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A SIMPLIFIED ENERGY DISSIPATION BASED MODEL OF HEAT TRANSFER FOR SUBCOOLED FLOW BOILING

Jarosław Mikielewicz[1], Dariusz Mikielewicz[2]

[1]Institute of Fluid-Flow Machinery, Polish Academy of Science,

80-231 Gdańsk, ul. Fiszera 14, Poland

[2]Gdansk University of Technology, Faculty of Mechanical Engineering,

80-233 Gdańsk, ul.Narutowicza 11/12, Poland

email: jarekm@imp.gda.pl, Dariusz.Mikielewicz@pg.gda.pl

Abstract

In the paper a model is presented based on energetic considerations for subcooled flow boiling heat transfer. The model is the extension of authors own model developed earlier for saturated flow boiling and condensation. In the former version of the model we used the heat transfer coefficient for the liquid single-phase as a reference level, due to the lack of the appropriate model for heat transfer coefficient for the subcooled flow boiling. That issue was a fundamental weakness of the that approach. The purpose of present investigation is to fulfil this drawback. Now the reference heat transfer coefficient for the saturated flow boiling in terms of the value taking into account the subcooled flow conditions. The wall heat flux is based on partitioning and constitutes of two principal components, namely the convective heat flux and partial evaporation heat flux of the liquid replacing the detached bubble. Both terms are accordingly modelled. The convective heat flux is regarding vapour bubbles travelling longitudinally and the liquid moving radially – liquid pumping. The results of calculations have been compared with some experimental data from literature showing a good consistency.

Keywords: subcooled flow boiling, heat transfer, modeling

NOMENCLATURE

A	- projection area, m^2
a	- thermal diffusivity [m/s^2]
B	– blowing parameter [−]
Bo	- Boiling number, $B=q/(G\ h_{lv})$
c	- specific heat [J/(kgK)]
C	- constant
D_h	- hydraulic diameter [m]
E	- enhancement factor
f	- departure frequency. [1/s]
F	- reduction factor, enhancement factor
g	- gravitational acceleration [m^2/s]
G	- mass velocity, [$kg/(m^2s)$]
h_{lv}	- latent heat [J/kg]
$\dot{m}$	- mass flow rate, kg/s
p	- pressure [N/m^2]
P	- empirical correction, perimeter
q	- heat flux [W/m^2]
R	-radius [m]
Re	- Reynolds number, $Re=G\ D_h/\mu_l$
S	- suppression factor,
T	- temperature [^oC]
t	- time [*s*]
u	- velocity [m/s]
w	- superficial velocity, [m/s]
V	- volume [m^3]
x	- quality [-]
z	- wall normal coordinate [m]

Greek symbols

α	- heat transfer coefficient [$W/(m^2K)$]
λ	- thermal conductivity [(W/mK)]
δ	- penetration depth, boundary layer thickness, [m]
ε	- q_a/q_{ev}, [-]
μ	- dynamic viscosity [kgm/s]
ρ	- density [kg/m^3]
Φ	- enhancement factors [−]
σ	- surface tension [kg/s^2]
τ	- shear stress [N/m^2]

subscripts

a	- agitated
b	- bulk
con	- convective
ev	- evaporative

in - inlet
l - liquid phase
lp - liquid pumping
onb - onset of boiling
OSV - onset of significant voids
O - reference
p - constant pressure
Pb - pool boiling
ref - reference
sat - saturation
S - subcooled
sub - subcooling
TP - two-phase
TPB - two-phase flow boiling
v - vapour
w - wall

1. INTRODUCTION

The subcooled flow boiling for a long time is perceived as one of the most effective ways of removal of large heat fluxes due to a large temperature difference and presence of boiling in the flow. The phenomenon found application in various areas of technology where efficient cooling is required. An example of such application is nuclear reactor cooling, medical applications where cooling of neutron generators used in treatment of tumors is necessary, testing of materials, cooling of electronic equipment or cooling of gas turbine nozzles. Understanding of the physics of local boiling in subcooled liquids flowing inside heated channels is still unsatisfactory. A number of papers in the literature are devoted to this issue but the complexity of the process makes the analysis of the issue very challenging. Several modeling approaches have been developed to predict the heat transfer rate during subcooled flow boiling. Such models can be generally divided into two categories, namely purely empirical correlations for heat flux calculations or the formulas based on mechanistic models. The empirical approaches express the wall heat flux or partitioning of the wall heat flux. Non-consistent empirical correlations for heat transfer coefficient are used for expressing a particular wall heat flux partitioning. Non-consistency partially stem from the fact that empirical correlations are generally limited to particular flow conditions. Hence empirical correlations do not include modeling of the heat transfer mechanisms. The alternative are the mechanistic models which are capable of determining the particular heat flux components individually. Usually two main aspects of the problem are studied, firstly, the inception of subcooled boiling and its distance from the inlet of the channel and, secondly, heat transfer from the wall to fluid. Hence empirical correlations for wall heat flux partitioning can only provide information regarding how the wall heat flux is to be partitioned. They cannot be used for the prediction of the wall heat flux itself. The mechanistic models, on the other hand, which are based on the relevant heat transfer mechanisms occurring during the boiling process, have the capability for individual determination of each of the relevant heat flux components. Hence the mechanistic models can be used for both the prediction of the wall heat flux and the partitioning of the wall heat flux between the liquid and vapor phases. An excellent review of literature on the topic of empirical correlations for heat flux, empirical correlation for partitioning of wall heat flux and mechanistic models for prediction of wall heat flux and partitioning can be found in Warrier and Dhir [1].

The objective of the present work is to devise a model for calculation of the convective part of heat transfer coefficient in subcooled flow boiling developed on the basis of energy dissipation in the flow. The presented approach belongs to the group of mechanistic treatments to determination of the contribution of convective heat transfer in subcooled flow boiling. The resultant model of subcooled flow model is a modification to the saturation flow boiling developed earlier by the authors, presented in detail in [2-4]. In addition the heat flux due to evaporation has been determined.

The beginning of the nucleate boiling starts at the location where vapour can exist in the steady state condition on the heated surface without condensation. The greater the fluid energy (longitudinally) the bubbles can grow until departure from the heated surface and penetration into the liquid (end of wall voidage region in Fig. 1).

The region of the subcooling zone can be either large or small in relation to the fluid properties, mass flux, pressure and heat flux. It is a non-equilibrium region in which the quality and void fraction are assuming positive non-zero values but the liquid temperature is below the saturation temperature. Modelling of such phenomenon represents significant difficulties.

One of the earliest models for empirical determination of partitioning of wall heat flux was developed by Griffith et al. [5]. Based on visual observations during experiments, they identified two distinct boiling regions, see Fig. 2, namely a highly subcooled region with a low void fraction, i.e. “region I”, and a slightly subcooled region with a significant void fraction - “region II”. Region I extends over the heated area between the onset of nucleate boiling (ONB) and the onset of significant voids (OSV) locations, while region II begins at OSV and extends until saturated boiling begins in the entire cross-section of the flow. They used the arithmetic superposition of single phase forced convection heat flux, q_l, and fully developed pool boiling heat flux, q_{Pb}:

$$q_w = q_l + q_{Pb} \tag{1}$$

A similar approach was subsequently presented by Bowring [6]. He provided however a different explanation to the mechanism of rapid increase in void fraction in the moderately subcooled region, i.e. “region II”. Contrary to Griffith et al. [5] that was not due to the “wall effect”, but the increase in void fraction was postulated to be due to the bubbles lifting off the heater surface, the location of the OSV. These bubbles are then assumed to slowly condense in the bulk liquid as they travel downstream. Bowring developed an empirical correlation for the location of the beginning of the bubble lift-off (OSV) in the form:

$$\Delta T_{sub,OSV} = q_w \frac{14 + 0.1p}{u} \tag{2}$$

In (2) the local subcooling $\Delta T_{sub,OSV}$ is expressed in Kelvins, whereas the wall heat flux q_w is in W/cm^2, p is pressure in atmospheres and u is the fluid velocity in cm/s. Bowring was the first to consider the phenomenon of bubble lift-off in region II. Subsequently for that region four heat transfer mechanisms have been identified, the objective of studies in subsequent approaches. These were the following: single phase convection in the areas not covered by the presence of bubbles, q_l, bubble evaporation, q_{ev}, convection heat transfer rendered by bubble agitation of the thermal boundary layer (corresponding to the sensible heating of liquid that occupies the volume vacated by a departing bubble – sometimes referred to as the pumping), q_a, and finally the condensation from the top of bubbles still attached to the heated surface (usually neglected term). The resulting partitioning of the wall heat flux therefore reads:

$$q_w = q_l + q_{ev} + q_a \tag{3}$$

The heat flux due to evaporation, q_{ev}, can be determined from the relation:

$$q_{ev} = \rho_l c_{p,l} V_b f N_a \Delta T \tag{4}$$

In eq. (4) $\Delta T = T_w - T_b$ is the effective temperature difference of liquid heating, f is the frequency of bubble existence, whereas N_a is a density of existence of active nucleation sites. As f, N_a and V_b are usually very difficult to be determined the term $\varepsilon = q_a/q_{ev}$ has been introduced into the analysis (usually determined in experimental manner) which can be calculated using the following formulas [6]:

$$\varepsilon = 1 + 3.2\frac{\rho_L C_{p,l}\Delta T_{sub}}{\rho_g h_{lv}} \qquad for \quad 1 \le p \le 9.5$$
$$\varepsilon = 2.3 \qquad for \quad 9. \le p \le 50 \tag{5}$$
$$\varepsilon = 2.6 \qquad for \quad p \ge 50$$

In equation (5) p is the pressure in bars. Therefore the wall heat flux described by [3] can now be divided into the three following components:

$$q_w = q_l + q_{ev} + q_a = \alpha_l(T_{sat} - T_l) + (1+\varepsilon)q_{ev} \tag{6}$$

Knowledge of ε, q_l and q_{ev} enables determination of q_a. There were several other studies based on the Bowring's model, such as the approaches due to Rouhani and Axelsson [7], Ahmad [8], Maroti [9], Lahey [10], Zeitoun [11] or Liu and Winterton [12], just to mention a few. These models, however, do not determine the q_{ev} directly, but are based on the knowledge of applied q_w and calculated q_l. The contribution of Zeitoun [11] was to postulate the expression for determination of the parameter $\varepsilon=q_a/q_{ev}$ in the form:

$$\varepsilon = \frac{A_b \delta_t \rho_l c_{p,l}\left(\frac{T_w + T_b}{2} - T_b\right)}{V_b \rho_v h_{lv}} \tag{7}$$

In equation (7) δ_t is the thermal boundary thickness calculated from the expression of laminar conduction through the liquid $\delta_t=\lambda_l(T_w-T_b)/q_w$, whereas A_b and V_b are the bubble area and volume respectively, calculated using the mean Sauter diameter from the correlation:

$$D_{b,s} = 1.85\varphi^{0.243}\left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{0.55}\left(\frac{G}{\mu_l}\right)^{0.1} \tag{8}$$

In (8) φ denote the void fraction, σ is the surface tension, whereas G is the total mass flux.

Liu and Winterton [12] postulated the following expression for the near wall heat flux in subcooled flow boiling:

$$q_w = \sqrt{\left[(F\alpha_O \Delta T_{sub})^2 + (S\alpha_{Pb}\Delta T_w)^2\right]} \tag{9}$$

In equation (9) α_O is calculated from the Dittus-Boelter correlation for the single phase flow. The remaining terms in equation, i.e. coefficients F and S are calculated from the Chen model, whereas the pool boiling heat transfer coefficient, α_{Pb}, from the Cooper model [16]. The correlation has been developed for the database of experimental points from the range of mass flux 12.4-8180 kg/m^{2}s, pressures 0.5-200 bar and liquid subcoolings varying from 0 to 173^{o}C.

The location of the incipience of boiling, z, can be found for a known value of $\Delta T_{w,ONB}$. As the moving bubbles agitate the liquid, so the heat transfer gets considerably enhanced. As a result the wall temperature reaches a constant value along the channel, matching the distribution of wall temperature obtained for the single-phase with no boiling conditions, in line with the relation:

$$T_w = T_b + \frac{q_w}{\alpha_0} \tag{10}$$

In equation (10) α_0 is the heat transfer coefficient for the single liquid flow, which can be calculated for example from the Dittus-Boelter equation for turbulent flow. The local condition of sufficient superheating T_w-T_{sat}=ΔT_{sat}, allowing for the existence of bubbles, must be obeyed for the incipience of subcooled flow boiling [1]. Considering the heat balance in the axial direction from the inlet to a given distance z in the form, we have:

$$q_w U z = G C_{p,l} A \left(T_b - T_{in}\right) \tag{11}$$

Re-arrangement of equation (11) enables determination of the bulk fluid temperature in the core T_b as:

$$T_b = T_{in} + \frac{q_w U z}{G C_{p,l} A} \tag{12}$$

For small values of wall superheat $\Delta T_{w,ONB}$ can be determined from the expression due to Hsu [13]:

$$\Delta T_{w,ONB} \cong T_{w,ONB} - T_{sat} = \frac{4\sigma T_{sat}}{D_c \rho_v h_{lv}} \tag{13}$$

In (13) D_c is the size of available cavity (D_c=D_c^0F), F is the reduction factor (depending on the static contact angle β, and D_c^0 is obtained from the minimum superheat criterion:

$$F = 1 - \exp\left[-\left(\frac{\pi\beta}{180}\right)^3 - 0.5\left(\frac{\pi\beta}{180}\right)\right] \tag{14}$$

$$D_c^0 = \left[\frac{8\sigma T_{sat}\lambda_l}{\rho_v h_{lv} q_w}\right]^{0.5} \tag{15}$$

$\Delta T_{w,ONB}$ can also be determined from other relevant models, for example models due to Bergles and Rohsenow [14], Sato and Matsumura [15] or others. These models are however applicable to water flows only.

2. SEMI-EMPIRICAL MODELLING OF FLOW BOILING

Presented analysis will be derived from the original concept of flow boiling modeling applied to saturated flow conditions, J. Mikielewicz (1973) [2]. In that approach the heat transfer coefficient in the saturated flow boiling was devised in terms of the simpler modes of heat transfer namely the single phase heat transfer and pool boiling heat transfer as well as a two-phase flow multiplier, which is a distinct feature of the model. The beginning of the process of flow boiling modeling was referenced to the forced convection value in the liquid phase flow. Such approach is not physically correct, as the boiling process starts not from the equilibrium quality equal zero, but earlier when the bubble nucleation is developing on the wall. Therefore the model presented in the following attempts to determine the reference heat transfer coefficient for the saturated flow boiling in terms of the value taking into account the subcooled flow conditions. In authors previous papers [2-4], concerning saturated flow boiling, we used the heat transfer coefficient for the liquid single-phase flow as the reference level, due to the lack of the appropriate model for heat transfer coefficient for the subcooled flow boiling. Therefore that issue was a fundamental weakness of the model developed in earlier approaches. The purpose of present investigation is to fullfil this drawback.

2.1. Fundamentals of semi-empirical method of determination of heat transfer coefficient in subcooled flow boiling.

In the case of subcooled flow boiling the same fundamental hypothesis can be applied as in the case of saturated flow boiling [2]. The saturated flow boiling model states that the total energy dissipation in the saturated flow boiling with bubble generation, treated as an equivalent flow of fluid with properties of the two-phase flow, can be modeled as a sum of two contributions, namely the energy dissipation due to subcooled shearing flow without bubbles, E_{TP}, and dissipation resulting from bubble generation in subcooled flow, E_{Pb}. Same terms can be considered in case of the subcolled flow boiling, but this time the respective terms contain reference to subcoled flow. Hence energy dissipation due to subcooled shearing flow without bubbles, $E_{TP,S}$, and dissipation resulting from bubble generation in subcooled flow, $E_{Pb,S}$ are considered to constitute the following relation:

$$E_{TPB,S} = E_{TP,S} + E_{Pb,S} \tag{16}$$

Energy dissipation under steady state conditions in the subcooled two-phase flow can be approximated by the energy dissipation in the laminar boundary layer, which dominates in heat and momentum transfer in the considered process. Analogically can be expressed the energy dissipation due to bubble generation in the subcooled two-phase flow. These energies are defined as the power lost in the control volume. Substituting the respective expressions for energies into (16) a geometrical summation between respective friction factors in subcooled flow is obtained:

$$\xi_{TPB,S}^2 = \xi_{TP,S}^2 + \xi_{Pb,S}^2 \tag{17}$$

Utilizing the analogy for separate processes, namely the convection and boiling between the momentum and heat transfer we can generalize the above result to extend it over to heat transfer coefficients to yield the heat transfer coefficient in flow boiling with bubble generation in the subcooled flow in terms of simpler modes of heat transfer, namely heat transfer coefficient in convective two phase flow without bubble generation in subcooled flow and heat transfer coefficient for only nucleate boiling in subcooled flow:

$$\alpha_{TPB,S}^2 = \alpha_{TP,S}^2 + \alpha_{Pb,S}^2 \tag{18}$$

Equation (18) presents a geometrical summation of convective and bubble generation components of the heat transfer model. Equation (18) can be rearranged to the form:

$$\frac{\alpha_{TPB,S}}{\alpha_{ref}} = \sqrt{\left(\frac{\alpha_{TP,S}}{\alpha_{ref}}\right)^2 + \left(\frac{\alpha_{Pb,S}}{\alpha_{ref}}\right)^2} \tag{19}$$

in which α_{ref} is the reference heat transfer coefficient. In case of subcooled flow boiling model α_{ref} is the heat transfer coefficient for the liquid only single-phase flow conditions α_O. For the case of saturated flow boiling the reference heat transfer coefficient α_{ref} should be the heat transfer coefficient at the end of zone of subcooled flow boiling, namely the location corresponding to the equilibrium quality x_{eq}=0.

2.2 Modifications of modelling for subcooled flow boiling

2.2.1. Bubble generation term in subcooled flow boiling

In case of subcooled flow boiling the heat transfer coefficient responsible for bubble generation should be the reduced pool boiling heat transfer coefficient referenced to the subcooled conditions ($\Delta T = T_w - T_f = \Delta T_{sat} + \Delta T_{sub}$). We usually have the methods for calculation of heat transfer coefficient for saturated pool boiling and not for subcooled pool boiling. Hence the approach is based on the equal heat fluxes in saturated pool boiling and subcooled pool boiling. In such case the product of heat transfer coefficient in saturated pool boiling and temperature difference between the wall and saturated conditions is the same as the product of subcooled pool boiling and temperature difference between the wall and bulk of fluid. Taking into account the total temperature drop responsible for heat transfer in subcooled flow boiling, the linearly reduced pool boiling heat transfer coefficient can be expressed as:

$$\alpha_{Pb,S} = \alpha_{Pb}(\Delta T_{sat}) \frac{\Delta T_{sat}}{\Delta T_{sat} + \Delta T_{sub}} \tag{20}$$

Similarly as for saturated conditions in case of subcooled pool boiling the corresponding heat transfer coefficient α_{Pb}, is to be calculated from the known correlations such as for example due to Cooper (1984) [16]. For the same difference between the wall and saturation temperature there is a different temperature gradient in the fluid in case of pool boiling and in case of flow boiling. In the case of flow boiling the boundary layer is thinner and hence the gradient of temperature is more pronounced, which suppresses generation of bubbles in flow boiling. In the original model for flow boiling under saturated conditions we introduced an empirical correction factor for this purpose.

The objective of this study for the case of subcooled flow boiling is to determine the convective component of the heat transfer coefficient as well as the bubble generation term. In the bubble generation term we introduce an empirical correction factor P, similarly as in the case of saturated flow boiling [2-4]. The heat transfer coefficient for subcooled flow boiling can be hence obtained from the following expression:

$$\frac{\alpha_{TPB,S}}{\alpha_{ref}} = \sqrt{\left(\frac{\alpha_{TP,S}}{\alpha_{ref}}\right)^2 + \frac{1}{1+P}\left(\frac{\alpha_{Pb}(\Delta T_{sat})}{\alpha_{ref}} \frac{\Delta T_{sat}}{\Delta T_{sat} + \Delta T_{sub}}\right)^2} \tag{21}$$

The modified correction term P in (21) does not feature the dependence on quality development, represented in [2-4] through the effect on the multiphase flow multiplier, R_{M-S}, but on the degree of subcooling $\frac{\Delta T_{sat}}{\Delta T_{sat} + \Delta T_{sub}}$ and reads:

$$P = 2.53 \times 10^{-3} \mathrm{Re}^{1.17} Bo^{0.6} \left(\frac{\Delta T_{sat}}{\Delta T_{sat} + \Delta T_{sub}}\right)^m \tag{22}$$

The influence of the subcooling is introduced in the form of the non-dimensional simplex $\frac{\Delta T_{sat}}{\Delta T_{sat} + \Delta T_{sub}}$ which is influencing the subcooled flow boiling. At the moment the exponent of the simplex has been assumed a value of m=2. The appropriate form of the correction may the task of future studies. In subsequent section the analysis will be devoted to devise a model for calculation of the term $\alpha_{TP,S}$.

2.2.2. Convective term

As has been described before, in the subcooled boiling regime the two regions can be distinguished, namely:

1. Local boiling with stationary bubbles on the surface and high subcooling,
2. Local boiling with low enough subcooling to allow bubble detachment and flow of vapor bubbles with liquid.

The maximum value of wall voidage occurs at the end of the first region. Second region starts at the point of detachment and ends at a position where the liquid subcooling becomes negligible. At high subcoolings the single phase heat transfer will still be effective but accompanied by the other mechanisms. As the subcooling decreases the heated surface will become more and more covered with bubbles and hence less accessible to the bulk liquid flow. The maximum value of wall voidage occurs at the end of the first region and can be calculated for the case of water as [7]:

$$\varphi_c = 2.435 \times 10^{-3} p^{-0.237} \frac{P_h}{A_c} \tag{23}$$

where pressure p is in N/m^2, heated perimeter P_h in meters and the core flow area in the channel, A_c in m^2. In case of calculation of the average void for steam the Zuber and Findlay relation [18] can be applied:

$$\varphi = \frac{x}{\rho_v} \left\{ C \left[\frac{x}{\rho_v} + \frac{1-x}{\rho_l} \right] + \frac{1.18}{G} \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2} \right]^{\frac{1}{4}} \right\}^{-1} \tag{24}$$

As results from equation (3) the wall heat flux on the heated surface constitutes of two principal components, namely the convective heat flux and partial evaporation of the liquid replacing the detached bubble. The convective heat flux, q_{conv}, can be modeled as due to vapour bubbles travelling longitudinally, q_l, and the liquid moving radially – liquid pumping, q_{lp}. Hence the total heat flux reads:

$$q_w = q_{conv} + q_{ev} = q_l + q_{lp} + q_{ev} \tag{25}$$

Eq. (25) is expressed therefore in terms of three components of applied wall heat flux. The convective component considers the fact that the surface is only partially covered with bubbles. The actual void fraction φ is used for that purpose. That value of void fraction should be referred to the maximum value which is possible to exist over the surface, φ_c [17]. The non-boiling fraction of heat flux $q_l=(1-\varphi/\varphi_0)\alpha_0(T_w-T_b)$ will gradually reduce with increasing wall voidage and it vanishes when the actual wall voidage reaches maximum value φ_c. The two remaining terms, i.e. the term responsible for transverse motion of fluid, q_{lp}, and evaporation, q_{ev}, are determined from the respective definitions:

$$q_w = q_l + q_{lp} + q_{ev} = \left(1-\frac{\varphi}{\varphi_c}\right)\alpha_0\left(T_w - T_b\right) + G_l C_{p,l}\left(T_w - T_{sat}\right) + G_{ev} h_{lv} \tag{26}$$

The amount of heat which going to the subcooled liquid is therefore the convective component of the applied heat flux:

$$q_{con} = q_l + q_{lp} = \left(1-\frac{\varphi}{\varphi_c}\right)\alpha_0 (T_w - T_b) + G_l C_{p,l}(T_w - T_{sat}) \tag{27}$$

On the other hand the volumetric flow rate of liquid which is agitated during the flow and then converted to vapor may be calculated assuming that the volume occupied by leaving bubbles is replaced by the volume of liquid warmed up from the saturated temperature to wall temperature $\dot{V}_l = \dot{V}_{ev}$. That result leads to the following relation:

$$\frac{q_{lp}}{C_{p,l}(T_w - T_{sat})} = \frac{q_{ev}}{h_{lv}}\frac{\rho_l}{\rho_v} \tag{28}$$

Hence the mass flux of liquid moving perpendicularly to the flow, the so called liquid pumping, can be expressed as:

$$G_l = \frac{q_{ev}}{h_{lv}}\frac{\rho_l}{\rho_v} \tag{29}$$

Subsequently, heat transfer caused by single phase convection can be expressed by:

$$q_{con} = \alpha_{con}\left(T_w - T_b\right) = \left(1-\frac{\varphi}{\varphi_c}\right)\alpha_0\left(T_w - T_b\right) + \frac{q_{ev}}{h_{lv}}\frac{\rho_l}{\rho_v} C_{p,l}\left(T_w - T_{sat}\right) \tag{30}$$

Combining equations (25) and (30) we can find the sought expression describing the heat flux due to evaporation:

$$q_{ev} = \frac{q_w - \left(1 - \frac{\varphi}{\varphi_c}\right)\alpha_0 (T_w - T_b)}{1 + \frac{C_{p,l}\rho_l (T_w - T_{sat})}{\rho_v h_{lv}} \frac{\rho_l}{\rho_v}} \tag{31}$$

Introducing (31) to (30) we can calculate the convective heat transfer coefficient in subcooled flow boiling, which in line with equation (21) corresponds to the sough term in the authors own model, i.e. $\alpha_{con}=\alpha_{TPS}$, which was the target of the research:

$$\left(\frac{\alpha_{TP,S}}{\alpha_0}\right) = 1 - \frac{\varphi}{\varphi_c} + \frac{\left[\frac{q_w}{\alpha_0 (T_w - T_b)} - \left(1 - \frac{\varphi}{\varphi_c}\right)\right] \frac{C_{p,l}(T_w - T_{sat})}{h_{lv}} \frac{\rho_l}{\rho_v}}{1 + \left(\frac{C_{p,l}\rho_l (T_w - T_{sat})}{h_{lv}\rho_v}\right)^{-1} \left(\frac{\rho_l}{\rho_v}\right)^{-1}} \tag{32}$$

Selected simplifications of equation (32) can be considered due to the value of void fraction or the Jakob number defined as $Ja = \frac{C_{p,l}(T_w - T_{sat})}{h_{lv}} \frac{\rho_l}{\rho_v}$.

In case of a very small void fraction, i.e. $\varphi \ll \varphi_c$ the approximation of (32) reads:

$$\left(\frac{\alpha_{TP,S}}{\alpha_0}\right) = 1 + \frac{\left[\frac{q_w}{\alpha_0 (T_w - T_b)} - 1\right] Ja}{1 + Ja} \tag{33}$$

In case of void fraction comparable in value to the maximum voidage in subcooled flow boiling, i.e. $\varphi \approx \varphi_c$, the expression (32) reduces to the form:

$$\left(\frac{\alpha_{TP,S}}{\alpha_0}\right) = \frac{\left[\frac{q_w}{\alpha_0 (T_w - T_b)} - 1\right] Ja}{1 + Ja} \tag{34}$$

Introducing equation (33) to the one describing authors own model of subcooled flow boiling (21) enables to calculate the local two-phase flow heat transfer coefficient for the subcooled flow boiling:

$$\left(\frac{\alpha_{TPB,S}}{\alpha_0}\right)=\sqrt{\left\{1-\frac{\varphi}{\varphi_c}+\frac{\left[\frac{q_w}{\alpha_0(T_w-T_b)}-\left(1-\frac{\varphi}{\varphi_c}\right)\right]Ja}{1+Ja}\right\}^2+\frac{1}{1+P}\left(\frac{\alpha_{Pb}(\Delta T_{sat})}{\alpha_0}\frac{\Delta T_{sat}}{\Delta T_{sat}+\Delta T_{sub}}\right)^2} \quad (35)$$

In case when φ=0 and Ja=0, equation (35) reduces to the very simple form:

$$\left(\frac{\alpha_{TPB,S}}{\alpha_0}\right)=\sqrt{1+\frac{1}{1+P}\left(\frac{\alpha_{Pb}(\Delta T_{sat})}{\alpha_0}\frac{\Delta T_{sat}}{\Delta T_{sat}+\Delta T_{sub}}\right)^2} \quad (36)$$

Eq. 36 presents the mathematical limit of equation (35). That assumption corresponds to the case of subcooled boiling in the liquid. Since the vapour mass fraction in subcooled boiling flow bears typically small values the effect of convection caused by bubbles agitation is negligibly small [19] such that $\alpha_{TPB,S}/\alpha_0$ can be set to unity. Examining relation (35) we find that for φ=0 and $Ja \to 0$ we have also $\frac{\alpha_{TPS}}{\alpha_0} \to 1$. Taking advantage of (31) one can find from (12) the true local vapor quality:

$$dx=\frac{q_{ev}U}{G_v h_{lv}}dz=\frac{q_w-\left(1-\frac{\varphi}{\varphi_c}\right)\alpha_0(T_w-T_b)}{1+\frac{C_{p,l}\rho_l(T_w-T_{sat})}{h_{lv}\rho_v}}\frac{U}{m h_{lv}}dz \quad (37)$$

Subcooled flow boiling terminates when the equilibrium quality reaches value zero. Having determined the true vapor quality x one can determine the slip, void fraction and two-phase fraction multiplayer from known two phase flow correlations for saturated two phase flow, which exist close to wall during subcooled flow boiling. Secondly, this allows to calculate heat transfer coefficient according to general formulas developed by the authors in earlier papers [2-4].

3 VALIDATION PROCEDURE

From (36) we can determine the local heat transfer coefficient for subcooled flow boiling. In subcooled boiling flow the forced convection and nucleate boiling mechanisms both contribute to heat transfer as depicted in Fig. 3. Generally the heat transfer coefficient for pool boiling can be expressed in a form:

$$\alpha_{Pb} = C_1 q^m \tag{38}$$

or assuming that m=n/(n+1) the pool boiling heat transfer coefficient can be expressed in terms of temperature difference:

$$\alpha_{Pb} = C_2 \left(T_w - T_{sat}\right)^{n/(n+1)} \tag{39}$$

Exponent m, according to the simplified theory, presented in appendix, is m=2/3 and hence n=2. Other values of m presented by various researchers are varying in the range from 2 to 4 and are worked out on the basis of experimental evidences. Constants C_1 in (38) and C_2 in (39) include information concerning properties of given fluid and parameters of boiling process.

The similar remarks concern relations describing coefficient α_0 for liquid single phase flow. For this purpose can be used for turbulent flow Dittus Boelter relation or other authors. Including relation (39) into (36) we obtain:

$$\left(\frac{\alpha_{TPB,S}}{\alpha_0}\right) = \sqrt{1 + \frac{1}{1+P}\left(\frac{1}{\alpha_0}\frac{C\Delta T_{sat}^{m+1}}{\Delta T_{sat} + \Delta T_{sub}}\right)^2} \tag{40}$$

Assuming m=2, we examine the distributions of heat transfer coefficient resulting from (40) in function of varying parameters a=C/α_0 for a given constant value of subcooling ΔT_{sub} =20°C, Fig. 4, and varying subcooling at a constant value of a=C/α_0=5, Fig. 5.

As can be seen from Fig. 4 increase of the parameter a causes the curve slope to be more steep, whereas increasing the subcooling also decreases the slope, Fig. 5.

3.1 Comparison with experimental data

For the sake of comparison of presented simple model with experimental the data available from literature have been selected due to Steiner [20], Warrier and Dhir [21], and finally Liu and Garimella [22]. These data provide some experimental evidences concerning heat transfer coefficients for subcooled flow boiling of water. To compare the derived model with experimental data we use formula (35). In (35) the heat transfer coefficient for pool boiling was calculated using equation (39) for m=2. Constant C and heat transfer coefficient for single phase flow of liquid in (35) was fitted to experimental data presented by the author's figures in their respective papers. Comparison of experimental data with present model is shown in figs. 6 to 13. As can be seen from figures the model exhibits a good consistency with the experimental data, especially in case of using equation (35) or in its simplified form using equation (36). In case of comparison with the data due to McAdams et al. (Fig. 6) we can observe that the model overpredicts the heat transfer for small superheats and underpredicts the heat transfer for superheats exceeding 20K. The effect of omitting the correction P in equation (35) is significant. Similar situation can be detected whilst examining Fig. 7. Also here at higher wall superheats the model (35) underpredicts heat transfer, however the error induced is not greater than 10%.

More detailed experimental evidence is provided by the data due to Steiner (2004). In that case we have two series of experiments at the constant subcooling, one executed at p=1.5bar and another one at p=2bar. In both series the bulk velocity was varied assuming three values, namely u_b = 0.05, 0.39 and 1.17m/s. The overall agreement between the predictions of the postulated model and the experimental data is very good. Particularly, in the case of small velocity (u_b = 0.05 and 0.2 m/s) the model predicts the shift of the onset of nucleate boiling to higher wall superheats accompanied by a relative reduction of the boiling component in the total heat flux very accurately. This indicates that the model is capable to capture the strong flow induced supression of the nucleate boiling in the low void regime quite well. For higher wall superheats, however, approaching the high void regime regime, the agreement becomes worse especially in case of the higher saturation pressure considered (p = 2bar). It is thought that the predictions of the model are less accurate once phenomena related to the very complex multi-bubble dynamics become important. Due to the high bubble number densities, which are typically found on the bubble-bubble interaction as well as a notable twoway coupling between the motion of the bubbles and the liquid phase. In such a regime bubbles tend to coalesce forming larger structures on the surface.

4. CONCLUSIONS

Basing on concept of dissipation of energy in partially flow boiling simple model of subcooling boiling was derived. Obtained results are quite satisfactory. But the main trends of parameters describing these complex phenomena are reflected by this model. Still are needed further investigations of the phenomenon in other to have more data which allow finding correction factor in model to describe phenomenon more closely qualitatively.

LITERATURE

1. G.R. Warrier, V.K. Dhir, Heat Transfer and Wall Heat Flux Partitioning During Subcooled Flow Nucleate Boiling—A Review, Journal of Heat Transfer, 128 (2006) 1243-1256.
2. J. Mikielewicz, Semi-empirical method of determining the heat transfer coefficient for subcooled saturated boiling in a channel, Int. J. Heat Transfer, 17 (1973) 1129-1134.
3. D. Mikielewicz, J. Mikielewicz, J. Tesmar, Improved semi-empirical method for determination of heat transfer coefficient in flow boiling in conventional and small diameter tubes, Int. J. of Heat and Mass Transfer, 50 (2007) 3949-3956.
4. D. Mikielewicz, A new method for determination of flow boiling heat transfer coefficient in conventional diameter channels and minichannels, Heat Transfer Engineering, 31(4) (2010) 276-287.
5. P. Griffith, A. Clark, W.W. Rohsenow, Void volumes in subcooled boiling systems, ASME 58-HT-19, 1958.
6. R.W. Bowring, Physical model of bubble detachment and void volume in subcooled boiling, OECD Halden Reactor Project Rep. HPR-10, 1962.
7. S.Z. Rouhani, E. Axelsson, Calculation of Void Volume Fraction in the Subcooled Volume and Quality Boiling Regions, Int. Journal of Heat Mass Transfer, 13 (1970) 383-393.
8. S.Y. Ahmad, Axial Distribution of Bulk Temperature and Void Fraction in a Heater Channel With Inlet Subcooling, ASME J. Heat Transfer, 92 (1970) 595–609.
9. L. Maroti, Axial Distribution of Void Fraction in Subcooled Boiling, Nuclear Technology, 34 (1977) 8–17.

10. R.T. Lahey, A Mechanistic Subcooled Boiling Model, Proceedings of the 6th International Heat Transfer Conference, Toronto, Canada, Hemisphere Publishing Corporation, Washington, 1 (1978) 293–297.
11. O. Zeitoun, Subcooled Flow Boiling and Condensation, Ph.D. thesis, McMaster, University, Hamilton, Ontario, Canada, 1994.
12. Z. Liu, R.H.S. Winterton, A General Correlation for Saturated and Subcooled Flow Boiling in Tubes and Annuli, Based on a Nucleate Pool Boiling Equation, Int. J. Heat Mass Transfer, 34 (1991) 2759–2766.
13. Y.Y. Hsu, On the size range of active nucleation cavities on heating surface J. Heat Transfer, 84 (1962) 207-213.
14. A.E. Bergles, W.M. Rohsenow, The determination of forced convection surface boiling heat transfer, Int. J. Heat Mass Transfer, 86 (1964) 365–372.
15. T. Sato, H. Matsumura, On the conditions of incipient subcooled boiling and forced-convection, Bull. JSME, 7 (1963) 392–398.
16. M.G. Cooper, Saturation Nucleation Boiling—A Simple Correlation, IChemE Symp. Ser., 86 (1984) 786–793.
17. W.M. Rohsenow, A method of correlating heat transfer data for surface boiling of liquids, Trans. ASME, 74, 1951.
18. N. Zuber, J. Findlay, Average volumetric concentration in two phase flow systems, J Heat Transfer 87, Ser. C (1965) 453-462.
19. P. Van Carey, Liquid –Vapor Phase Change Phenomena, Hemisphere 1992.
20. H. Steiner, A. Kobor, L. Gebhard, A wall heat transfer model for subcooled boiling flow, Proceeding at the ASME-ZSIS International Thermal Science Seminar II, Bled, Slovenia, 2004.

21. N. Basu, G.R. Warrier, V.K. Dhir, Subcooled flow boiling Part1, Model Development, J of Heat Transfer, 27 (2005) 131-139.

22. D. Liu, S.V. Garimella, Flow Boiling Heat Transfer in Microchannels, ASME Journal of Heat Transfer, 129 (2007) 1321-1332.

23. M.Z. Podowski, Modeling and analysis of subcooled boiling dynamics, ANS Proceedings, National Heat Transfer Conference, 1988.

24. Y.Y. Hsu, Boiling heat transfer equations. In *Thermohydraulics of Two-Phase Systems for Industrial Design and Nuclear Engineering*, Delhaye, J. M., Giot, M., and Riethermuller, M. L., Eds., McGraw-Hill, New York, 255–296, 1981.

25. V. Chatoorgoon, G.R. Dimmick, M.B. Carver, W.N. Selander, M. Shoukri., Application of generation and condensation models to predict subcooled boiling void at low pressures, Nuclear Technology, 98 (1992) 366-378.

26. S.S. Kutateladze, Boiling heat transfer, Int. J. of Heat and Mass Transfer, 14 (1961) 31-45.

27. J.C. Chen J.C., A correlation for boiling heat transfer to saturated fluids in convective flow, Ind. Eng. Chem. Process Des. Dev., 5(3) 1966 322–329.

Appendix A

Proof of existence for the relation (39) for the assumption of n=2/3.

In a thin thermal sublayer close to the wall during boiling process there exists a number of bubbles. The minimum radius of such bubbles in equilibrium with boundary layer is approximately equal [19]:

$$R_{\min} = \frac{2\sigma \vartheta_{lv} T_s}{h_{lv} \Delta T_s} \tag{A1}$$

These bubbles agitate in the boundary sublayer causing that its thickness is chaotically changing. The period of change can be estimated from:

$$\tau = \frac{R_{\min}}{w_v} \qquad \text{(A2)}$$

In (A2) w_v is the superficial velocity of vapor phase leaving the wall:

$$w = \frac{q}{h_{lv}\rho_v} \qquad \text{(A3)}$$

Change of the effective thickness of sublayer is caused by spreading disturbances and can be evaluated from the penetration theory in semi-infinite space as [19]:

$$\delta_{ef} = \sqrt{\pi a \tau} \qquad \text{(A4)}$$

Heat transfer coefficient for a thin sublayer can be determined from conduction:

$$\alpha = \frac{\lambda}{\delta_{ef}} \qquad \text{(A5)}$$

On the other hand the heat transfer coefficient can be determined from Newton's equation:

$$\alpha = \frac{q}{\Delta T_s} \qquad \text{(A6)}$$

Combining (A5) and (A6) the ΔT_s can be determined. Subsequently the result to (A6) the expression for the heat transfer coefficient in function of the applied wall heat flux is obtained in the form:

$$\alpha = \left(\frac{\lambda^2}{2\pi a \sigma T_s}\right)^{\frac{1}{3}} q^{\frac{2}{3}} = C q^{\frac{2}{3}} \qquad \text{(A7)}$$

In (A7) $C = \left(\frac{\lambda^2}{2\pi a \sigma T_s}\right)^{\frac{1}{3}}$ we obtain the expression (26) for n=2/3.

Figure captions:

Fig. 1. Incipience of flow boiling.

Fig. 2. Variation of void fraction with axial distance

Fig. 3. Boiling curve with specified different modes of heat transfer.

Fig. 4. Effect of parameter a on heat transfer coefficient for subcooled flow boiling, ΔT_{sub}=20^{o}C.

Fig. 5. Effect of varying subcooling on heat transfer coefficient for subcooled flow boiling, a=5.

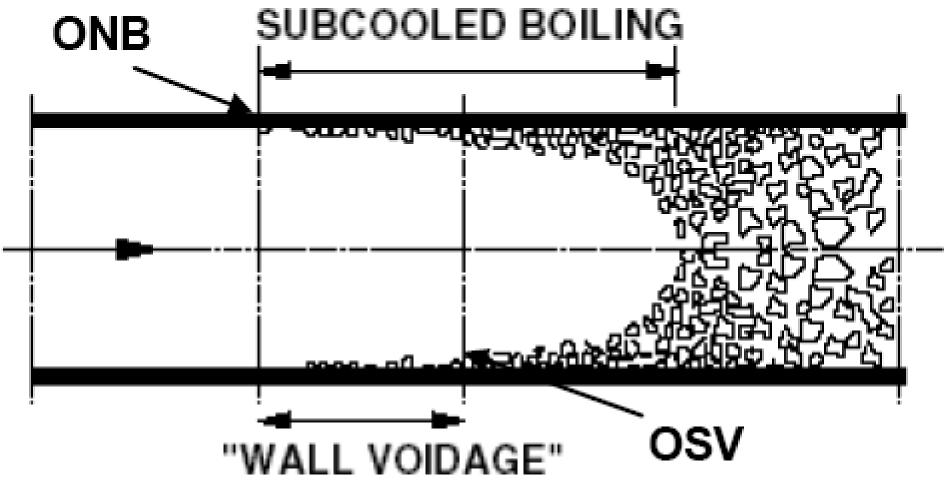
Fig. 6. Comparison with experimental data due to McAdams at al. (1949), p=4.14bar, ΔT_{sub}=27.8^{o}C, u_b=0.95m/s

Fig. 7. Comparison with experimental data due to Liu and Garimella (2007), p=0.56bar, ΔT_{sub}=6.7^{o}C, u_b=0.96m/s

Fig. 8. Comparison with experimental data due to Steiner at al. (2004), p=1.5bar, ΔT_{sub}=16^{o}C, u_b=0.05m/s.

Fig. 9. Comparison with experimental data due to Steiner at al. (2004), p=1.5bar, ΔT_{sub}=16^{o}C, u_b=0.39m/s.

Fig. 10. Comparison with experimental data due to Steiner at al. (2004), p=1.5bar, ΔT_{sub}=16^{o}C, u_b=1.17m/s.



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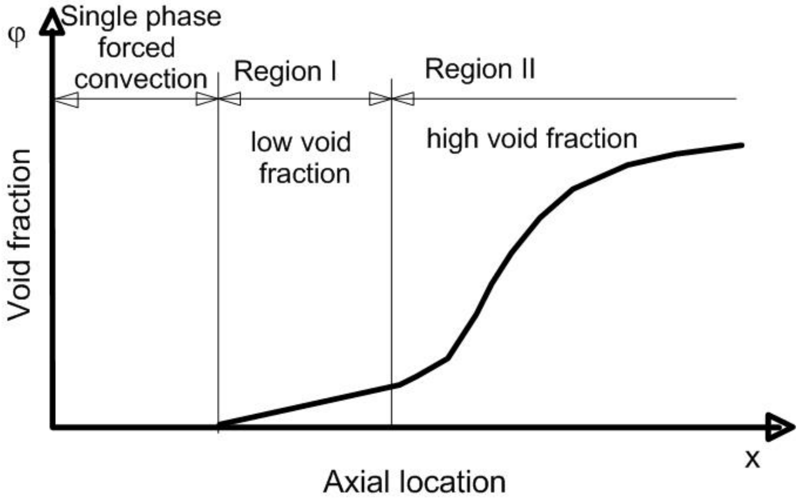


Fig. 2. Variation of void fraction with axial distance

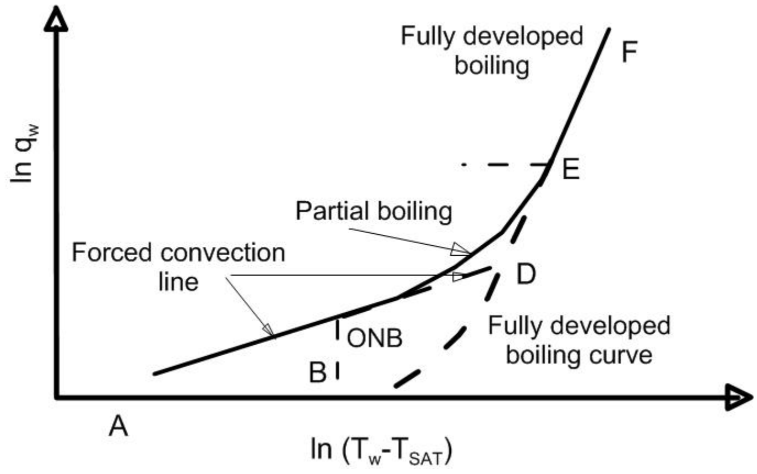
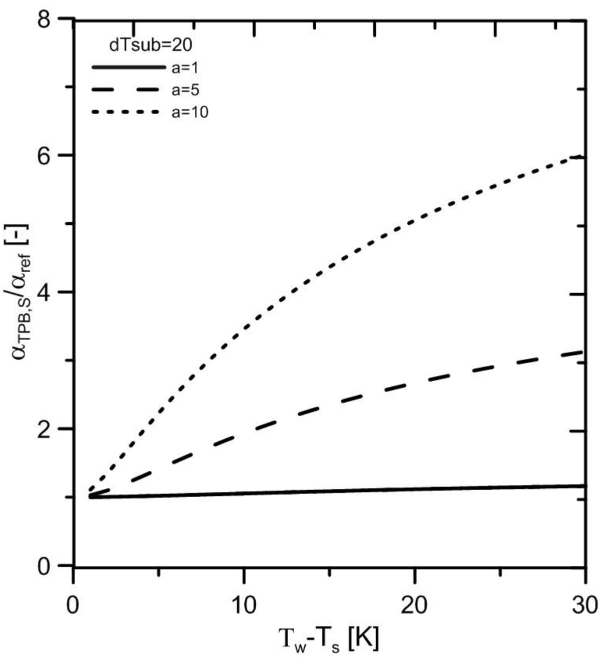


Fig. 3. Boiling curve with specified different modes of heat transfer.



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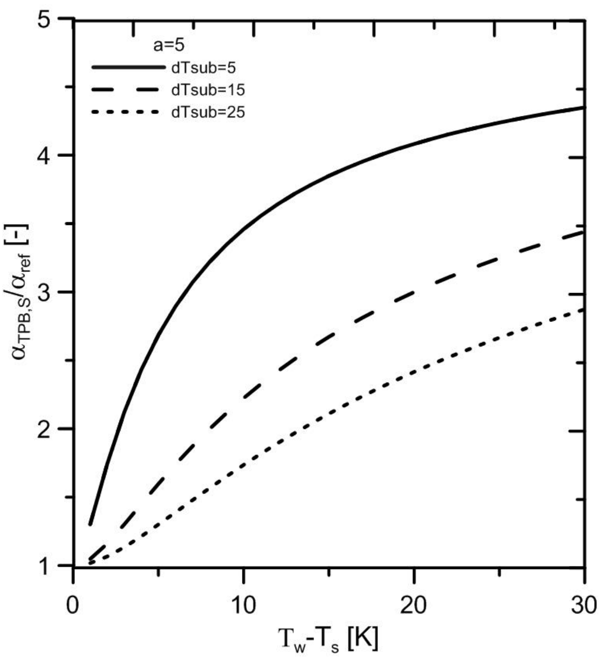

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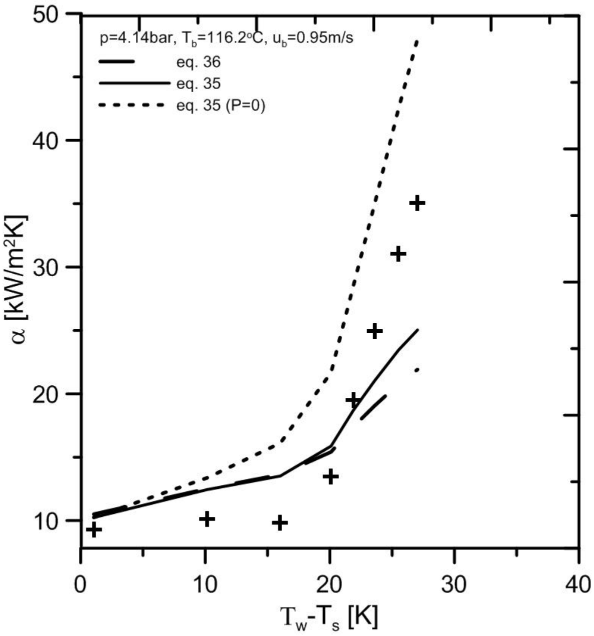

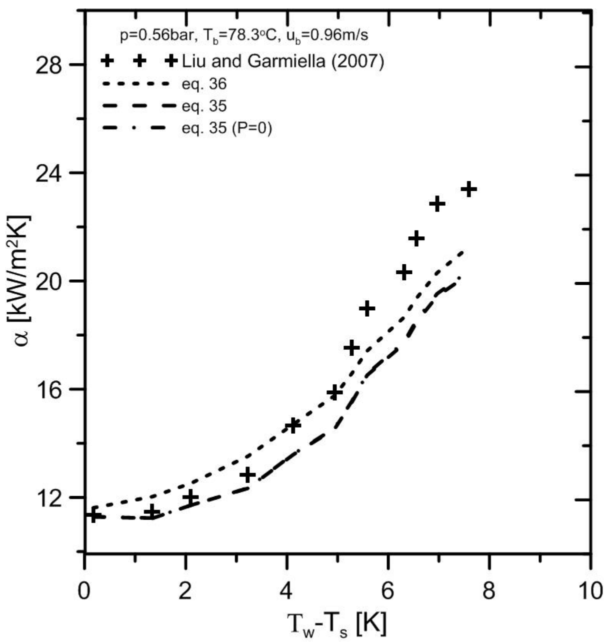

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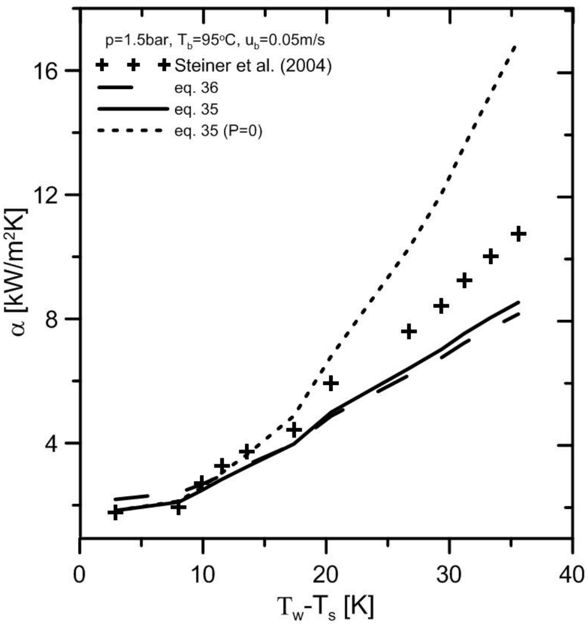

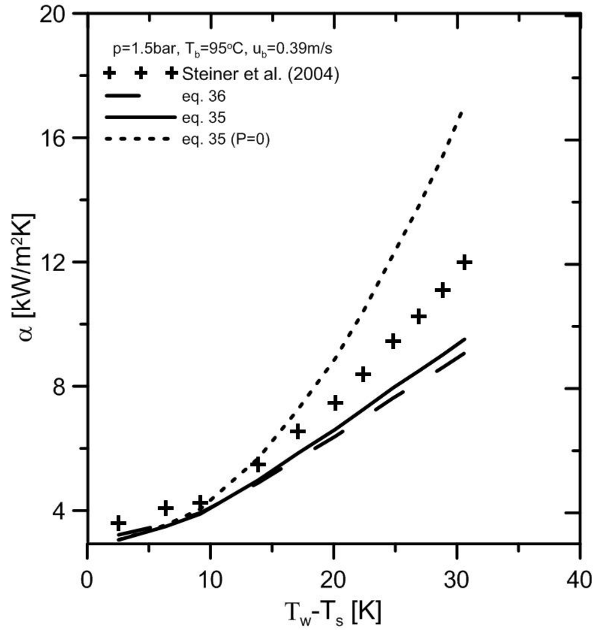

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Fig. 9. Comparison with experimental data due to Steiner at al. (2004), p=1.5bar, ΔT_{sub}=16^{o}C, u_b=0.39m/s.

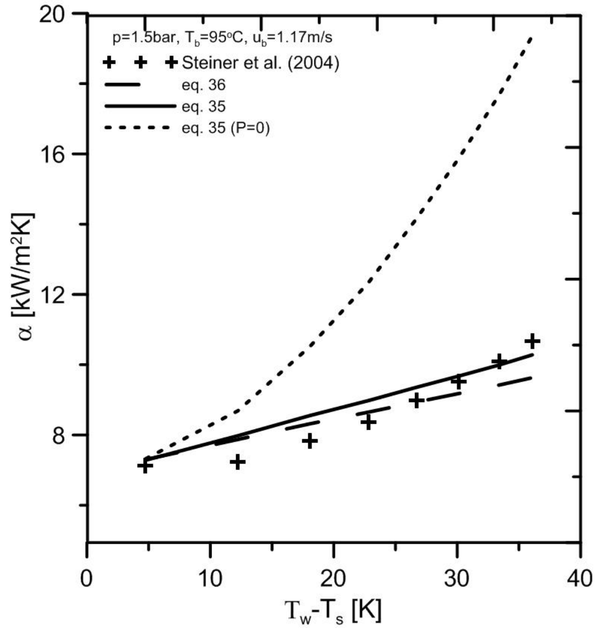

Fig. 10. Comparison with experimental data due to Steiner at al. (2004), p=1.5bar, ΔT_{sub}=16°C, u_b=1.17m/s.

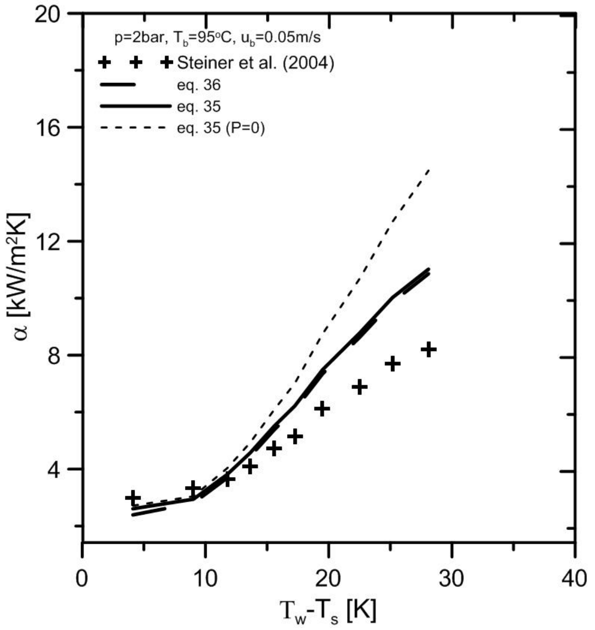

Fig. 11. Comparison with experimental data due to Steiner at al. (2004), p=2bar, ΔT_{sub}=16°C, u_b=0.05m/s.

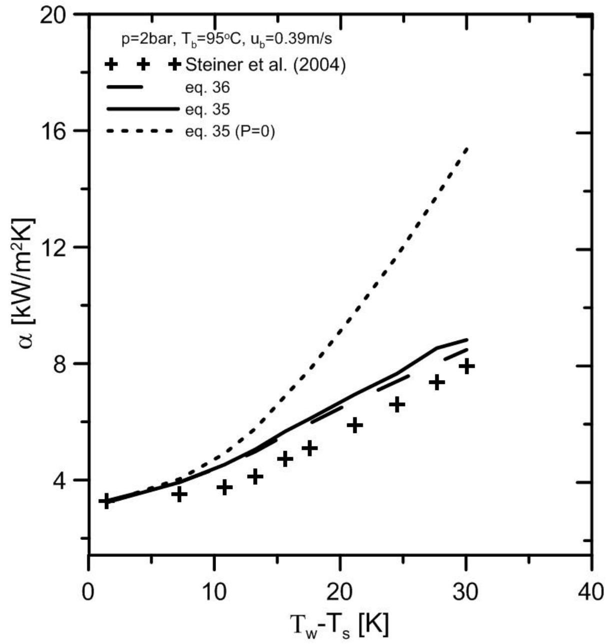

Fig. 12. Comparison with experimental data due to Steiner at al. (2004), p=2bar, ΔT_{sub}=16°C, u_b=0.39m/s.

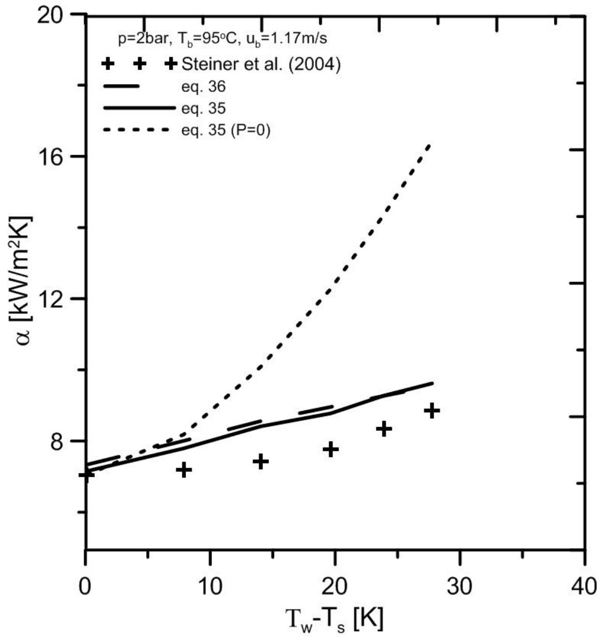

Fig. 13. Comparison with experimental data due to Steiner at al. (2004), p=2bar, ΔT_{sub}=16°C, u_b=1.17m/s.