the viscous resistance depends on the pressure distribution around the hull, which depends on wave-making [17]. Thus, an interaction term $C_{INT}(Re,Fr)$, depending on both numbers, is non-zero, i.e.

$$C_T(\mathscr{Re},\mathscr{Fr})=C_V(\mathscr{Re})+C_W(\mathscr{Fr})+C_{INT}(\mathscr{Re},\mathscr{Fr}). \quad (17)$$

Therefore, the resistance breakdown is an assumption made for the scaling practice rather than an exact physical representation. A detailed outline of the scaling effects and evidence supporting the existence of an interaction term is given by Terziev [18]. Nevertheless, the overall error caused by the resistance coefficient breakdown assumption (16) is sufficiently small. The form factor method proposed by Hughes and adopted by the ITTC is still an extremely valuable tool in predicting ships' energy requirements.

For the dynamical scaling purpose of this paper, certain assumptions, as mentioned above, will also be made; that is, the viscous friction coefficient C_V depends only on the Reynolds number *Re*, the wave-making coefficient C_W only on the Froude number *Fr*, and the interaction term will be neglected.

SCALING PROCEDURE FOR CONSTANT SPEED TOWING TESTS

Before explaining the scaling procedure from the acceleration stage, let us look at the constant speed stage towing tests because most assumptions will be the same for both approaches. To perform a scaling procedure for constant speeds, first, a geometric scale λ is set as the ratio of the full-scale ship length L_S to the model vessel length L_M, i.e.,

$$\lambda = \frac{L_S}{L_M} \quad (18)$$

Then for equal Froude numbers of both the full-scale ship and the model vessel: $Fr_M = Fr_S$, Froude's law of similarity sets the corresponding speeds:

$$\frac{v_S}{v_M} = \lambda^{0.5}. \quad (19)$$

Here v_S is the full-scale ship speed and v_M denotes the model vessel speed. Newton's second law of dynamics (1) for constant speeds takes the following form for both the full-scale and the model vessel:

$$0 = F_P(v, 0) - R_T(v). \quad (20)$$

Therefore, for constant vessel speeds, the towing force is equal in magnitude to the total hull resistance force; thus, Eq. (3) holds. Moreover, the propulsion force needed to assess the energy requirement for constant full-size vessel speeds is equal to the total resistance force acting on the full-size hull. In general, the scaling procedure for determining the total hull resistance of a full-scale ship from constant speed towing experiments on a geometrically scaled model vessel may be described in the following steps:

Step 1: Setting the range of the full-scale ship speed v_S, from the minimum to the desired maximum ship speed.
Step 2: Calculating the corresponding towing speeds for the model v_M using Froude's law of similarity (19).
Step 3: Recording, from the constant speed stage, the total hull resistance force $R_T(v_M)$ of the model vessel towed in a series of tests at each speed v_M.
Step 4: Determining the coefficient of the total hull resistance for the model at each speed $C_T(v_M)$ from formula (9).
Step 5: Determining the coefficient of the viscous resistance for the model vessel at each speed $C_V(v_M)$ using the ITTC recommended formulas (14) and (15).
Step 6: Calculating the wave-making coefficient for the model vessel at each speed $C_W(v_M) = C_T(v_M) - C_V(v_M)$.
Step 7: The wave-making resistance coefficients for the full-scale and the model vessel are equal: $C_W(v_S) = C_W(v_M)$.
Step 8: Determining the coefficient of the viscous resistance for the full-scale ship $C_V(v_S)$, at speeds corresponding to the model towing speeds, with the use of the ITTC recommended formulas (14) and (15).
Step 9: Calculating the dimensionless coefficient of the total hull resistance for the full-scale vessel at each speed: $C_T(v_S) = C_W(v_S) + C_V(v_S)$.
Step 10: Determining the total hull resistance of the full-scale vessel for each speed using formula (9).

PROPOSED SCALING PROCEDURE FROM THE ACCELERATION STAGE TOWING TESTS

The proposed scaling procedure for accelerated motion has the same methodology as Froude's scaling for constant speed mentioned above. The difference is that we are going to take a step back from the equation of motion (20) to full dynamics (1) because, in accelerated motion, the towing force is needed to overcome the total hull resistance and to accelerate the model vessel.

Table 1 presents the basic assumptions for the scaling rules needed in accelerated motion. Subscripts *S* correspond to the full-size vessel and *M* to the model vessel.

The geometric similarity and the Froude number for the full-size and model vessels remain the same for the acceleration stage scaling proposition. The difference is that, when accelerated, the mass of the vessel and the added mass of water have to be taken into account and scaled. Moreover, since the acceleration is the derivative of speed, for geometric scale λ, the acceleration scales as

$$\frac{\mathrm{dv}_S}{\mathrm{d}t} = \frac{\mathrm{d}(\lambda^{0.5}\mathrm{v}_M)}{\mathrm{d}t} = \lambda^{0.5}\,\frac{\mathrm{dv}_M}{\mathrm{d}t}. \tag{21}$$

To derive the scaling formula for the acceleration stage, let us start from Newton's second law of dynamics in the following form:

$$m\mathrm{v}' = F_P(\mathrm{v}, \mathrm{v}') - \tfrac{1}{2}\rho S C_T(\mathcal{F}r, \mathcal{R}e)\mathrm{v}^2. \tag{22}$$

Further, the breakdown of the resistance coefficients (16) is assumed, i.e.,

$$m\mathrm{v}' = F_P(\mathrm{v}, \mathrm{v}') - \tfrac{1}{2}\rho S\big(C_W(\mathcal{F}r) + C_V(\mathcal{R}e)\big)\mathrm{v}^2. \tag{23}$$

Tab. 1. Scaling rules and basic assumptions

Physical quantity	Scaling rule	Assumptions
Length at the water line	$L_S = \lambda L_M$	Geometric similarity
Wetted surface area of hull	$S_S = \lambda^2 S_M$	Geometric similarity
Immersed volume	$V_S = \lambda^3 V_M$	Geometric similarity
Mass of the vessel	$m_{vS} = \lambda^3 m_{vM}$	The load of the model is prepared in such a way that the wetted volumes correspond to the geometric scaling.
Hydrodynamic added mass	$m_{addS} = \lambda^3 \frac{\rho_S}{\rho_M} m_{addM}$	The accelerating vessel moves a specific volume of the surrounding water and this volume scales with respect to geometric similarity.
Froude number	$\mathcal{F}r_S = \mathcal{F}r_M$	The ratio of the inertia forces to the gravitational forces related to the mass of water displaced by a floating vessel is the same for the model and full-scale ship.
Reynolds number	$\mathcal{R}e_S = \lambda^{0.5}\frac{\mu_M}{\mu_S}\frac{\rho_S}{\rho_M}\mathcal{R}e_M$	Same Froude number and geometric similarity
Speed	$\mathrm{v}_S = \lambda^{0.5}\mathrm{v}_M$	Same Froude number and geometric similarity
Acceleration	$a_S = \lambda^{0.5} a_M$	Same Froude number and geometric similarity

The wave resistance coefficient $C_W(Fr)$ is assumed to depend only on the Froude number and may be derived from Eq. (23), i.e.,

$$C_W(\mathcal{F}r) = \frac{2}{\rho S \mathrm{v}^2}F_P(\mathrm{v},\mathrm{v}') - \frac{2m}{\rho S \mathrm{v}^2}\mathrm{v}' - C_V(\mathcal{R}e). \tag{24}$$

When the Froude number is set to be the same for both the full-size and the model vessel, the partial dynamic similarity of the wave resistance coefficient $C_W(Fr)$ can also be used for accelerated motion; therefore

$$C_W(\mathcal{F}r) = \frac{2}{\rho_M S_M \mathrm{v}_M^2}\big(F_{PM}(\mathrm{v}_M,\mathrm{v}_M') - m_M\mathrm{v}_M'\big) - C_V(\mathcal{R}e_M), \tag{25}$$

$$C_W(\mathcal{F}r) = \frac{2}{\rho_S S_S \mathrm{v}_S^2}\big(F_{PS}(\mathrm{v}_S,\mathrm{v}_S') - m_S\mathrm{v}_S'\big) - C_V(\mathcal{R}e_S). \tag{26}$$

In formula (25), $F_{PM}(\mathrm{v}_M, \mathrm{v}_M')$ is the towing force from the acceleration stage towing tank test on the model vessel. Just one towing test up to the maximum speed is needed to access such information. $F_{PS}(\mathrm{v}_S, \mathrm{v}_S')$ in (26) is the propulsion force needed to predict the ship's energy requirement for accelerated motion. Further, it is assumed that the gravitational field g is the same for both the model and the full-scale vessels. Then, one can write the wave-making coefficient $C_W(Fr)$ for the full-scale vessel (26) using the scaling rules in Table 1:

$$C_W(\mathcal{F}r) = \frac{2}{\rho_S M \mathrm{v}_M^2 \lambda^3}\Big(F_{PS}(\mathrm{v}_S,\mathrm{v}_S') - \lambda^{3.5}\big(m_{vM} + \tfrac{\rho_S}{\rho_M}m_{addM}\big)\mathrm{v}_M'\Big) - C_V\Big(\lambda^{1.5}\,\frac{\mu_M}{\mu_S}\frac{\rho_S}{\rho_M}\,\mathcal{R}e_M\Big). \tag{27}$$

Below, let us write an equation where the upper part is the wave-making coefficient $C_W(Fr)$ for the full-size vessel with the scaling rules applied (27), and the bottom part is the wave-making coefficient for the model vessel (25):

$$\frac{2}{\rho S_M \mathrm{v}_M^2\lambda^3}\Big(F_{PS}(v_S,v_S') - \lambda^{3.5}\big(m_{vM} + \tfrac{\rho_S}{\rho_M}m_{addM}\big)v_M'\Big) - C_V\Big(\lambda^{1.5}\,\frac{\mu_M}{\mu_S}\frac{\rho_S}{\rho_M}\,\mathcal{R}e_M\Big) \tag{28}$$

$$=$$

$$\frac{2}{\rho S_M \mathrm{v}_M^2}\Big(F_{PM}(v_M,v_M') - (m_{vM} + m_{addM})v_M'\Big) - C_V\big(\mathcal{R}e_M\big). \tag{29}$$

Then, after basic transformations on the above equation, the following scaling rules for obtaining the propulsion force for the full-scale vessel from the acceleration stage towing experiments on the scaled model are derived:

$$\mathrm{v}_S = \lambda^{0.5}\mathrm{v}_M,$$

$$\mathrm{v}_S' = \lambda^{0.5}\mathrm{v}_M',$$

$$m_S = \lambda^3\big(m_{vM} + \tfrac{\rho_S}{\rho_M}m_{addM}\big),$$

$$F_{PS}(\mathrm{v}_S,\mathrm{v}_S') = \lambda^3 F_{PM}(\mathrm{v}_M,\mathrm{v}_M') \tag{30}$$

$$+\,\lambda^3(\lambda^{0.5} - 1)m_{vM}\mathrm{v}_M' \tag{31}$$

$$+\,\lambda^3\big(\lambda^{0.5}\tfrac{\rho_S}{\rho_M} - 1\big)m_{addM}\mathrm{v}_M' \tag{32}$$

$$+\,\lambda^3\,\frac{\rho S_M \mathrm{v}_M^2}{2}\Big(C_V\big(\lambda^{1.5}\,\tfrac{\mu_M}{\mu_S}\tfrac{\rho_S}{\rho_M}\,\mathcal{R}e_M\big) - C_V\big(\mathcal{R}e_M\big)\Big). \tag{33}$$

Here, terms (30) and (33) are equivalent to the standard scaling procedures for constant speed when $v' = 0$, i.e.,

$$F_{PS}(\mathrm{v}_S,0) = \lambda^3 F_{PM}(\mathrm{v}_M,0)+$$

$$\lambda^3 \frac{\rho S_M v_M^2}{2}\left(C_\mathrm{V}(\lambda^{1.5}\,\frac{\mu_M}{\mu_S}\frac{\rho_S}{\rho_M}\,\mathscr{R}e_M) - C_\mathrm{V}(\mathscr{R}e_M)\right). \tag{34}$$

Term (31) is the part of the propulsion force needed to accelerate a full-scale vessel. This part is equal to the part of the towing force needed to accelerate the model vessel with corresponding acceleration v_M' times the scaling factor $\lambda^3(\lambda^{0.5}-1)$. Finally, the term (32) is the part of the propulsion force that is needed to accelerate the added water mass of the full-scale vessel, and this is equal to the part of the towing force needed to accelerate the added water mass of the model vessel with the corresponding acceleration v_M' times the scaling factor $\lambda^3(\lambda^{0.5}-1\rho M/\rho M-1)$. Different water densities for the full-scale and model vessels were considered for the scaling factor in (32).

Therefore, the scaling approach for calculating the propulsion force of the full-scale vessel from the accelerated stage towing experiment on a scale model is proposed below:

Step 1: Setting the range of the full-scale ship speed v_S, from the minimum to the desired maximum speed.

Step 2: Calculating the towing speeds for the model v_M using Froude's law of similarity (19).

Step 3: Recording the towing force $F_{PM}(\mathrm{v}_M,\mathrm{v}_M')$ of the model vessel from the acceleration stage towed up to the maximum speed.

Step 4: Calculating the added mass by extrapolating the towing force to zero speed and using formula (7).

Step 5: Determining the propulsion force $F_{PS}(\mathrm{v}_S,\mathrm{v}_S')$ using formulas (30)–(33).

It should be noted that no time scale has been used in the proposed scaling procedure. The equations of motion for any profile can be derived from the second law of dynamics after determining the propulsion force of a full-size vessel $F_{PS}(\mathrm{v}_S,\mathrm{v}_S')$.

CONCLUSIONS

This work derives a dynamical scaling proposition for the propulsion force required to estimate the full-scale vessel energy requirement. The towing force can be measured experimentally using the acceleration stage tests on a scale model vessel. This theoretical analysis demonstrates that such an approach may have advantages over constant speed towing tests. From the acceleration stage, it is possible to obtain information about the hydrodynamic added mass, which should also be considered when predicting the ship's energy consumption. Furthermore, all information about the constant speed stage is accessible from only one acceleration test done up to the maximum speed. Finally, the proposed testing and scaling procedure can be used for dynamic models when simulating various profiles of motion, including constant speed, accelerating, decelerating, and gliding.

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