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3D Buckling Analysis of a Truss with Horizontal Braces

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The present research is devoted to the study of out–of–plane buckling of a truss with horizontal braces. The truss is a model of real roof truss scaled by factor 1/4. A linear buckling and a non–linear analysis with geometric and material non–linearity were carried out. The truss buckling and limit load for different stiffnesses and number of braces are found. Numerical analysis are verified by experiment. Threshold bracing stiffness condition for full bracing of the truss is proposed.

Keywords: Buckling load, limit load, brace stiffness, imperfection.

1. Introduction

Steel trusses have a much greater strength and stiffness in their plane than out of their plane, and therefore should be braced against lateral deflection and twisting. The problem of bracing requirements necessary to provide lateral stability of compressed structural members is present in codes [1, 2]. The simplified design code requirements allow one to reduce the problem of the truss stability to the analysis of compressed chord or diagonals that are separated from the other truss elements. The effect of the lower chord, verticals and diagonals on the truss stability is neglected. Verticals and diagonals are considered only as vertical supports to the upper truss chord, side bracing of the truss chords is considered as rigid side–support and normal forces in the truss chords are assumed to be constant along their length. As a result of above described simplifications the requirements concerning the number and the stiffness of braces are not precise, because analysis of the whole structure is not taken into account. The stability of trusses with elastic bracing was investigated in experimental research [3, 4] or in numerical analysis [5], where the relation between the truss buckling load and the bracing stiffness was investigated. The basic problem was devoted to investigating the required bracing stiffness that ensures that the out–of the truss plane buckling occurs between braces, or is prevented, so the buckling occurs in the plane of the truss. The full bracing condition was defined as the bracing stiffness that causes the maximal buckling load of the truss, or when an increase in bracing stiffness doesn't result in a further increase in the buckling load. The study was limited to linear buckling analysis. A numerical analysis of 1D truss model and experimental test of a truss stability was presented in paper [6].

In the present research the study [6] is extended to non–linear analysis of 3D model with geometric and material non–linearity of an imperfect truss. The numerical analysis is verified and extended by experimental tests of the truss with side braces. The truss top chord full bracing condition is found for the truss buckling and a non–linear analysis with geometric and material non-linearity taken into account.

2. Description of the Model

In the present parametric study the truss illustrated in Fig.1 is considered. The length of the truss is $L = 6.0$ m, the depth in the middle is $h = 0.44$ m, and 0.3 m near the supports. The compression chords consists of 2 × L20 × 3 rolled profiles. Two compression diagonals near the supports are made of square cross–section (15 mm × 15 mm × 1.5 mm). Other diagonals are made of profile 15 mm × 10 mm with thickness 1.5 mm. The truss is made of steel with yield strength of 350 MPa.

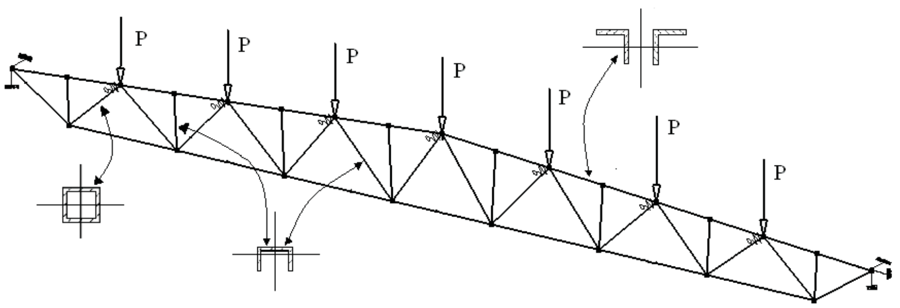

Figure 1 Truss with lateral braces

Figure 2 Experimental set-up of truss with elastic braces

The connections between the truss chord, the diagonal, and the vertical elements are rigid, so the bottom chord, the diagonals and the verticals interact together with the truss top chord and partially restrain the top chord against out–of–plane buckling. The built–up top chord section is battened every 0.37 m to avoid buckling of individual members. The batten consists of profile 15 mm × 10 mm with thickness 1.5 mm located between profiles of the truss top chord. The truss is a model of a real roof truss sized according to code [1] with scale factor 1/4 (Fig. 2). In the numerical analysis it is assumed that the load is applied as concentrated forces at three, or seven top chord joints, depending on the analyzed model of braces. In the case of the truss with 3 braces the distance between braces was 1.5 m. In the experiment the truss was loaded by concentrated force in the middle of the span.

The main purpose of the investigations was to determine the load–deflection relationship for different stiffnesses of braces. The truss is simply supported without any additional torsional restraints that prevent the truss against twisting at the supports (Fig. 3a, b). Due to this assumption the truss braces provide the stability of the structure. The lateral bracing was modeled in the form of springs situated in the truss top chord joints. The springs characteristics were determined using a separate testing procedure. A detail of the brace attachment to the top truss chord is presented in Fig. 3c.

a) b) c)

Figure 3 a), b) Truss details at the supports, c) truss brace and force application set–up detail

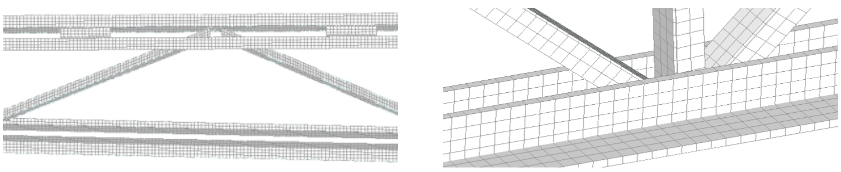

Figure 4 FEM model of the truss

In the numerical analysis the truss was modelled by shell elements and program [7]. The 4–node shell elements QUAD4 (with 6 degrees of freedom in node) were employed. In the numerical model there are battens between the truss top chord profiles. It was out of scope of the present analysis to investigate the influence of the battens length on the truss stability, but it should be stressed that those braces are important in the buckling resistance of the truss. The battens length was 0,07 m. The total amount of finite elements was about 58 000. The minimum 4 shell elements were used to describe the walls of the chord cross–sections and 2 elements on the walls of the U–diagonals cross–section (Fig. 4). The element size on the truss top chord and on the U–diagonals was about 5,0×5,0 mm^2. Connections between the truss elements are modeled by rigid links between the adjacent members.

Linear buckling analysis and non–linear static analysis with geometric and material non–linearity of the imperfect truss model with imperfection in the shape of the buckling modes were carried out by means of the program [7]. Considered were three different types of initial geometric imperfections shown in Fig. 6a. and Fig. 6b. and Fig. 6d. For the truss with global imperfection (Fig. 6a. and Fig. 6d) the imperfection magnitude was equal to $L/500 = 0.012$ m according to the code [2]. For the truss with local imperfection (Fig. 6b) the maximum value of the diagonal total displacement was equal to 0,006 m, due to code [8, 9]. In the experimental research a stability of truss braced by three braces and loaded in the middle of the span was investigated.

3. Results of Numerical Analysis

3.1. *Linear buckling analysis*

The relationship between first buckling load at one top chord joint, due to the bracing stiffness for trusses with different brace locations is presented in Fig. 5.

It is worth noting that the truss buckling load for the model with 7 braces depends on the number of battens between the truss top chord profiles. There is a large dispersion in the truss buckling resistance 4 kN - 7 kN depending on the number of battens between the profile elements. For the truss with 3 braces and for the truss with 7 braces without battens at the threshold condition for full bracing the truss buckles between braces Fig..6e, Fig. 6c. In the case of the truss with 7 braces at the threshold condition for full bracing the local buckling of the most compressed diagonal occurs (Fig. 6b).

3.2. *Non–linear analysis*

For different stiffnesses of braces a non–linear relation between the truss load and displacement has been obtained. The load at the one top chord joint increases with an increase of the bracing stiffness (Fig. 7, 8, 9). The truss deformation corresponding to the limit state is presented for the truss with 3 and 7 braces in Fig.10. The truss limit load (sum of all concentrated forces) for the analysed imperfect models and for different stiffness of braces is presented in Fig.11. In the case of the truss with 3 braces the limit load is constant for braces of higher stiffness than about 200 kN/m.

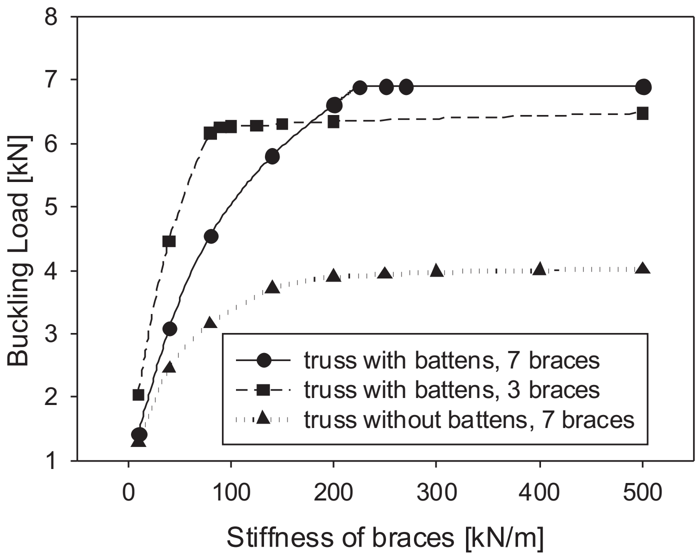

Figure 5 Comparison between the first buckling load at one top chord joint, with respect to the stiffness of braces for a different number of braces

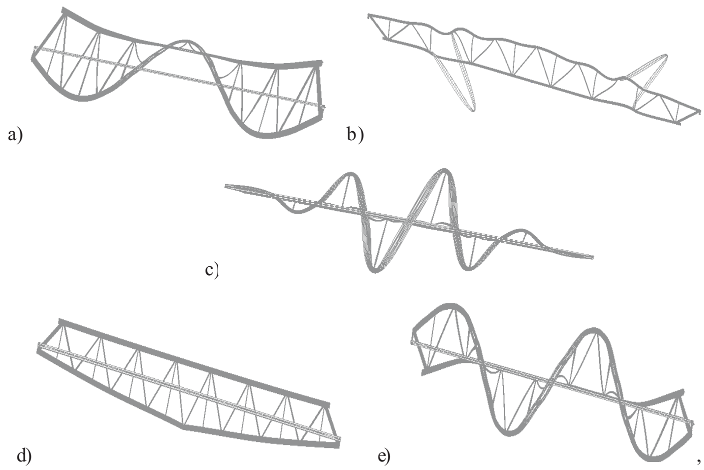

Figure 6 Buckled shape of the truss for: a) 7 braces with stiffness 40 kN/m, b) 7 braces with stiffness 250 kN/m, c) 7 braces with stiffness 250 kN/m - truss without battens, d) 3 braces with stiffness 10 kN/m e) 3 braces with stiffness 100 kN/m

This stiffness may be treated as the threshold condition for full bracing. In the case of the truss with 7 braces and global imperfection (Fig.6a) the increase of limit load was obtained in the whole range of the braces stiffness, but the increase is lower for higher brace stiffnesses. The limit loads of the truss with local imperfection (Fig.6b) are lower than in the case of global imperfection. Threshold bracing stiffness was in this case 40 kN/m. The influence of the assumed imperfection magnitude on the limit truss load studied by the authors in papers [8, 9] was in this case very important.

4. Results of experimental research

For braces of stiffnesses $k = 10$ kN/m the relation between the truss load due to the truss displacements determined in the numerical analysis and in experiment is presented in Fig.12. In this research the truss was loaded only in the midspan. The limit load found in the numerical analysis was about 5 kN for the truss with imperfection corresponding to the first buckling load (a torsional deformation of the truss – imperfection I). Deformation at the limit state of the truss obtained in experiment may be described as a combination of two half–waves of the truss top chord and torsional deformation of the whole structure. Due to the fact that the results found in the numerical analysis and the experiment are different additional imperfection in a form of combination of two half–waves deformation and torsional deformation of the truss (Imperfection II) was taken into account (Fig.12). The differences confirms that the numerical model is not precise. The possible explanation of this discrepancies may be explained by different performance of the springs.

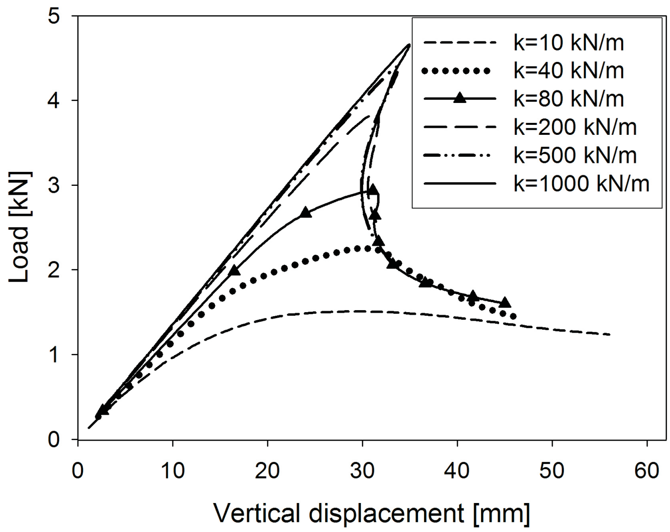

Figure 7 The truss load at one top chord joint vs. the vertical displacement (at 0,75 m from midspan) of truss with 7 braces and global imperfection (Fig. 6a) for different stiffnesses of braces

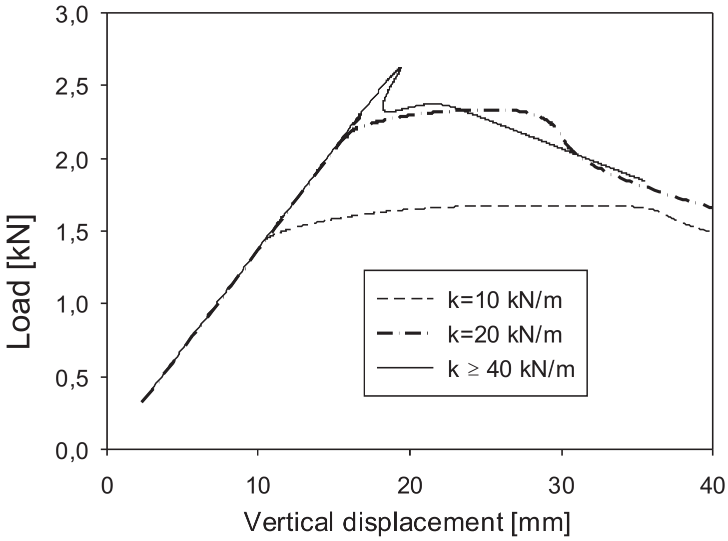

Figure 8 The truss load at one top chord joint vs. the vertical displacement (at 0,75 m from midspan) of truss with 7 braces and local imperfection (Fig. 6b) for different stiffnesses of braces

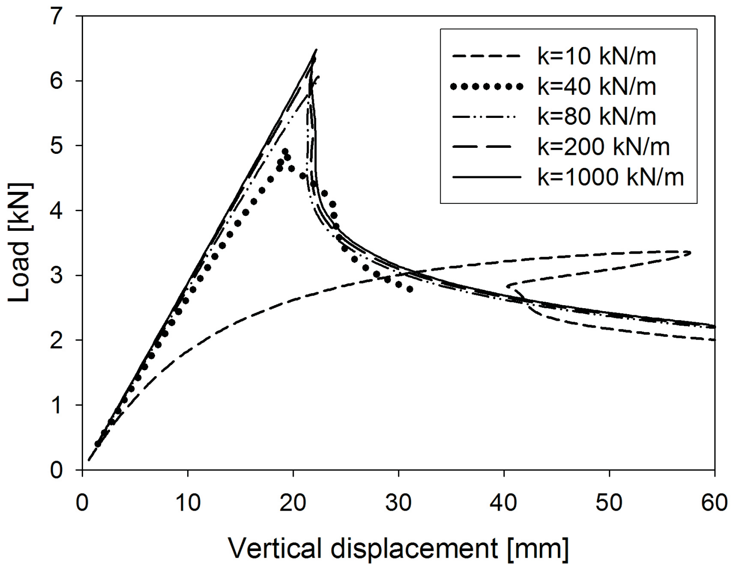

Figure 9 The truss load at one top chord joint vs. the vertical displacement (at 0,75 m from midspan) of truss with 3 braces and global imperfection (Fig. 6d) for different stiffnesses of braces

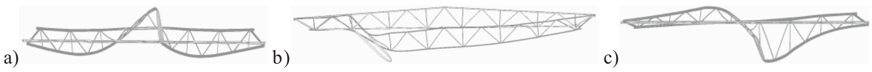

Figure 10 Deformation of the truss after limit load for truss with braces of stiffness 40 kN/m a) 7 braces, imperfection Fig. 6a (a view from the top), b) 7 braces, imperfection Fig. 6b, c) 3 braces, imperfection Fig. 6d (a view from the top)

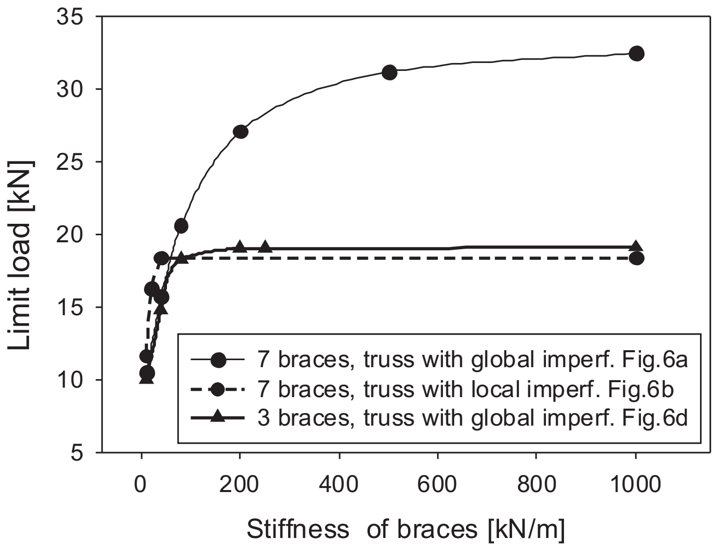

Figure 11 Relation between the limit load of the truss and the braces stiffness

In the numerical model it was assumed that the braces are modeled as horizontal springs while in the deformed model at the limit state (Fig. 13) it is shown that the springs are bent. This effect may result in some rotational stiffness that was not taken into account in the numerical model. The application of the truss load in the test setup may be also responsible for the differences between the numerical and laboratory test results. The load setup partially prevented out of plane displacements of the truss. Solving of these problems will be a subject of future research.

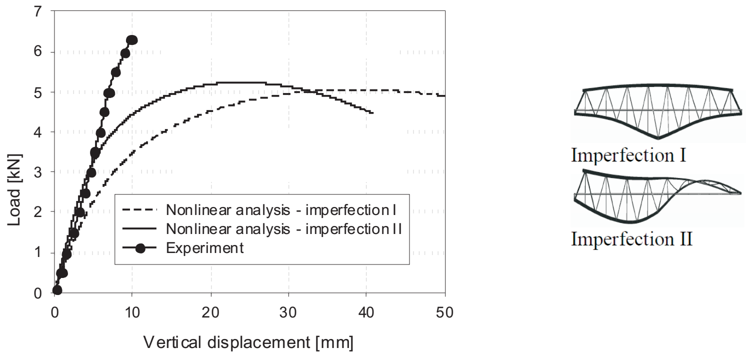

Figure 12 The truss loading at one top chord joint vs. the vertical displacement (at 0,75 m from the mid–span) of truss with 3 braces k = 10 kN/m

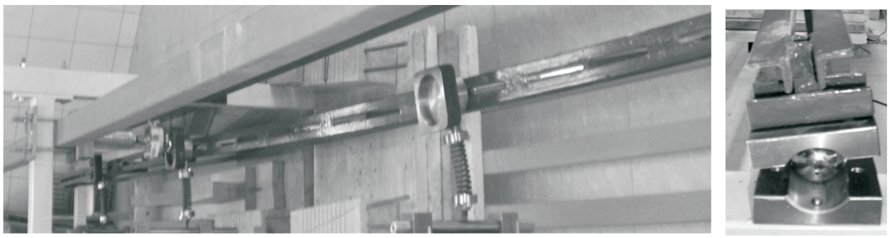

Figure 13 Deformation at the limit state of the truss obtained in experiment

5. Conclusions

The results of the performed numerical analyses and experiment provide a basis for drawing some conclusions regarding the effect of bracing stiffness on the buckling and limit load.

The buckling load and the limit load of the truss depends on the stiffness and spacing of braces.

The truss buckling and limit load are comparable for the truss with 3 braces.

In case of the truss with 7 braces and global imperfection the limit load is about 60–70% of the linear buckling truss load. It is even lower (about 38%) for the truss with local imperfection. In this case it is worth noting that the imperfection amplitude was relatively large (0,006 m [8, 9]) in relation with the diagonal length $l = 0,49$ m.

The threshold stiffness of braces necessary to obtain maximal limit load depends on the truss imperfection shape and amplitude.

The battens between the truss top chord profile are important in the buckling resistance of the structure. The differences between the buckling resistance due to battens are up to about 45%.

The experiment and the numerical results are not coincidental. The discrepancies may be caused by different performance of the springs. Modification of the truss load set–up will be a subject of planed research in the future.

References

[1] **PN–90/B–03200**: Konstrukcje stalowe. Obliczenia statyczne i projektowanie, Warszawa, *PKN*, **1990**.

[2] **PN–EN 1993–1–1 2006 Eurocode 3**: Projektowanie konstrukcji stalowych, Część 1–1: Reguły ogólne i reguły dla budynków.

[3] **Kołodziej, J. and Jankowska–Sandberg, J.**: Badania doświadczalne zwichrzenia sprężystego kratownicy stalowej z uwzględnieniem podatności stężeń bocznych, LII Konferencja Naukowa Komitetu Inżynierii Lądowej i Wodnej PAN i Komitetu Nauki PZITB, Krynica, *Zeszyty Naukowe Politechniki Gdańskiej, nr 601, Budownictwo Lądowe*, 58, 123–129, **2006**.

[4] **Iwicki, P. and Krajewski, M.**: Analiza stateczności i nośności granicznej kratownicy ze stężeniami, LVIII Konferencja Naukowa Komitetu Inżynierii Lądowej i Wodnej PAN i Komitetu Nauki PZITB, Krynica, *Zeszyty Naukowe Politechniki Rzeszowskiej, Budownictwo i Inżynieria środowiska*, 59, 169–176, *2012*.

[5] **Iwicki, P.**: Sensitivity analysis of critical forces of trusses with side bracing, *Journal of Constructional Steel Research*, 66, 923–930, **2010**.

[6] **Iwicki, P. and Krajewski, M.**: Analiza numeryczna i badania doświadczalne kratownicy ze stężeniami, LVII Konferencja Naukowa Komitetu Inżynierii Lądowej i Wodnej PAN i Komitetu Nauki PZITB, Krynica, *Zeszyty Naukowe Politechniki Rzeszowskiej*, Budownictwo i Inżynieria Środowiska, 58, 233–240, 57, **2011**.

[7] **Femap with NX Nastran. Version 10.1.1. Siemens Product Lifecyde Management Software Inc.**, **2009**.

[8] **PN–B–06200**: Konstrukcje stalowe budowlane. Warunki wykonania i odbioru. Wymagania podstawowe, Warszawa: PKN **2002**.

[9] **PN–EN 1090–2+A1**: Wykonanie konstrukcji stalowych i aluminiowych. Część 2: Wymagania techniczne dotyczące konstrukcji stalowych. Warszawa: PKN **2012**.