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About definition of modes and magnetosonic heating in a plasma's flow. Especial cases of perpendicular and nearly perpendicular wave vector and magnetic field

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Summary

Dynamics of hydrodynamic perturbations in a plasma depend strongly on an angle between the wave vector and equilibrium straight magnetic field. The case of perpendicular propagation is especial. There are only two (fast) magnetosonic modes since two (slow) ones degenerate into the stationary one with zero speed of propagation. This demands individual definition of wave modes by the links of hydrodynamic relations. These links are not limiting case of the relations in the case of non-zero angle. The nonlinear excitation of the entropy mode in the field of intense magnetosonic perturbations is also unusual. Bulk and shear viscosity and thermal conduction are considered as the damping mechanisms in a weakly nonlinear flow. The leading-order dynamic equation is derived which governs perturbation of density in the entropy mode. The links of magnetosonic perturbations and magnetosonic heating may be indicators of plasma-β, geometry of a flow, damping coefficients and type of wave motion. The "almost resonant" character of magnetosonic heating excited by the slow magnetosonic wave in the course of nearly perpendicular wave propagation, is discussed.

1 Introduction

The wave processes in a plasma and associated non-linear phenomena are of growing importance. This concerns laboratory and astrophysical applications (see e.g. [1, 2, 3]). Links of small-signal perturbations determine every wave or non-wave mode on a pair with the corresponding dispersion relation in a linear flow. The dispersion relations for non-wave modes are degenerative in an ideal fluid flow in the absence of magnetic field with zero thermal conduction and viscosity. Three-dimensional flow specify three non-wave modes with zero dispersion relations, $\omega(k) = 0$. That leads to ambiguously defined non-wave eigenvectors corresponding to these roots. One of them is the entropy mode with any stationary perturbation in density and zero velocity and perturbation of pressure. The second is vorticity mode determined by condition of incompressibility for the velocity field, $\vec{\nabla} \cdot \vec{v} = 0$ and zero perturbations of density and pressure. Any linear combination of these modes also represents a mode and specifies non-wave stationary fluid flow. Including in consideration thermal conduction eliminates degeneracy of the entropy mode, and including in consideration shear viscosity eliminates degeneracy of the vorticity mode. As for the two branchs of sound, they rely to different dispersion relations in both ideal and damping flow.

The flows of plasma are much more complex compared to the fluid flow in the absence of magnetic field. Presence of magnetic field introduces non-isotropy of a flow and brings slow and fast magnetosonic modes with variety of propagation speeds which leads to unusual degenerative dispersion relations and ambiguously defined modes. In particular, slow wave modes may degenerate into non-wave ones with zero speed of propagation. This never happens to the homogeneous flows where sound speed does not vary and degeneration may be caused only by zero damping coefficients. The special case of degeneracy connected with transformation of wave modes into non-wave ones, should be considered individually. The key issue is proper re-definition of modes; that yields also unusual nonlinear phenomena.

The nonlinear effects are still unresolved issue in many applications of plasma dynamics but are recognized as having a key influence (inter alia, the energy transfer from the transition region into the corona is understood as a nonlinear phenomenon [4, 5]). Nonlinear excitation of the entropy mode by intense sound [6, 7], that is, magnetoacoustic heating, may be confidently observed remotely. In astrophysical applications, the slow evolution of entropy perturbations is the main and often the only source of information concerning equilibrium parameters of a plasma, geometry of a flow and a wave driver. It is especial if excited by perpendicularly or nearly perpendicularly propagating wave. The mathematical content of the study has been used in description of various linear and weakly nonlinear fluid flows [8]. The initial point is to determine links between hydrodynamic perturbations specifying any mode on a pair with the dispersion relation. The next issue is to evaluate the projecting operators which subdivide the individual modes from the total field of perturbations. They subdivide also dynamic equations for perturbations in a specific mode when operating on the system of conservation equations. Going to investigation of weakly nonlinear dynamics in the field of intense wave, we still make use of the specific links and corresponding projecting. Projecting is in fact a linear combination of the conservation equations which result in the system of coupling evolutionary equations for the interacting modes in a weakly nonlinear flow.

2 Finite-magnitude perturbations in a plasma's flow with mechanical and thermal losses

We start from a set of MHD (magnetofydrodynamic) equations describing dynamics of fully ionized gas which take into account thermal conduction and mechanical viscosity. It consists of the continuity equation, the momentum equation, the energy balance equation and electrodynamic equations in the differential form (e.g. [9, 10]):

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0,$$

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$$\rho\frac{D\vec{v}}{Dt} = -\vec{\nabla}p + \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\times\vec{B} + \eta\Delta\vec{v} + \left(\frac{1}{3}\eta+\xi\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v}), \tag{1}$$

$$\frac{Dp}{Dt} - \gamma\frac{p}{\rho}\frac{D\rho}{Dt} = (\gamma-1)\left[\vec{\nabla}\cdot(\chi\vec{\nabla}T) + \xi(\vec{\nabla}\cdot\vec{v})^2 + \frac{\eta}{2}\sum_{i,j=1,2,3}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{i,k}\vec{\nabla}\cdot\vec{v}\right)^2\right],$$

$$\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}),$$

$$\vec{\nabla}\cdot\vec{B} = 0,$$

where p, ρ, $\vec{v}$, $\vec{B}$, are hydrostatic pressure and density of a plasma, its velocity, the magnetic field, and μ_0 is the permeability of free space. The third equation incorporates the continuity equation and the energy balance. It refers to an ideal gas with the adiabatic index $\gamma = C_P/C_V$ (C_P and C_V denote the specific heats per unit mass under constant pressure and constant density, respectively), $T = \frac{p}{\rho(C_P-C_V)}$ is the temperature of a plasma. We make use of the Navier-Stokes form of the viscous tensor [6, 10]. The viscous coefficients η and ξ are referred to as shear (first) and bulk (second) viscosity in classic fluid mechanics. Thermal conduction coefficient consists of two parts,

$$\chi = \chi_{||}\cos^2(\theta) + \chi_{\perp}\sin^2(\theta), \tag{2}$$

where here $\chi_{||}$ and $\chi_{\perp}$ are the thermal conduction coefficients parallel and perpendicular to the magnetic field. The "perpendicular thermal conductivity" in strongly magnetized plasmas is much less than "parallel thermal conductivity" [11]. The transport coefficients in some studies are assumed to be isotropic in view of microturbulence [11, 12, 13, 14]. This concerns also the thermal conduction χ [11, 15] (particularly, isotropic transport coefficients is a good approximation in studies of dynamics of the solar corona [16]). All transport parameters are treated as constants which do not vary with coordinates.

We consider the geometry of a flow accepted in Refs [15, 17]: the straight equilibrium magnetic field $\vec{B}_0$ forms the constant angle θ ($0 \le \theta \le \pi$) with the wave vector which is directed along axis z. The y-component of $\vec{B}_0$ equals zero, so as

$$B_{0,x} = B_0\sin(\theta), \quad B_{0,z} = B_0\cos(\theta), \quad B_{0,y} = 0.$$

All thermodynamic quantities are expanded around the equilibrium thermodynamic state as $f(z,t) = f_0 + f'(z,t)$. The bulk flow is absent, $\vec{v}_0 = \vec{0}$. The leading-order system which describes weakly nonlinear phenomena, includes quadratically nonlinear terms

$$\frac{\partial\rho'}{\partial t} + \rho_0\frac{\partial v_z}{\partial z} = -\rho'\frac{\partial v_z}{\partial z} - v_z\frac{\partial\rho'}{\partial z},$$

$$\frac{\partial v_x}{\partial t} - \frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z} - \frac{\eta}{\rho_0}\frac{\partial^2 v_x}{\partial z^2} = -v_z\frac{\partial v_x}{\partial z} - \frac{B_{0,z}}{\rho_0^2\mu_0}\rho'\frac{\partial B_x}{\partial z}, \tag{3}$$

$$\frac{\partial v_y}{\partial t} - \frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_y}{\partial z} - \frac{\eta}{\rho_0}\frac{\partial^2 v_y}{\partial z^2} = -v_z\frac{\partial v_y}{\partial z} - \frac{B_{0,z}}{\rho_0^2\mu_0}\rho'\frac{\partial B_y}{\partial z},$$

$$\frac{\partial v_z}{\partial t} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z} - \left(\frac{4\eta}{3\rho_0}+\frac{\xi}{\rho_0}\right)\frac{\partial^2 v_z}{\partial z^2} = \frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0^2\mu_0}\rho'\frac{\partial B_x}{\partial z} - \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{B_x^2+B_y^2}{2\mu_0}\right) -$$

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$$v_z\frac{\partial v_z}{\partial z}-\left(\frac{4\eta}{3\rho_0}+\frac{\xi}{\rho_0}\right)\frac{\rho'}{\rho_0}\frac{\partial^2 v_z}{\partial z^2},$$

$$\frac{\partial p'}{\partial t}+c^2\rho_0\frac{\partial v_z}{\partial z}-\frac{\chi}{\rho_0 C_P}\frac{\partial^2\gamma p'}{\partial z^2}+\frac{\chi c_0^2}{\rho_0 C_P}\frac{\partial^2\rho'}{\partial z^2}=-\gamma p'\frac{\partial v_z}{\partial z}-v_z\frac{\partial p'}{\partial z}+(\gamma-1)\left(\frac{4\eta}{3}+\xi\right)\left(\frac{\partial v_z}{\partial z}\right)^2+$$

$$(\gamma-1)\eta\left(\left(\frac{\partial v_x}{\partial z}\right)^2+\left(\frac{\partial v_y}{\partial z}\right)^2\right)-\frac{\chi}{\rho_0^2 C_P}\frac{\partial^2(\gamma p'\rho'-c_0^2\rho'^2)}{\partial z^2},$$

$$\frac{\partial B_x}{\partial t}+\frac{\partial}{\partial z}\left(B_{0,x}v_z-B_{0,z}v_x\right)=-B_x\frac{\partial v_z}{\partial z}-v_z\frac{\partial B_x}{\partial z},$$

$$\frac{\partial B_y}{\partial t}-\frac{\partial}{\partial z}\left(B_{0,z}v_y\right)=-B_y\frac{\partial v_z}{\partial z}-v_z\frac{\partial B_y}{\partial z}.$$

It represents the Taylor series expansions of Eqs(1) in powers of the magnetosonic Mach number M up to the second order, where M is a ratio of velocity magnitude to the speed of magnetosonic perturbations. Hence, there are seven unknown variables and the same number of dispersion relations and modes of a flow. The linear damping may be described by the generic small dimensionless parameter, say, λ, which ensures weak attenuation of magnetosonic perturbations in the course of propagation. The terms of order M and $M\lambda$ are collected on the left of Eqs(3), and the terms $O(M^2)$, $O(M^2\lambda)$ are collected on its right. The nonlinear terms of order $O(M^2\lambda)$ are of importance in the context of magnetosonic heating although they are smallest among other terms.

The dispersion relations follow from linearized Eqs(3) if one looks for all hydrodynamic perturbations in the form of a sum of planar waves with the wave vector k and frequency ω (that is, proportional to $\exp(i\omega(k)t-ikz)$). C is the magnetosonic speed satisfying the equation (e.g.,[18, 19, 15])

$$C^4-C^2(c_0^2+C_A^2)+c_0^2C_{A,z}^2=0, \tag{4}$$

and C_A and c_0

$$C_A=\frac{B_0}{\sqrt{\mu_0\rho_0}},\quad c_0=\sqrt{\frac{\gamma p_0}{\rho_0}}$$

designate the Alfvén speed and the acoustic speed in non-magnetized gas in equilibrium, $C_{A,z}=C_A\cos(\theta)$. Eq.(4) represents two branches (fast and slow) propagating in the positive direction of axis z and two branches propagating in the negative direction of axis z with the same modules of speeds. The exception is the case $\theta=\frac{\pi}{2}$ which leads to the double non-wave zero root and speeds of fast modes $C=\pm\sqrt{c_0^2+C_A^2}$. Hence, there are only two wave modes which specify this case. The case $\theta=0$ yield degeneration of two magnetosonic p-modes into the Alfvén modes with $C=\pm C_A$. These two cases should be considered individually. Neither the definition of modes nor nonlinear effects are limiting cases of the more general ones for θ differing from 0 and $\pi/2$. All evaluations in this study (dispersion relations, links between specific perturbations, dynamic equations) are leading-order, that is, they contain terms up to $M\lambda$ in regard to linear phenomena and $M^2\lambda$ in regard to nonlinear phenomena. The dispersion relations which are valid apart from the specific cases, are well-known. They correspond to two Alfvén branches

$$\omega=\pm C_{A,z}k+i\frac{\eta}{2\rho_0}k^2, \tag{5}$$

four magnetosonic p-branches

$$\omega=Ck+i\frac{\alpha}{2}k^2, \tag{6}$$

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where

$$\alpha = \alpha_\eta \frac{4\eta}{3\rho_0} + \alpha_\xi \frac{\xi}{\rho_0} + \alpha_\chi \frac{(\gamma-1)\chi}{C_p \rho_0},$$

$$\alpha_\eta = \frac{C^2(C^4 + C^2(6c_0^2 - C_A^2) - 3c_0^2(c_0^2 + C_A^2))}{4c_0^2(C^4 - c_0^2 C_{A,z}^2)}, \quad \alpha_\xi = \frac{C^4(C^2 - C_A^2)}{c_0^2(C^4 - c_0^2 C_{A,z}^2)}, \quad \alpha_\chi = \frac{C^2(C^2 - C_A^2)}{C^4 - c_0^2 C_{A,z}^2},$$

and the entropy mode

$$\omega = i \frac{\chi}{C_P \rho_0} k^2. \tag{7}$$

Eq.(6) has been established by Nakariakov, Chin et al.[15, 17] in the flows of a plasma with only one damping factor, that is, thermal conduction, and that due to the shear viscosity was derived in Ref.[20]. Links of perturbations in any individual mode are determined by the corresponding dispersion relation. In particular, four magnetosonic modes, two Alfvén modes and the entropy mode apart from the specific cases are determined by relations

$$\psi_{ms} = \begin{pmatrix} \rho' \\ v_x \\ v_y \\ v_z \\ p' \\ B_x \\ B_y \end{pmatrix}_{ms} = \tag{8}$$

$$\begin{pmatrix} 1 \\ -\frac{C_{A,z}(C^2-c_0^2)}{C_{A,x} C \rho_0} + \frac{C^2 C_{A,z}(C^2-c_0^2)(C^2-2c_0^2-C_A^2)}{2\rho_0 C_{A,x} c_0^2 (C^4-c_0^2 C_{A,z}^2)} \left(\frac{\xi}{\rho_0} + (\gamma-1)\frac{c_0^2}{C^2}\frac{\chi}{C_P \rho_0} \right) \frac{\partial}{\partial z} + \\ \eta \frac{C_{A,z}(C^2-c_0^2)(C^4+C^2(4c_0^2-C_A^2)-3c_0^2(c_0^2+C_A^2))}{6\rho_0 C_{A,x} c_0^2 (C^4-c_0^2 C_{A,z}^2)} \frac{\partial}{\partial z} \\ 0 \\ \frac{C}{\rho_0} - \frac{C^4(C^2-C_A^2)}{2\rho_0 c_0^2 (C^4-c_0^2 C_{A,z}^2)} \left(\frac{\xi}{\rho_0} + (\gamma-1)\frac{c_0^2}{C^2}\frac{\chi}{C_P \rho_0} \right) \frac{\partial}{\partial z} - \\ \frac{\eta}{\rho_0} \frac{C^2(C^4+C^2(6c_0^2-C_A^2)-3c_0^2(c_0^2+C_A^2))}{6\rho_0 c_0^2 (C^4-c_0^2 C_{A,z}^2)} \frac{\partial}{\partial z} \\ c_0^2 - (\gamma-1)\frac{\chi}{C_P \rho_0}\frac{c_0^2}{C}\frac{\partial}{\partial z} \\ \frac{(C^2-c_0^2)\mu_0}{B_{0,x}} + \frac{C C_{A,z}^2 (C^2-c_0^2)\mu_0}{B_{0,x}(C^4-c_0^2 C_{A,z}^2)} \left(\frac{\xi}{\rho_0} + \frac{\eta}{3\rho_0} + (\gamma-1)\frac{c_0^2}{C^2}\frac{\chi}{C_P \rho_0} \right) \frac{\partial}{\partial z} \\ 0 \end{pmatrix} \rho_{ms},$$

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$$\psi_A = \begin{pmatrix} \rho' \\ v_x \\ v_y \\ v_z \\ p' \\ B_x \\ B_y \end{pmatrix}_A = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \mp\frac{B_0}{C_A} - \eta\frac{B_0}{2\rho_0 C_A C_{A,z}}\frac{\partial}{\partial z} \end{pmatrix} v_{y,A}, \quad \psi_{ent} = \begin{pmatrix} 1 \\ -\frac{\chi C_{A,x}}{C_{A,z} C_P \rho_0^2}\frac{\partial}{\partial z} \\ 0 \\ -\frac{\chi}{C_P\rho_0^2}\frac{\partial}{\partial z} \\ 0 \\ 0 \\ 0 \end{pmatrix} \rho_{ent}, \tag{9}$$

where ρ_{ms}, $v_{y,A}$, ρ_{ent} are the reference perturbations for the magnetosonic, Alfvén and entropy modes. The links accounting shear viscosity and thermal conductivity without account of bulk viscosity have been derived in [21]. Evidently, these links are not valid if $\theta = 0$ or $\theta = \pi/2$ (denominators in some expressions equal zero) and should be determined individually in these specific cases. As for the case $\theta = 0$, it is well studied regarding weakly nonlinear dynamics magnetosonic perturbations (e.g., [22, 23]). We focus on the perpendicular propagation in this study.

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3 Dispersion relations and specific perturbations. Case $\theta = \pi/2$

In this case, $C_{A,z} = 0$, $C = \pm C_\perp$ ($C_\perp = \sqrt{c_0^2 + C_A^2}$), and the links for the Alfvén and the entropy modes given by Eqs(9) are not longer valid, as well as the links for the magnetosound modes (8) (there are only two fast wave modes, since slow modes degenerate into the stationary one with zero roots of dispersion relation). The properties of a flow, including definition of modes as links of specific perturbations and nonlinear phenomena, change abruptly. We will number modes, corresponding relations and the reference variables inherent to each mode for convenience. The reference perturbations are designated as $B_{y,1}$, $B_{x,2}$, ρ_3, $v_{x,4}$, $v_{y,5}$, ρ_6, ρ_7. The corresponding links for two degenerate stationary modes with the dispersion relations

$$\omega_{1,2} = 0 \tag{10}$$

sound

$$\psi_1 = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1)B_{y,1}, \quad \psi_2 = (-\frac{\gamma C_A^2\rho_0}{c_0^2 B_0} \quad 0 \quad 0 \quad 0 \quad -\frac{C_A^2\rho_0}{B_0} \quad 1 \quad 0)B_{x,2}. \tag{11}$$

ψ_1, ψ_2 are very especial types of motion which appear only in the case $\theta = \pi/2$. They do not remind the only non-wave entropy mode in the case $\theta \neq \pi/2$. ψ_1 relies on one non-zero stationary perturbation B_y, and ψ_2 connects stationary perturbations ρ', p' and B_x. This mode specifies zero perturbation in temperature, $\frac{T'}{T_0} = \frac{p'}{p_0} - \frac{\rho'}{\rho_0} = 0$. Any linear combination of ψ_1, ψ_2 also represents the stationary mode, so we face with ambiguous definition of the stationary modes. The entropy mode is determined by the dispersion relation

$$\omega_3 = i\frac{(c_0^2 + \gamma C_A^2)k^2}{c_0^2 + C_A^2}\frac{\chi}{C_P\rho_0} \tag{12}$$

and the links

$$\psi_3 = (1 \quad 0 \quad 0 \quad -\frac{\chi}{C_P\rho_0^2}\frac{c_0^2 + \gamma C_A^2}{c_0^2 + C_A^2}\frac{\partial}{\partial z} \quad -C_A^2, \quad \frac{B_0}{\rho_0} \quad 0)\rho_3. \tag{13}$$

Evidently, ω_3, ψ_3 is not a special case of ω, ψ_{ent} (Eqs (7), (9)) apart from the case without magnetic field with $B_0 = 0$. There are two degenerate modes which associate with the dynamical viscosity,

$$\omega_{4,5} = ik^2\frac{\eta}{\rho_0},$$

$$\psi_4 = (0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)v_{x,4}, \quad \psi_5 = (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0)v_{y,5}. \tag{14}$$

Neither dispersion relation nor ψ_4, ψ_5 are limiting cases of Alfvén modes (Eqs(5),(9)). ψ_4, ψ_5 represent diffusion of transversal components of velocities with coefficient of damping η/ρ_0 which is two times larger than that of Alfvén modes. Any linear combination of linearly independent vectors ψ_4 and ψ_5 also represents a mode corresponding to $\omega_{4,5}$. Two magnetosound modes are determined by relations:

$$\omega_{6,7} = \pm C_\perp k + i\left(\frac{\xi}{2\rho_0} + \frac{2\eta}{3\rho_0} + (\gamma-1)\frac{c_0^2}{2(c_0^2+C_A^2)}\frac{\chi}{C_P\rho_0}\right)k^2. \tag{15}$$

The magnetosonic links take the forms

$$\psi_{6,7} = \left(1 \quad 0 \quad 0 \quad \pm\frac{C_\perp}{\rho_0} - \left(\frac{\xi}{2\rho_0} + \frac{2\eta}{3\rho_0} + (\gamma-1)\frac{c_0^2}{2\rho_0 C_\perp^2}\frac{\chi}{C_P\rho_0}\right)\frac{\partial}{\partial z} \quad c_0^2 \mp (\gamma-1)\frac{c_0^2}{C_\perp}\frac{\chi}{C_P\rho_0}\frac{\partial}{\partial z} \quad \frac{B_0}{\rho_0} \quad 0\right)\rho_{6,7}. \tag{16}$$

$\omega_{6,7}$, $\psi_{6,7}$ are described in the frames of (6),(8). It is remarkable that different scenarios of attenuation impose different definition of modes. In the case of only non-zero bulk viscosity ξ, there are five zero roots of dispersion equation, ω_1,. . . ,ω_5. That brings degenerate five modes which are defined ambiguously. Any linear independent combination of them also represents a mode. The case of only non-zero shear viscosity η brings three zero dispersion relations and three degenerate modes, and the case of only non-zero thermal conduction χ brings four zero dispersion relations and four degenerate modes.

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4 Nonlinear effects

4.1 Magnetoacoustic heating. Case $\theta = \pi/2$

Applicability of the projecting in the weakly nonlinear fluid flows is indicated in Ref.[8]. In regard to magnetohydrodynamics, it is described in details in Ref.[24]. The fundamental issue is to evaluate the projecting operators in a linear fluid flow. The projecting operator which distinguishes an excess density in the entropy mode from the total vector of perturbations

$$\psi = \sum_{i=1}^{7}\psi_i, \tag{17}$$

so as

$$P_{ent}\psi = \rho_3 \equiv \rho_{ent}, \tag{18}$$

takes the leading-order form

$$P_{ent} = \left(\frac{c_0^2}{c_0^2+\gamma C_A^2} \quad 0 \quad 0 \quad -\frac{(\gamma-1)\chi}{C_P}\frac{c_0^2}{C_\perp^4}\frac{\partial}{\partial z} \quad -\frac{1}{C_\perp^2} \quad \frac{c_0^2 C_A^2(\gamma-1)\rho_0}{B_0 C_\perp^2(c_0^2+\gamma C_A^2)} \quad 0\right). \tag{19}$$

It is not a special case of P_{ent} in the non-degenerate case

$$P_{ent,\theta\neq\pi/2}=\begin{pmatrix}1 & -\frac{(\gamma-1)\chi C_{A,x}}{C_{A,z}c_0^2C_P}\frac{\partial}{\partial z} & 0 & -\frac{(\gamma-1)\chi}{C_Pc_0^2}\frac{\partial}{\partial z} & -\frac{1}{c_0^2} & 0 & 0\end{pmatrix}. \qquad (20)$$

For definiteness, we consider the dominant magnetosonic mode propagating in the positive direction of axis z, that is, with $C_\perp>0$. The dominance of a mode means that its specific perturbations are much larger than that of other modes, and in particular, of the entropy mode. The dynamic equation for the density perturbation in the magnetosonic mode which propagates in the positive direction of axis z, takes the form

$$\frac{\partial\rho_{ms}}{\partial t}+C_\perp\frac{\partial\rho_{ms}}{\partial z}+\frac{3C_A^2+(\gamma+1)c_0^2}{2C_\perp^2}\rho_{ms}\frac{\partial\rho_{ms}}{\partial z}-\left(\frac{\xi}{2\rho_0}+\frac{2\eta}{3\rho_0}+(\gamma-1)\frac{c_0^2}{2C_\perp^2}\frac{\chi}{C_P\rho_0}\right)\frac{\partial^2\rho_{ms}}{\partial z^2}=0. \qquad (21)$$

Application of the projector (19) at the system (3), which may be briefly represented as

$$\frac{\partial\psi}{\partial t}+L\psi=\widetilde{\psi} \qquad (22)$$

(where $\widetilde{\psi}$ is the vector which consists of nonlinear terms), distinguishes the dynamic equation for specific perturbation of density in the entropy mode.

$$P_{ent}\left[\frac{\partial\psi}{\partial t}+N\psi\right]=\frac{\partial\rho_{ent}}{\partial t}-\frac{\chi}{C_p\rho_0}\frac{c_0^2+\gamma C_A^2}{C_\perp^2}\frac{\partial^2\rho_{ent}}{\partial z^2}=P_{ent}\widetilde{\psi}=Q_{ms,\perp}. \qquad (23)$$

Among all nonlinear terms in $\widetilde{\psi}$, we consider only terms belonging to the magnetosonic mode which propagates in the positive direction of axis z, hence $Q_{ms,\perp}$ represents the nonlinear source due to this magnetosonic mode exclusively,

$$Q_{ms,\perp}=\frac{(\gamma-1)\chi}{C_p\rho_0^2}\left(A_1\left(\frac{\partial\rho_{ms}}{\partial z}\right)^2+A_2\rho_{ms}\frac{\partial^2\rho_{ms}}{\partial z^2}\right)- \qquad (24)$$
$$(\gamma-1)\left(\frac{4\eta}{3\rho_0^2}+\frac{\xi}{\rho_0^2}\right)\left(\frac{\partial\rho_{ms}}{\partial z}\right)^2,$$

where

$$A_1=-\frac{\gamma\beta(8\gamma-12+\beta^2(\gamma-3)\gamma+\beta(2\gamma^2-\gamma-7))}{(2+\beta)(2+\beta\gamma)^2} \qquad (25)$$
$$A_2=\frac{\gamma\beta(8-4\gamma+2\beta^2\gamma+(\gamma+5)\beta)}{2+\beta)(2+\beta\gamma)^2},$$

and plasma-β is determined as the ratio $\frac{2c_0^2}{\gamma C_A^2}$. In the case of periodic or nearly periodic acoustic exciter, in view that magnetosonic perturbations in the leading order are functions of $x-C_\perp t$,

$$\langle Q_{ms,\perp}\rangle\approx-\frac{\gamma-1}{\rho_0}\left(\frac{(\gamma-1)\chi}{C_P\rho_0}\frac{c_0^2}{C_\perp^2}+\frac{4\eta}{3\rho_0}+\frac{\xi}{\rho_0}\right)\left\langle\left(\frac{\partial\rho_{ms}}{\partial z}\right)^2\right\rangle, \qquad (26)$$

where the angular brackets designate average over the period of perturbations. In the linear flow (that is, if $\widetilde{\psi}=0$), $Q_{ms,\perp}$ equals zero and an excess entropy density evolutes independently on

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other modes in accordance to the dispersion relation (12). There is no leading-order nonlinear coupling between magnetosonic and other non-wave modes.

The dynamic equation which governs perturbation of density in the entropy mode excited by the periodic magnetosonic wave which propagates in the positive direction of axis z ($\theta = 0$), is the same than in the absence of magnetic field ($C_A = 0$):

$$\frac{\partial \rho_{ent}}{\partial t} - \frac{\chi}{C_p \rho_0} \frac{\partial^2 \rho_{ent}}{\partial z^2} = Q_{ms,||}, \tag{27}$$

$$Q_{ms,||} = \frac{(\gamma-1)\chi}{C_P \rho_0^2} \left(-(\gamma-3) \left(\frac{\partial \rho_{ms}}{\partial z} \right)^2 + 2\rho_{ms} \frac{\partial^2 \rho_{ms}}{\partial z^2} \right) - (\gamma-1) \left(\frac{4\eta}{3\rho_0^2} + \frac{\xi}{\rho_0^2} \right) \left(\frac{\partial \rho_{ms}}{\partial z} \right)^2,$$

so in the case of periodic or nearly periodic exciter,

$$\langle Q_{ms,||} \rangle \approx -\frac{(\gamma-1)}{\rho_0} \left(\frac{(\gamma-1)\chi}{C_P \rho_0} + \frac{4\eta}{3\rho_0} + \frac{\xi}{\rho_0} \right) \left\langle \left(\frac{\partial \rho_{ms}}{\partial z} \right)^2 \right\rangle . \tag{28}$$

This coincides with the results of Ref.[7]. In general, equations (23),(27) do not refer to special kinds of exciters nor to average quantities. Since the difference in equations (23),(27) is brought by thermal conduction, we focus on the effects associated with the thermal conduction exclusively. Fig.1 shows coefficients in the magnetosonic sources of dynamic equations for perturbation in density inherent to the entropy mode (23),(27). In evaluations, $\gamma = 5/3$. Eq.(27) is a limiting case of Eq.(23) when $\beta \to \infty$ ($C_A \to 0$). The magnetosonic source of heating $Q_{ms,\perp}$ tends to zero if $\beta \to 0$.

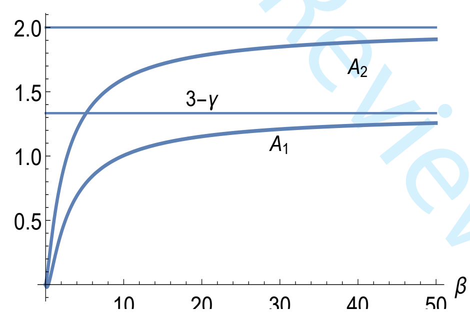

Fig.1. Coefficients which determine sources $Q_{ms,\perp}$, $Q_{ms,||}$ in dynamic equations for perturbation in density inherent to the entropy mode (Eqs(23),(27)). Cases of perpendicular and parallel wave vector and equilibrium magnetic field.

4.2 Excitation of the entropy mode by slow magnetosonic wave in the course of nearly perpendicular propagation

The magnetosonic heating in the case $\theta \neq 0$, $\theta \neq \pi/2$ in the case of slow and fast modes is described by equation

$$\frac{\partial \rho_{ent}}{\partial t} - \frac{\chi}{C_p \rho_0} \frac{\partial^2 \rho_{ent}}{\partial z^2} = Q_{ms} = \frac{\partial F(z - Ct)}{\partial z}. \tag{29}$$

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In the leading order, all magnetoacoustic perturbations and magnetosonic source Q_{ms} depend on $z - Ct$. Analysis may be done making use of results discussed in the review of Rudenko, Makov [25]. In the leading order, perturbations in the excited mode propagate with its own speed and the speed of exciting mode [26, 25, 27]. This last part contributes to the exciting mode. The magnitude of excited perturbation increases infinitely when linear speeds of both modes coincide. If thermal conduction in its left hand side is neglected, Eq.(29) has a solution satisfying zero initial condition

$$\rho_{ent} = \frac{1}{C}(F(z) - F(z - Ct)) \tag{30}$$

which if $C \to 0$ rearranges readily into

$$\rho_{ent} = t\frac{\partial F}{\partial z} \tag{31}$$

which reflects linear growth with time of magnitude of entropy perturbation in the course of resonant excitation. (This resonant excitation concerns impulsive exciters. If there is non-zero average of the period compound of magnetic source, $\langle Q_{ms}\rangle \neq 0$, excitation by source moving with any speed, is always resonant. Taking in mind that dynamics of entropy perturbations is slow, Eq.(29) after averaging over wave period transforms into $\rho_{ent} = t\,\langle Q_{ms}\rangle$.)

The "almost resonant" excitation of the entropy mode by the slow mode propagating nearly perpendicular to the magnetic field with the speed close to zero may take place. The solution to (29) for non-zero C which satisfies zero initial condition, takes the form [25]:

$$\rho_{ent} = \int_{-\infty}^{\infty} \frac{G(k)}{-C + ik\frac{\chi}{C_p\rho_0}}\left(\exp(ikCt) - \exp(-\frac{\chi}{C_p\rho_0}k^2t)\right)\exp(-ikz)dk, \tag{32}$$

where

$$G(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(z)dz \tag{33}$$

is the spatial spectrum of the magnetosound source. For large time domains starting from the beginning of evolution, the module of perturbation is

$$|\rho_{ent}| = \frac{|G(k_0)|k_0}{\sqrt{C^2 + \left(\frac{\chi}{C_p\rho_0}k_0\right)^2}}. \tag{34}$$

If the spectrum represents a single wave number k_0,

$$G(k) = G(k_0)\delta(k - k_0). \tag{35}$$

This example shows that the thermal conduction eliminates the resonant peculiarity when C tends to zero and determines maximum magnitude of perturbation of density inherent to the entropy mode in the course of magnetosonic excitation if $C \to 0$,

$$|\rho_{ent,max}| = \frac{C_p\rho_0}{\chi}|G(k_0)|, \tag{36}$$

It may achieve considerable values in view of small $\chi \approx \chi_\perp$. The similar behavior takes place for other kinds of $F(z)$.

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5 Remarks and conclusions

The special cases of a plasma flow with the wave vector perpendicular or nearly perpendicular to the magnetic field and the related nonlinear phenomena require proper definition of modes as the relations between specific small-magnitude perturbations. In general, there are two slow and two fast magnetosonic p-modes (Eq.(8)) with the speeds determined by Eq.(4) apart from $\theta = 0$ and $\theta = \pi/2$. The properties of a flow change abruptly when approaching $\theta = \pi/2$. The case $\theta = \pi/2$ is of especial interest, where slow modes degenerate into two stationary ones with zero speed (ψ_1,ψ_2, Eqs(11)), and two Alfvén modes degenerate into non-wave diffusion modes (ψ_4,ψ_5, Eqs(14)). These modes are determined by unusual dispersion relations and links of specific perturbations which are not limiting cases of the general ones if $\theta \neq \pi/2$. Also, dispersion relation and links for the entropy mode are unusual. The entropy mode relies on the dispersion relation (12) and the corresponding mode (13). Neither the dispersion relation nor the entropy mode are limiting cases of the more general one. In particular, the entropy mode possesses unusual non-zero perturbations of pressure and magnetic field, p' and B_x, and zero v_x. The entropy mode is not longer isobaric but specified by non-zero perturbation in pressure $p' = -C_A^2\rho'$, so as the dimensionless variation of temperature equals

$$\frac{T'}{T_0} = \frac{p'}{p_0} - \frac{\rho'}{\rho_0} = -\frac{c_0^2 + \gamma C_A^2}{c_0^2}\frac{\rho'}{\rho_0}.$$

The diffusion coefficient in the case of perpendicular propagation equals $\frac{(c_0^2+\gamma C_A^2)}{c_0^2+C_A^2}\frac{\chi}{C_P\rho_0}$ (Eq.(12)) but in the case $\theta \neq \pi/2$ it equals $\frac{\chi}{C_P\rho_0}$ (Eq.(7)). Hence, it is not a special case but a quantity always larger than that in the more general case (1 to γ times larger). The projector into the perturbation of density inherent to the entropy mode (19) is not a limiting case of more general one (20) as well.

The links between specific perturbation are unfairly underestimated in the fluid dynamics. They may be referred to as "constitutive equations" or "polarization relations" [28] and are the foundation for linear and weakly nonlinear analysis. They allow to subdivide different modes in the total field of perturbations by projectors and to undertake analysis of nonlinear dynamics. They also point at the character of irreversible processes in a flow. In particular, curves in the plane perturbations of pressure versus perturbation of density in wave processes (Eq.(8))

$$\frac{p'}{c_0^2} = \rho' - (\gamma-1)\frac{\chi}{C_P\rho_0 C}\frac{\partial}{\partial z}\rho' \approx \rho' + (\gamma-1)\frac{\chi}{C_P\rho_0 C^2}\frac{\partial}{\partial t}\rho' \qquad (37)$$

reveal hysteretic behavior due to thermal conduction and indicate speed of sound in a plasma since $|C|/c_0 > 1$ for fast magnetosonic modes and $|C|/c_0 < 1$ for slow magnetosonic modes. When $\theta \to \pi/2$ and speed of slow mode C tends to zero, the term responsible for thermal conduction may achieve considerable values even for small $\chi_\perp$. Eq.(37) may indicate $\chi_\perp$, C (hence, equilibrium parameters of a plasma and θ) in the case of nearly perpendicular wave vector and magnetic field. This concerns also links of v_z and ρ' which depends on all damping coefficients but with growing contribution of χ in a slow mode when $\theta \to \pi/2$. With zero thermal conduction and the first viscosity, the case $\theta = \pi/2$ yields five degenerate modes with zero dispersion relation. The links of perturbations for the degenerate modes are uncertain and have no analogues with the more general case.

Nonliner effects of sound in the particular case, $\theta = \pi/2$, are also especial. Nonlinear dynamics of perturbations cannot be Fourier analyzed but the linear links may be used in order

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to determine modes and to derive dynamic equations in a weakly nonlinear flow. Eqs (21),(23) describe propagation of magnetosonic excess density and excitation of perturbation in density which specifies heating in this particular cases of geometry of a flow. In the case $\theta \neq \pi/2$, the corresponding enlargement of a medium temperature and decrease in its density due to magnetosound heating is isobaric (ψ_{ent}, (9)), so as the variation in the background temperature associates with the perturbation of density as $\frac{T'}{T_0} = -\frac{\rho'}{\rho_0}$. The excited entropy mode in the case $\theta = \pi/2$ is not longer isobaric and specifies zero v_x. The part of the source associating with the thermal conductivity in the case of periodic excitation is smaller than that in the more general case $\theta \neq \pi/2$ ($\frac{c_0^2+C_A^2}{c_0^2}$ times smaller). The magnetosonic source $Q_{ms,\perp}$ includes contribution of mechanical viscosity which does not depend on θ, but the part connected with thermal conduction may be indicator of plasma-β (Eq.(24), Fig.1). Evidently, the deviation from the non-magnetic case increases with enlargement of C_A and leads to smaller magnitudes of perturbations of density specifying the entropy mode ceteris paribus.

"Almost resonant" excitation of the entropy mode is theoretically possible in the course of nearly perpendicular propagation of the slow impulsive magnetosonic wave with a speed close to zero. The entropy mode excited by the magnetosonic wave represents slow non-wave motion and may be confidently detected. The character of excitation points also at the equilibrium and transport parameters of a plasma, kind of a wave exciter and, generally, an angle between the equilibrium magnetic field and the wave vector. The theoretical results which concern links of specific perturbation and excitation of magnetosonic heating may be useful in both laboratory and astrophysical applications. They may be included in indirect methods such as "coronal seismology" and be helpful in estimation of coronal properties and coefficients of dissipation.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.

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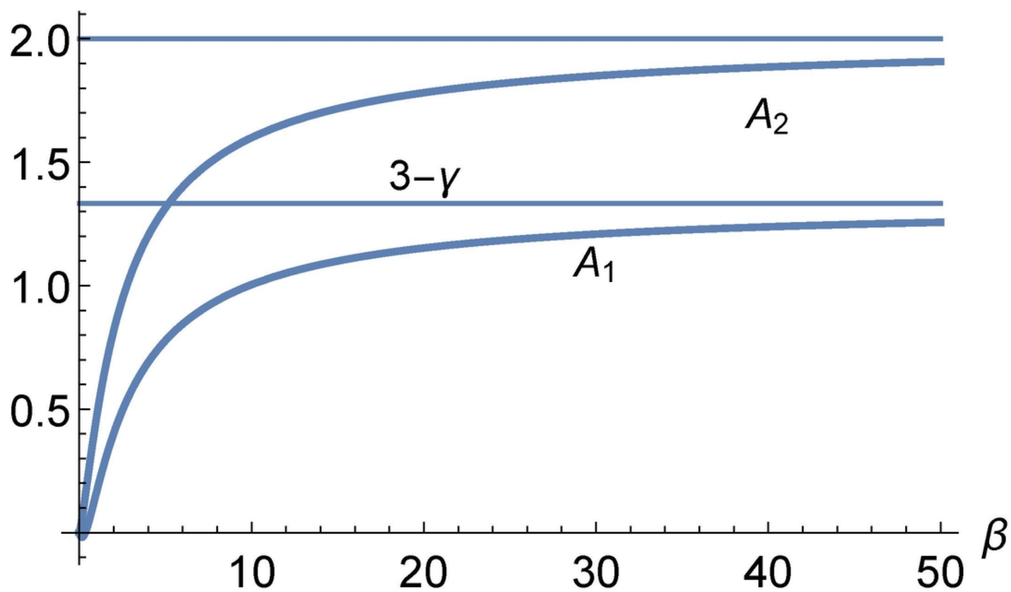

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