

# All graphs with restrained domination number three less than their order

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## Abstract

For a graph  $G = (V, E)$ , a set  $S \subseteq V$  is a restrained dominating set if every vertex not in  $S$  is adjacent to a vertex in  $S$  as well as another vertex in  $V - S$ . The restrained domination number of  $G$ , denoted by  $\gamma_r(G)$ , is the smallest cardinality of a restrained dominating set of  $G$ . In this paper we find all graphs  $G$  satisfying  $\gamma_r(G) = n - 3$ , where  $n$  is the order of  $G$ .

## 1 Introduction

All graphs considered in this paper are finite, undirected, without loops and multiple edges. Let  $G = (V, E)$  be a graph with the vertex set  $V$  and the edge set  $E$ . Then we use the convention  $V = V(G)$  and  $E = E(G)$ . The number of vertices of  $G$  is called the *order* of  $G$  and is denoted by  $n(G)$ . When there is no confusion we can use the abbreviation  $n(G) = n$ . Let  $C_n$  and  $P_n$  denote the cycle and the path of order  $n$ , respectively. The *open neighborhood* of a vertex  $v \in V$  in  $G$  is denoted  $N_G(v) = N(v)$  and defined by  $N(v) = \{u \in V : vu \in E\}$  and the *closed neighborhood*  $N[v]$  of  $v$  is  $N[v] = N(v) \cup \{v\}$ . For a set  $S$  of vertices the *open neighborhood*  $N(S)$  is defined as the union of open neighborhoods  $N(v)$  of vertices  $v \in S$ , the *closed neighborhood*  $N[S]$  of  $S$  is  $N[S] = N(S) \cup S$ . The *degree*  $d_G(v) = d(v)$  of a vertex  $v$  in  $G$  is the number of edges incident to  $v$  in  $G$ ; by our definition of a graph, this is equal to  $|N(v)|$ . A *leaf* in a graph is a vertex of degree one, however a *stem* is a vertex adjacent to a leaf.

In the present paper we continue the study of restrained domination. Problems related to restrained domination in graphs appear in [1–5]. A set  $S \subseteq V$  is a *restrained dominating set*, denoted **RDS**, if every vertex in  $V - S$  is adjacent to a vertex in  $S$  and another vertex in  $V - S$ . The *restrained domination number* of  $G$ , denoted by  $\gamma_r(G)$ , is the minimum cardinality of a restrained dominating set of  $G$ . We will call a set  $S$  a  $\gamma_r$ -*set* if  $S$  is a restrained dominating set of cardinality  $\gamma_r(G)$ . The solutions

$G$  of the graph-equations  $\gamma_r(G) = n$  and  $\gamma_r(G) = n - 2$  are well-known (see [1]). In this paper we describe all graphs  $G$  satisfying

$$\gamma_r(G) = n - 3. \quad (1)$$

For this purpose we need the following statements.

**Fact 1.** *Let a graph  $G$  be a subgraph of a graph  $G'$ , written as  $G \subseteq G'$ . If  $S$  is a **RDS** in  $G$  then  $S' = S \cup (V(G') - V(G))$  is a **RDS** in  $G'$ .*

**Proof.** By definition of **RDS** and the equality  $V(G) - S = V(G') - S'$ , we obtain the thesis of this statement.  $\square$

**Fact 2.** *If  $G \subseteq G'$  and  $\gamma_r(G) = n - k$ , then  $\gamma_r(G') \leq n' - k$ , where  $n = |V(G)|$ ,  $n' = |V(G')|$ ,  $0 \leq k \leq n - 1$ ,  $k \neq 1$ .*

**Proof.** Assume that  $S$  is a  $\gamma_r$ -set in  $G$ ; thus  $|V(G) - S| = k$ . It follows from Fact 1 that  $S' = S \cup (V(G') - V(G))$  is a **RDS** in  $G'$ . Moreover, we have  $|S'| = n' - |V(G') - S'| = n' - |V(G) - S| = n' - k$ . Hence we obtain  $\gamma_r(G') \leq |S'| = n' - k$ .  $\square$

## 2 The equation $\gamma_r = n - 3$

The main purpose of this paper is to find all graphs  $G$  for which  $\gamma_r(G) = n - 3$ . Assume at first that  $G$  is a tree. In this case we have the following result.

**Theorem 2.1.** *If  $T$  is a tree of order  $n \geq 4$ , then  $\gamma_r(T) = n - 3$  if and only if  $T$  is obtained from  $P_5$  or  $P_6$  by adding zero or more leaves to the stems of the path and adding either (1) at least one leaf or (2) exactly one stem to  $v_3$  for  $P_5 = (v_1, v_2, v_3, v_4, v_5)$  and to exactly one of  $v_i$ ,  $i = 3, 4$ , for  $P_6 = (v_1, v_2, v_3, v_4, v_5, v_6)$ .*

**Proof.** Let  $T$  be a tree described above. We shall verify that  $\gamma_r(T) = n - 3$ . Denote by  $x$  the stem appearing in (2). If  $T$  is obtained from  $P_5$  by the above construction, then  $V(T) - \{v_2, v_3, v_4\}$  in case (1) and  $V(T) - \{v_2, v_3, v_4\}$ ,  $V(T) - \{v_2, v_3, x\}$ ,  $V(T) - \{x, v_3, v_4\}$  in case (2) are  $\gamma_r$ -sets of size  $n - 3$ . However, if  $T$  is obtained from  $P_6$ , then  $V(T) - \{v_2, v_3, v_4\}$  (for  $i = 3$ ),  $V(T) - \{v_3, v_4, v_5\}$  (for  $i = 4$ ) in case (1) and  $V(T) - \{v_2, v_3, v_4\}$ ,  $V(T) - \{v_2, v_3, x\}$  (for  $i = 3$ ),  $V(T) - \{v_3, v_4, v_5\}$ ,  $V(T) - \{x, v_4, v_5\}$  (for  $i = 4$ ),  $V(T) - \{v_3, v_4, x\}$  (for  $i = 3, 4$ ) in case (2) are  $\gamma_r$ -sets of size  $n - 3$ .

Conversely, let  $T$  be a tree of order  $n$  such that  $\gamma_r = n - 3$ . If  $\text{diam}(T) \geq 6$ , then  $T$  contains an induced  $P_7 = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$ . But then  $V(T) - \{v_2, v_3, v_5, v_6\}$  is a **RDS** in  $T$  of size  $n - 4$ , a contradiction. Thus, consider the following cases.

*Case 1:*  $\text{diam}(T) = 5$ .

Then  $T$  contains an induced  $P_6 = (v_1, v_2, v_3, v_4, v_5, v_6)$ .

*Case 1.1:* Neither  $v_3$  nor  $v_4$  have neighbors not on  $P_6$ .

Then  $V(T) - \{v_2, v_3\}$ ,  $V(T) - \{v_3, v_4\}$ ,  $V(T) - \{v_4, v_5\}$  are  $\gamma_r$ -sets of size  $n - 2$ , which contradicts the assumption that  $\gamma_r(T) = n - 3$ .



*Case 1.2:* Both  $v_3, v_4$  have neighbors not on the path.

Then  $V(T) - \{v_2, v_3, v_4, v_5\}$  is a **RDS** of  $T$ , therefore  $\gamma_r(T) \leq n - 4$ , a contradiction.

*Case 1.3:* Exactly one of the vertices  $v_3, v_4$ , say  $v_3$ , has neighbors not on  $P_6$ .

With respect to the condition  $\text{diam}(T) = 5$ , new neighbors of  $v_3$  can be only leaves or stems.

*Case 1.3.1:* All neighbors of  $v_3$  not on  $P_6$  are leaves.

In this case,  $V(T) - \{v_2, v_3, v_4\}$  is a  $\gamma_r$ -set of  $T$  of size  $n - 3$ .

*Case 1.3.2:* Neighbors of  $v_3$  not on  $P_6$  are at least one leaf and stems  $w_1, \dots, w_m$ ,  $m \geq 1$ .

Then  $V(T) - \{v_2, v_3, v_4, w_1, \dots, w_m\}$  is a **RDS** of  $T$ , so  $\gamma_r(T) \leq n - 4$ , a contradiction.

*Case 1.3.3:* Neighbors of  $v_3$  not on  $P_6$  are only stems  $w_1, w_2, \dots, w_m$ ,  $m \geq 2$ .

But then  $V(T) - \{v_2, v_3, v_4, w_1, \dots, w_{m-1}\}$  is a **RDS** of  $T$ ; hence  $\gamma_r(T) \leq n - 4$ , which is a contradiction.

*Case 1.3.4:* A unique neighbor of  $v_3$  not on  $P_6$  is a stem  $w_1$ .

We now can deduce that  $V(T) - \{v_2, v_3, v_4\}$ ,  $V(T) - \{v_2, v_3, w_1\}$ ,  $V(T) - \{v_3, v_4, w_1\}$  are  $\gamma_r$ -sets in  $T$ , and thus  $\gamma_r(T) = n - 3$ .

One can obtain similar cases when  $v_4$  has neighbors not on  $P_6$ .

*Case 2:*  $\text{diam}(T) = 4$ .

Then  $T$  has an induced  $P_5 = (v_1, v_2, v_3, v_4, v_5)$ .

*Case 2.1:* The open neighborhood of  $v_3$  in  $T$  is  $N(v_3) = \{v_2, v_4\}$ .

In this position  $V(T) - \{v_2, v_3\}$ ,  $V(T) - \{v_3, v_4\}$  are  $\gamma_r$ -sets in  $T$  of size  $n - 2$ , which contradicts the assumption that  $\gamma_r(T) = n - 3$ .

*Case 2.2:* Vertex  $v_3$  has neighbors not on  $P_5$ .

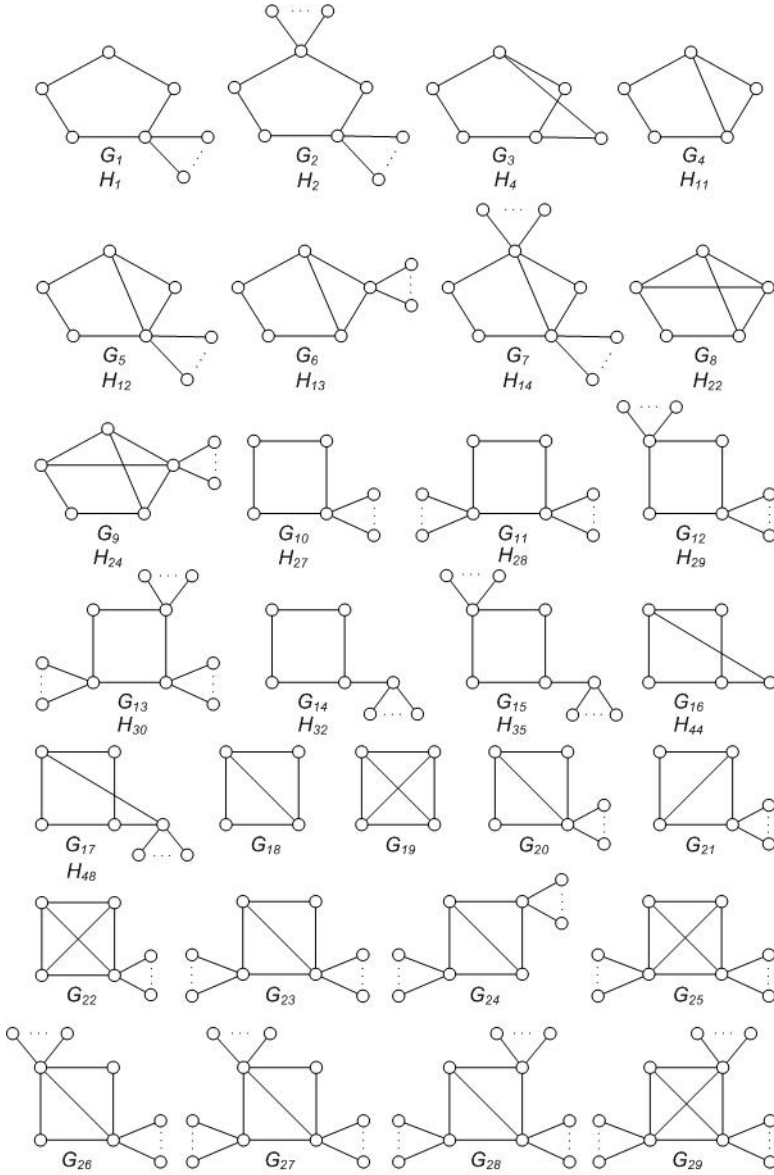
According to Case 2, new neighbors of  $v_3$  can only be leaves or stems.

Now let us replace  $P_6$  by  $P_5$  in cases 1.3.1, 1.3.2, 1.3.3, 1.3.4. In this way we obtain cases 2.2.1, 2.2.2, 2.2.3, 2.2.4, respectively.

The proof is complete.  $\square$

Now consider connected graphs which contain a cycle. Let  $\mathcal{G}$  be the collection of graphs in Figure 1.





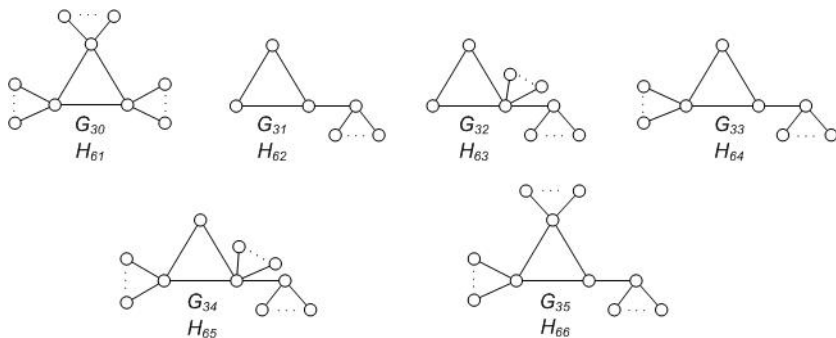


Figure 1: Graphs in family  $\mathcal{G}$ .

We shall show that the graphs in family  $\mathcal{G}$  are the unique connected graphs containing a cycle and satisfying (1).

**Theorem 2.2** *Let  $G$  be a connected graph of order  $n \geq 4$  containing a cycle. Then  $\gamma_r(G) = n - 3$  if and only if  $G \in \mathcal{G}$ .*

**Proof.** Let  $G$  be a connected graph of order  $n$  containing a cycle and satisfying (1). We shall prove that  $G \in \mathcal{G}$ . Our aim is to find all graphs  $G$  for which (1) holds. For finding graphs  $G$  we shall apply the following method.

**Procedure (P):** *For a connected  $H$  such that  $\gamma_r(H) = n - 2$  or  $\gamma_r(H) = n - 3$  find a connected  $G$ ,  $H \subseteq G$ , satisfying (1). We start with  $H := C_i$ ,  $i = 3, 4, 5$ .*

Note that by Fact 2 in the above procedure it suffices to consider  $\gamma_r(H) = n - 2$  or  $n - 3$ .

We first observe that  $G$  cannot have a cycle of length at least 6. In fact, if  $v_1, v_2, \dots, v_m$ , where  $m \geq 6$ , are consecutive vertices on a cycle, then  $V(G) - \{v_1, v_2, v_4, v_5\}$  is a **RDS** for  $G$  and  $\gamma_r(G) \leq n - 4$ , which is a contradiction. Thus, consider the following cases.

*Case 1:*  $C_5$  is the longest cycle in  $G$ .

Then in (P) we put  $H := C_5$  for and find  $G$  satisfying (1). Note that  $\gamma_r(H) = n - 2$ .

*Case 1.1:* Assume that there exists an induced  $C_5$  in  $G$ .

*Case 1.1.1:* Assume that the induced  $C_5$  has neighbors not on the cycle, i.e.  $I = N[V(C_5)] - V(C_5) \neq \emptyset$ .

*Case 1.1.1.1:* Suppose all  $v \in I$  are leaves.

Let  $H_1$  be a graph obtained from  $C_5$  by adding one or more leaves to one vertex of the cycle.



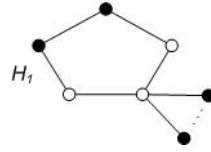


Figure 2:  $H_1 \cong G_1$  satisfies (1).

Figure 2 illustrates the graph  $H_1$ , where the shaded vertices form a  $\gamma_r$ -set. We shall continue to use this convention in our proof. For  $H_1$  we have  $\gamma_r(H_1) = n - 3$  and  $H_1 \cong G_1$ , where  $G_1 \in \mathcal{G}$ . Now, let us add leaves to at least two vertices of  $C_5$ .

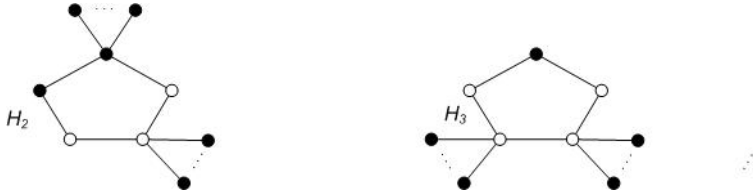


Figure 3:  $H_2 \cong G_2$  satisfies (1) but  $H_3$  does not.

Hence, by Fact 2 and case exhaustion, we can conclude that any graph obtained from  $C_5$  by adding leaves to at least 3 vertices does not satisfy (1).

Case 1.1.1.2: Assume that there exists  $v \in I$  which is not a leaf.

Case 1.1.1.2.1: Let  $|N(v) \cap V(C_5)| \geq 2$ .

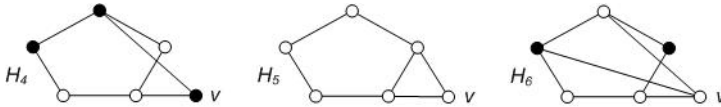


Figure 4: For  $H_4 \cong G_3$  (1) holds; for  $H_5$  and  $H_6$  (1) does not hold.

It follows from Fact 2 and  $C_6 \subseteq H_5$  that (1) does not hold for  $H_5$ . Now, by setting in (P)  $H := H_4$  and adding a new vertex, we have the graphs in Fig. 5, which do not satisfy (1).

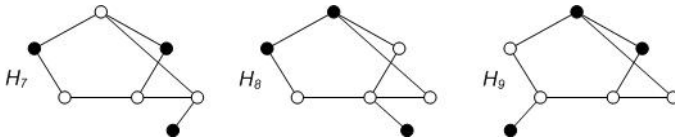


Figure 5:  $H_7, H_8, H_9$  do not satisfy (1).

Case 1.1.1.2.2: Let  $|N(v) \cap V(C_5)| = 1$ .

Then we have the graph shown in Fig. 6, which does not satisfy (1).



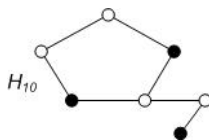


Figure 6:  $\gamma_r(H_{10}) = n - 4$ .

We now can deduce that  $H_1, H_2$  and  $H_4$  are the unique connected graphs which contain the induced  $C_5$  and satisfy (1).

We next shall reapply the procedure (P) by adding chords to  $C_5$ .

Case 1.2: Each  $C_5$  in  $G$  has a chord.

Case 1.2.1: Some  $C_5$  has exactly one chord.

Case 1.2.1.1: Let us consider the graph which consists of  $C_5$  with exactly one chord. In Fig. 7 below,  $\gamma_r(H_{11}) = n - 3$ .

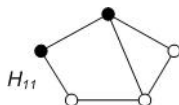


Figure 7: (1) holds for  $H_{11} \cong G_4$ .

Setting in (P)  $H := H_{11}$  we now obtain graphs  $G$  by adding new vertices.

Case 1.2.1.2:  $I = N[V(H_{11})] - V(H_{11}) \neq \emptyset$ .

Case 1.2.1.2.1: Assume that all  $v \in I$  are leaves.

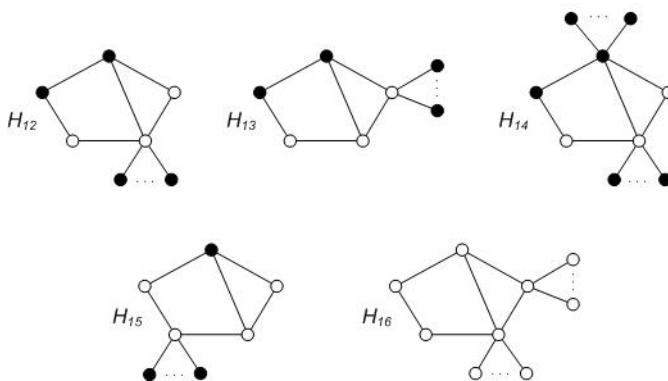


Figure 8:  $H_{12} \cong G_5, H_{13} \cong G_6, H_{14} \cong G_7$  satisfy (1) and  $H_{15}, H_{16}$  do not.

It is easy to deduce that among all graphs obtained from  $H_{11}$  by adding leaves, the unique graphs satisfying (1) are  $H_{12}, H_{13}$  and  $H_{14}$  of Fig. 8. Note that (1) does not hold for  $H_{15}$ . Furthermore,  $H_3 \subseteq H_{16}$  and hence (1) does not hold for  $H_{16}$ .



Case 1.2.1.2.2: Assume that there exists  $v \in I$  which is not a leaf.

Case 1.2.1.2.2.1:  $|N(v) \cap V(H_{11})| \geq 2$ .

In the procedure  $(P)$  we put  $H := H_{12}, H_{13}$  or  $H_{14}$  with one leaf. In this way we obtain  $H_{17}, H_{18}, H_{19}$  and  $H_{20}$  from  $H_{12}$  and  $H_{21}$  from  $H_{13}$  or  $H_i, i = 17, 18, 19, 20$ , from  $H_{14}$  (see Fig. 9).

Clearly,  $H_{19}, H_{20}$  and  $H_{21}$  are the supergraphs of  $C_6$  and hence (1) is false. Furthermore,  $\gamma_r(H_{17}) = \gamma_r(H_{18}) = n - 4$ .

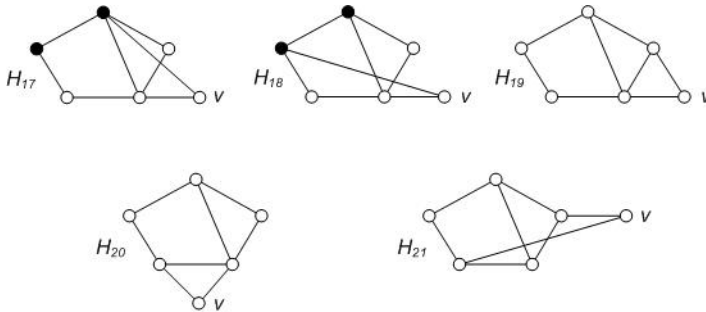


Figure 9: For  $H_{17}, \dots, H_{21}$  obtained from  $H_{12}$  and  $H_{13}$ , the equality (1) is false.

Case 1.2.1.2.2.2:  $|N(v) \cap V(H_{11})| = 1$ .

Then we have a supergraph of  $H_{10}$  and hence (1) does not hold.

Case 1.2.2: Each  $C_5$  has at least two chords.

Case 1.2.2.1: Let us consider graphs with the vertex set  $V(C_5)$ .

One can see that it suffices to consider the two graphs of Fig. 10. We have that  $\gamma_r(H_{22}) = n - 3$  and  $\gamma_r(H_{23}) = n - 4$ .

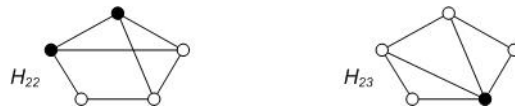


Figure 10: (1) holds for  $H_{22} \cong G_8$  and fails for  $H_{23}$ .

Case 1.2.2.2: There exists a vertex not on  $C_5$ , i.e.  $I = N[V(C_5)] - V(C_5) \neq \emptyset$ .

Then in  $(P)$  we put  $H := H_{22}$ .

Case 1.2.2.2.1: Suppose all  $v \in I$  are leaves.

Then we only consider graphs based on  $H_{12}, H_{13}, H_{14}$ . It follows that (1) holds for  $H_{24}$ , but not for  $H_{25}$  and  $H_{26}$ . Note that  $H_{25} \subseteq H_{26}$  (see Fig. 11).





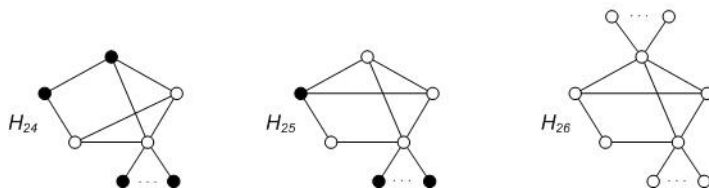


Figure 11:  $H_{24} \cong G_9$  satisfies (1) and  $H_{25}, H_{26}$  do not.

Case 1.2.2.2.2: Assume that there exists  $v \in I$  which is not a leaf.

In this case we obtain supergraphs of  $H_{10}, H_{17}, H_{18}, H_{19}, H_{20}$  or  $H_{21}$ , and (1) does not hold.

Case 2:  $C_4$  is the longest cycle in  $G$ .

Case 2.1: Assume that there exists an induced  $C_4$  in  $G$ .

Since  $\gamma_r(C_4) = n - 2$  we now put  $H := C_4$  in  $(P)$  and we have the following cases.

Case 2.1.1: Assume that the induced  $C_4$  has neighbors not on the cycle, i.e.  $I = N[V(C_4)] - V(C_4) \neq \emptyset$ .

Case 2.1.1.1: Suppose all  $v \in I$  are leaves.

Then (1) holds for  $H_{27}, H_{28}, H_{29}$  and  $H_{30}$  of Fig. 12.

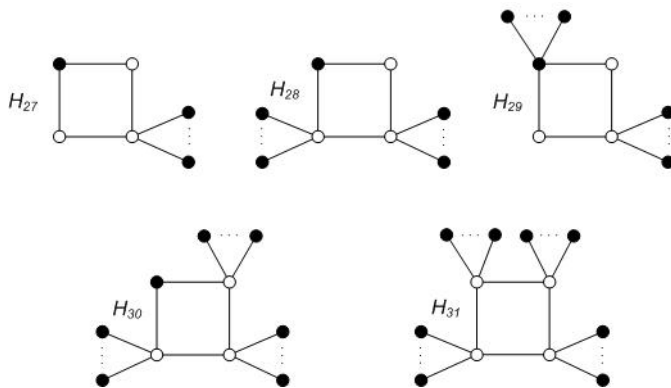


Figure 12: (1) holds for  $H_{27} \cong G_{10}, H_{28} \cong G_{11}, H_{29} \cong G_{12}, H_{30} \cong G_{13}$  and fails for  $H_{31}$ .

Case 2.1.1.2: Assume that there exists  $v \in I$  which is not a leaf.

Case 2.1.1.2.1: Suppose that  $|N(v) \cap V(C_4)| = 1$ .

At first we study the graphs of Fig. 13. For  $H_{32}$  and  $H_{35}$ , (1) holds.

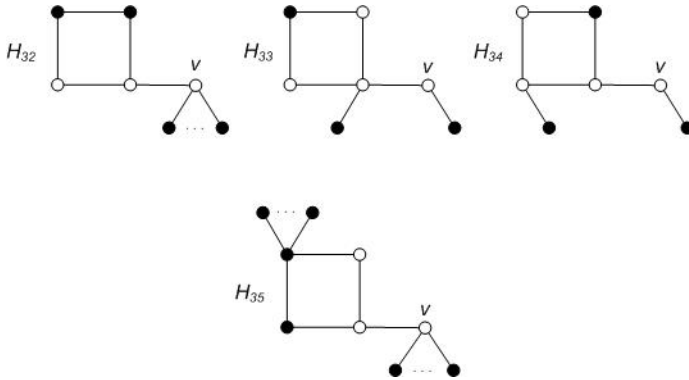


Figure 13: Only  $H_{32} \cong G_{14}$  and  $H_{35} \cong G_{15}$  satisfy (1).

We now consider supergraphs of  $H_{32}$  and  $H_{35}$ , depicted in Fig. 14. Let  $H_0$  be any connected graph. Notice that  $C_6 \subseteq H_{38}$ ,  $H_{39} \cong H_4$ ,  $H_{36} \subseteq H_{41}$ , where  $H_0$  is of order 1, and  $H_{37} \subseteq H_i$ ,  $i = 42, 43$ .

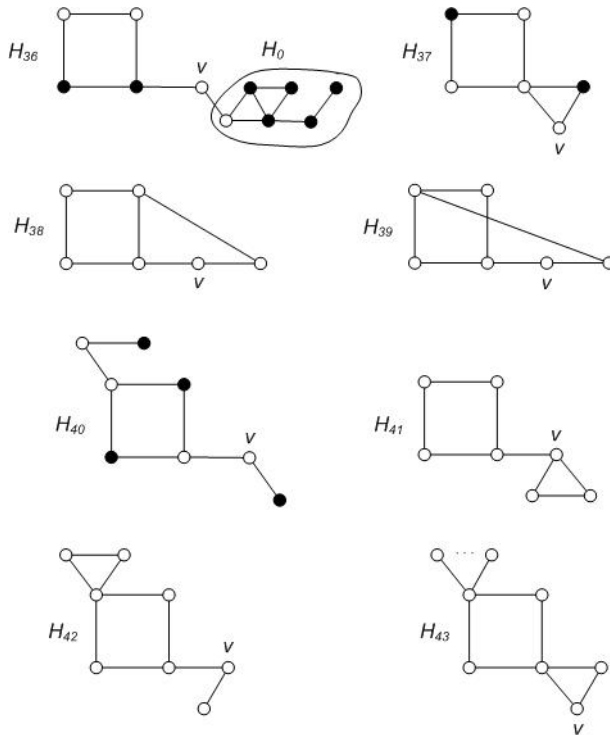


Figure 14: Only  $H_{39} \cong G_3$  satisfies (1).



Case 2.1.1.2.2: Suppose that  $|N(v) \cap V(C_4)| \geq 2$ .

According to case 2 it suffices to consider the respective equality.

At first we check (1) for the graphs  $H_{44}$ ,  $H_{45}$ ,  $H_{46}$  and  $H_{47}$  of Fig. 15, which are supergraphs of  $H_{27}$ ,  $H_{28}$ ,  $H_{29}$ ,  $H_{30}$ . It follows that (1) is true for  $H_{44} \cong G_{16}$ , but false for  $H_{45}$ ,  $H_{46}$  and  $H_{47}$ . Note that  $C_6 \subseteq H_{46}$ .

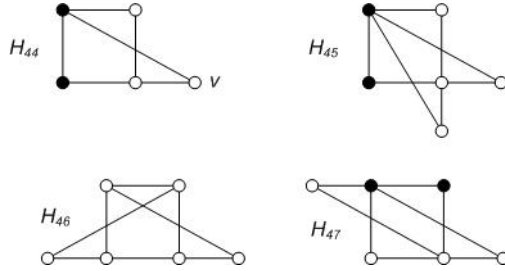


Figure 15: (1) is true for  $H_{44} \cong G_{16}$  and is false for  $H_{45}$ ,  $H_{46}$  and  $H_{47}$ .

Next, according to the procedure (P), we investigate supergraphs of  $H := H_{44}$ . Observe that it suffices to consider the graphs  $H_{48}$ ,  $H_{49}$  and  $H_{50}$  of Fig. 16. For  $H_{48} \cong G_{17}$  (1) holds, but for  $H_{49}$  and  $H_{50}$  it does not.

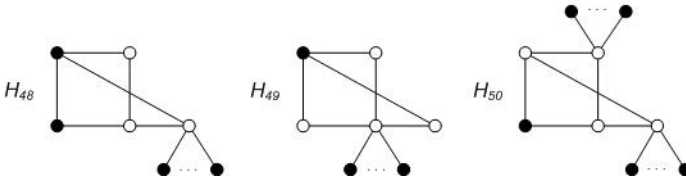


Figure 16: For  $H_{48} \cong G_{17}$  (1) holds but for  $H_{49}$ ,  $H_{50}$ , it does not.

We now put  $H := H_{48}$  in (P) and study supergraphs of  $H_{48}$  (Fig. 17). It follows that (1) fails for  $H_{51}$  and  $H_{52}$ . Note that  $H_{37} \subseteq H_{52}$  and  $H_{51} \subseteq H_{52}$ .



Figure 17: (1) fails for  $H_{51}$  and  $H_{52}$ .

We next consider supergraphs of  $H_{32}$  and  $H_{35}$  (Fig. 18). Since  $H_{53} \cong H_{48}$ , (1) holds. For  $H_{54}$  and  $H_{55}$ , it does not. Note that  $H_{54}$  and  $H_{55}$  are supergraphs of  $H_{49}$ .



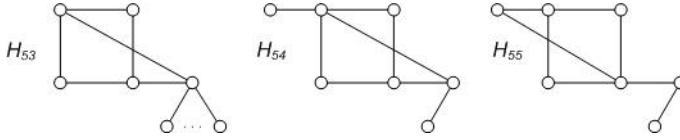


Figure 18:  $H_{53}$ ,  $H_{54}$  and  $H_{55}$ .

Case 2.2: Each  $C_4$  in  $G$  has a chord.

Let us consider the family  $\mathcal{G}'$  of graphs obtained from  $C_4$  by adding one or two chords and zero or more leaves to at least three vertices. It is easy to verify that for each  $G \in \mathcal{G}'$ , (1) holds, since an endvertex of a chord and the leaves form a  $\gamma_r$ -set. In this way, we obtain the graphs  $G_{18}, G_{19}, \dots, G_{29}$  in the family  $\mathcal{G}$ .

We now must determine whether supergraphs of  $H_i$ ,  $i = 18, \dots, 29$ , satisfy (1). One can see that it suffices to consider the graphs  $H_{56}, H_{57}$  and  $H_{58}$ , for which (1) is false. See Fig. 19.

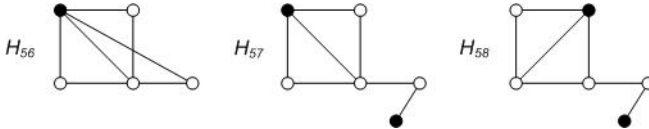


Figure 19: (1) is false for  $H_{56}$ ,  $H_{57}$  and  $H_{58}$ .

It follows that each supergraph  $H'$  of  $H_i$  ( $i = 18, \dots, 29$ ), such that  $H' \notin \mathcal{G}'$ , contains  $H_{31}, H_{37}, H_{56}, H_{57}, H_{58}$  or  $C_m$ , where  $m \geq 5$ .

Case 3:  $C_3$  is the longest cycle in  $G$ .

Observe that in this case we have  $I = N[V(C_3)] - V(C_3) \neq \emptyset$  and  $|N(v) \cap V(C_3)| = 1$  for each  $v \in I$ . Otherwise we could obtain a cycle  $C_m$ ,  $m \geq 4$ , in  $G$ , contrary to our assumption in this case. Therefore, let us study the graphs of Fig. 20.



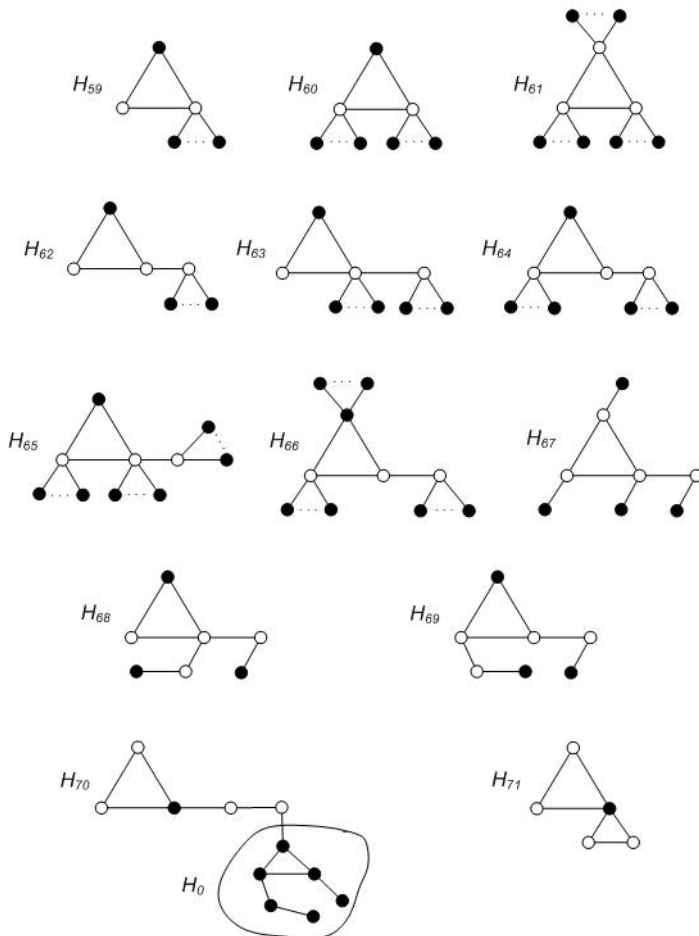


Figure 20:  $H_i \cong G_{i-31}$ ,  $i = 61, \dots, 66$ , are the unique graphs for which (1) is true ( $H_0$  is any connected graph).

Conversely, let  $G$  be any graph of the family  $\mathcal{G}$ . It follows from the former investigations that (1) holds for  $G$ .

This completes the proof of the theorem.  $\square$

We end this paper with the following statement.

**Corollary 2.1.** *If  $G$  is a graph of order  $n \geq 4$ , then  $\gamma_r(G) = n - 3$  if and only if exactly one of the components of  $G$  is isomorphic to a graph given in Theorems 2.1 or 2.2 and every other component is a star or  $K_1$ .*



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