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APPLICATION OF SEMI-MARKOV PROCESSES FOR EVALUATION OF DIESEL ENGINES RELIABILITY WITH REGARDS TO DIAGNOSTICS

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Abstract

The paper presents semi-Markov models of technical state transitions for diesel engines, useful for determination of their reliability, as a result of the conducted statistical empirical studies. Interpretation of technical states provided for this sort of engines refers to ship main engines, i.e. engines employed in propulsion systems of sea-going ships. The considerations recognize diesel engine as a diagnosed system (SDN), of which state can be identified by a diagnosing system (SDG). Both of the systems: SDN and SDG compose a diagnostic system (SD). Examples of three-state semi-Markov models were applied to demonstrate that in case of use of proper diagnosing systems (SDG) for identification of technical states of such engines as SDN, by classification of the states to the relevant class of the reference states, it is possible to make use of a Markov model to determine reliability of the engines. For developing a Markov model of state transitions for the engines, there were applied functions of the risk of damage: λ_{12} that causes transition from state s_1 to state s_2, and λ_{13} that causes transition from state s_1 to state s_3, as well as intensity functions of recovery (restitution): λ_{21} that causes transition from state s_2 to state s_1, and λ_{31} that causes transition from state s_3 to state s_1.

Keywords: diagnostics, reliability, semi-Markov process, diesel engine, diagnostic system

1. Introduction

Knowledge of reliability of diesel engines, especially ship main engines, i.e. engines employed in propulsion systems of ships, is indispensable in the phase of their operation, if the process of ship exploitation is to be rational. Lack of the knowledge on reliability, of main engines in particular, considerably raises the risk of failing to perform a task undertaken by the ship. Such knowledge instead allows planning transport tasks in the phase of ship operation. However, while implementing the approved plan of operation, it is significant to have a complete diagnosis (instantaneous diagnosis, prognosis and genesis) that enables prediction of engine failures, which may occur during its work. This always leads to increase the engine reliability. Taking into account the definitions of reliability for machines, provided in many publications [3, 5, 9, 10, 14], the reliability of ship main engines as well as other diesel engines, can be defined as the capability of the engines to convert energy in the full range of loads, which they were fit to in the design and manufacture phases [3]. For such understood reliability of this kind of engines, the measure can be recognized as the probability of proper energy conversion, for the all range of performances, at specified time and in defined operating conditions [3, 4, 5, 10, 11, 12, 14].

It can be assumed that any diesel engine works reliably when stays in full ability state (s_1) [2, 3, 5]. When the engine is in partial ability state (s_2), cannot work properly during performance of the task [3, 5, 12, 14, 15]. Furthermore, an engine being in disability state (s_3) is unfit to continue the work.

For identification of technical states s_i (i = 1, 2, 3) of ship engines, the following diagnosing systems (*SDG*) may be applied: *CoCoS* (*Computer Controlled Surveillance System*) of *MAN company*, or *CBM* (*Condition-Based Maintenance*) of *Wartsila company*, or others [14]. Information, obtained through employment of the systems, on duration of state s_1 and the time of its loss, as well as on the time of occurrence of states s_2 and s_3, and their duration, enables application of the theory of semi-Markov processes to determine reliability of the engines [1, 2, 3, 5, 6, 9].

2. Possible semi-Markov models of engine technical state transitions

Development of a model $\{W(t): t \geq t\}$ of a real process of technical state transitions $\{W^*(t): t \geq t\}$ that proceed in the phase of diesel engine operation, is a prerequisite to apply the theory of semi-Markov processes. Such a model can be developed when for the real process $\{W^*(t): t \geq t\}$ the following can be recognized[1, 3, 5, 6, 9]:

1) the Markov condition is satisfied, i.e. that future evolution of any real process $\{W^*(t): t \geq t\}$ of technical state changes during operation of any object, for which the semi-Markov model $\{W(t): t \geq t\}$ was built, depends only on the state of the given process at the specified time t, not on the states being in its past, thus that the future states of the process do depend not on the process states recorded earlier, but on the *currently existing one*,
2) random variables T_i (denoting the time duration of state s_i regardless of which state is next) and T_{ij} (denoting the time duration of state „s_i" provided that the next state of the process is state „s_j") have distributions different than exponential.

For ship main engines, the assumption can be made that the Markov condition is satisfied because the following hypothesis was proved true through empirical research [3, 5]: *prediction of technical state of any diesel engine at the moment* $\tau_n + t$, *when only its state at the moment* τ_n *is known, is possible, because the engine state considered at any moment* τ_n(n = 0, 1, ..., m; $\tau_0 < \tau_1 < ... < \tau_m$) *depends significantly on the directly preceding state, not on the states that were before nor their time duration.*

The paper [3] presents a possibility of estimation of engine reliability in the case when random variables, such as time of the proper operation (TPP) and recovery time (To) for such engines, can be described with gamma and normal distributions, as well as with the Weibull-Gniedenko distribution [3, 5, 14]. Such distributions can be used when engine failures are the consequence of cumulative loads. However, when safety of the ship is particularly important during the trip, description of reliability of the engines requires application of the exponential distribution. In this case, as it is shown in paper [3], the semi-Markov process can also be fully defined if the function matrix is known [1, 2, 5, 6, 8]

$$\mathbf{Q(t)} = [Q_{ij}(t)], \tag{1}$$

whose non-zero elements are interpreted as follows:

$$Q_{ij}(t)=P\{W(\tau_{n+1}) = s_j,\ \tau_{n+1}-\tau_n < t \mid W(\tau_n) = s_i\};\ s_i,\ s_j \in S;\ i, j = 1, 2, \ldots, n;\ i \neq j$$

and when the initial distribution is given

$$p_i = P\{W(0) = s_i\},\ s_i \in S;\ i = 1, 2, \ldots, n \tag{2}$$

When in the operation phase of ship main engines, some technical diagnostics with relevant diagnosing systems (*SDG*) are to be applied to identify the technical state of them as diagnosed systems (*SDN*), and to record the moments of occurrence of different types of states and their duration, there is a need to establish a three-state set of technical states [3]:

$$S = \{s_1, s_2, s_3\} \tag{3}$$

of the following interpretation [3]:

- s_1 - state of engine full ability
- s_2 - state of engine partial ability
- s_3 - state of engine disability.

The technical states $s_i \in S (i = 1, 2, 3)$ are the values of the process of state transitions $\{W(t): t \geq 0\}$. A graph of the state transitions for the process $\{W(t): t \geq 0\}$ for the engines is shown in Fig. 1.

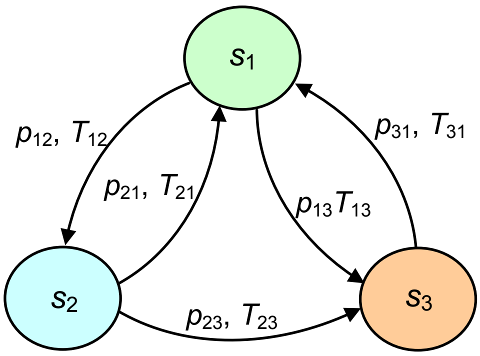

Fig. 1. Graph of engine state transitions:
p_{ij} – probability of process transition from state s_i to state s_j, T_{ij} – duration of state s_i provided that the process transits to state s_j; $i \neq j$; $i, j = 1, 2, 3$ [3]

The graph does not consider a possibility of the process transition from state s_3 to state s_2, because realization of rational operation should exclude performance of such preventive service that would lead only to partial recovery of the ship main engine. For this reason, the function $Q_{32}(t)$ is assumed to be zero, i.e. $Q_{32}(t) = 0$,

The initial distribution of the process is represented by the formula [3]:

$$P_i = P\{W(0) = s_i\} = \begin{cases} 1 & \text{dla} \quad i = 1 \\ 0 & \text{dla} \quad i = 2,3 \end{cases} \tag{4}$$

However, when the time of proper operation (T_{pp}) and the recovery time (T_o) of the engine can be described with gamma and normal distributions, or the Weibulla-Gniedenko distribution, the function matrix is as follows [3]:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & 0 & 0 \end{bmatrix} \tag{5}$$

Therefore, the limiting distribution for the process $\{X(t): t \geq 0\}$, with the function matrix defined by the formula (5), can be derived as follows [3]:

$$P_1 = \frac{E(T_1)}{M};\quad P_2 = \frac{p_{12}E(T_2)}{M};\quad P_3 = \frac{(1-p_{12}p_{21})E(T_3)}{M} \tag{6}$$

while:

$$M = E(T_1) + p_{12}E(T_2) + (1-p_{12}p_{21})E(T_3)$$

where:

P_1, P_2, P_3 – probabilities that diesel engine stays respectively in the states: s_1, s_2, s_3;
p_{ij} – probability of the process $\{W(t): t \geq 0\}$ transition from state s_i to state s_j;
$E(T_j)$ – expected value of duration of state s_j.

Application of *SDG* enabling development of a reliable diagnosis, allows implementation of the operation strategy including preventive maintenance of the main engine when its state is recognized as s_2. This prevents damage to the engine during its work. Therefore, the stochastic process $\{W(t): t \geq 0\}$ considered as a model of the process of engine state transitions can be simplified due to $p_{23} = 0$, so no random value T_{23} does exist (Fig. 2).

3. Simplified semi-Markov model of engine technical state transitions

Currently applied diagnosing systems (*SDG*) for identification of technical condition of ship main engines considered diagnosed systems (*SDN*) of MAN or *Wartsila* [15] are designed to disclose the most important states recognized as s_2 which are the results of engine wear [5, 12, 15]. Occurrence of state s_2 can be assumed to be a consequence of some damage to the engine. Such sort of damage allows further operation of the engine, but does not ensure performance of the task Z_d which must be carried out. Disclosure of the state s_2 in the engine enables application of some adequate preventive service and full engine recovery, and in consequence the return to state s_1. When recovery of engine being in state s_2 is impossible (due to stay of the ship at sea), its further operation brings damages which cause occurrence of state s_3.

It follows from the considerations that the process of changes in states of main engines is a stochastic process $\{X(t): t \geq 0\}$ with a set of states $S = \{s_i\,;\, i = 1, 2, 3\}$, and a graph of state transitions depicted in Fig. 2

Changes in states occur at moments $\tau_0 = 0,\ \tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5,\ \ldots,\ \tau_n$ (Fig. 3). Due to the fact that the process $\{X(t): t \geq 0\}$ is a semi-Markov process, the moments are random variables that satisfy the condition [5, 7]:

$$P\{X(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < \tau \,|\, X(\tau_n) = s_i, X(\tau_{n-1}), \ldots\ldots X(\tau_1), X(\tau_0), \tau_n - \tau_{n-1}, \ldots, \tau_1 - \tau_0, \tau_0\} =$$
$$= P\{X(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < \tau \,|\, X(\tau_n) = s_i\}$$

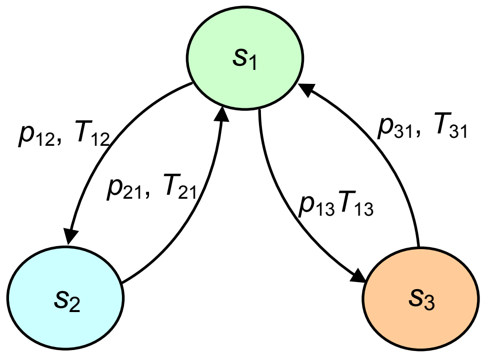

Fig. 2. Graph of engine state transitions:
p_{ij} – probability of process transition from state s_i to state s_j, T_{ij} – duration of state s_i provided that the process transits to state s_j; $i \neq j$; $i, j = 1, 2, 3$ [3]

This model of engine technical state transitions is a simplified model when comparing to the model with the graph of engine state transitions depicted in Fig. 1. The simplification, as mentioned above, consists in that the graph in Fig 2 does not include a curve illustrating a possibility of transition from state s_2 to s_3, due to the taken assumption that $p_{23} = 0$. Accordingly, the function matrix (7) for the process $\{X(t)\colon t \geq 0\}$ is simplified when comparing to the function matrix (5) for the process $\{X(t)\colon t \geq 0\}$. The matrix (10) does not comprise the function $Q_{23}(t)$.

An exemplary process with the graph of state transitions shown in Fig. 2, is presented in Fig. 3.

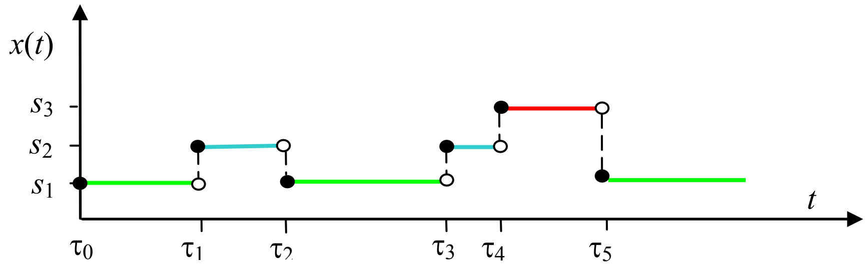

Fig. 3. An exemplary process $\{X(t)\colon t \geq 0\}$ for the engine: $\{x(t)\colon t \geq 0\}$ – process of technical state transitions, t – operating time; s_1 – state of full ability, s_2 – state of partial ability, s_3 – state of disability [3]

The initial distribution for the process is defined by the formula (4), while its function matrix is as follows [3]:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & 0 \\ Q_{31}(t) & 0 & 0 \end{bmatrix} \tag{7}$$

Therefore, for the presented process $\{X(t)\colon t \geq 0\}$ with the function matrix defined by formula (7), the limiting distribution can be derived from the formula [2, 3, 8]:

$$P_j = \frac{\pi_j \cdot E(T_j)}{\sum_{k=1}^{3} \pi_k \cdot E(T_k)}, \quad j = 1,\ 2,\ 3 \tag{8}$$

The distribution $\pi_j (j = 1, 2, 3)$ in the formula (8) is a limiting distribution of the Markov chain $\{Y(\tau_n)\colon n = 0, 1, 2, 3, \ldots\}$ embedded in the process $\{X(t)\colon t \geq 0\}$. When the time of correct work (T_{pp}) and the recovery time (T_o) of the engine can be described with an exponential distribution, the matrix of transition probabilities for the embedded Markov chain $\{Y(\tau_n)\colon n = 0, 1, 2, 3, \ldots\}$takes a form:

$$\mathbf{P} = \left\{ \begin{matrix} 0 & \frac{\lambda_{12}}{\Lambda} & \frac{\lambda_{13}}{\Lambda} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix} \right\} \tag{9}$$

Thus, determination of probabilities P_k $(k = 1, 2, 3)$ for engine being in states s_k $(k = 1, 2, 3)$ demands solution of the following system of equations [3, 8]:

$$\left.\begin{array}{l}[\pi_1,\pi_2,\pi_3]\cdot\begin{Bmatrix}0 & \frac{\lambda_{12}}{\Lambda} & \frac{\lambda_{13}}{\Lambda}\\ 1 & 0 & 0\\ 1 & 0 & 0\end{Bmatrix}=[\pi_1,\pi_2,\pi_3]\\ \pi_1+\pi_2+\pi_3=1\end{array}\right\} \quad (10)$$

while:

$$\Lambda=\sum_{k=2}^{3}\lambda_{1k}$$

where:

λ_{12} – function of risk of partial damage resulting in occurrence of state s_2,
λ_{21} – intensity function of recovery causing change from state s_2 to state s_1,
λ_{13} – function of risk of total damage resulting in occurrence of state s_3,
λ_{31} – intensity function of recovery causing change from state s_3 to state s_1.

The solution of the system of equations (10), with regard to the formula (8), gives the probabilities:

$$P_1=\frac{1}{H},\ P_2=\frac{\lambda_{12}\cdot E(T_2)}{H},\ P_3=\frac{\lambda_{13}\cdot E(T_3)}{H} \quad (11)$$

while:

$$H=1+\lambda_{12}\cdot E(T_2)+\lambda_{13}\cdot E(T_3)$$

where:

P_1, P_2, P_3 – probabilities that diesel engine is respectively in states: s_1, s_2, s_3;
π_j – limiting probability of a Markov chain embedded in the process $\{X(t): t\geq 0\}$ that describes possibility of occurrence of state s_j, $j = 1, 2, 3$;
p_{ij} – probability of the process $\{X(t): t\geq 0\}$ transition from state s_i to state s_j;
$E(T_j)$ – expected value of duration of state s_j.

When performance of the task by the main engine is possible only if it finds itself in a state of full ability (i.e. state s_1), its reliability is defined by the probability P_1, regardless of the type of the model selected for the considerations. However, when the task can be performed by the main engine, even when it is in a state of partial ability, the reliability of the engine can be determined by the sum of probabilities of existence of the both types of states. The probabilities are essential for setting a plan of engine operation, as implementation of it requires assurance of financial resources, fuel and lubricating oil supplies, and spare parts.

4. Remarks and conclusions

Employment of semi-Markov models for ship main engines is justified by that the Markov condition is satisfied, which was shown through empirical studies, and the random variables denoting time intervals, after which changes of states of such engines proceed, have distributions other than exponential. Semi-Markov processes are increasingly applied, and not only for solving different issues concerning reliability and diagnostics of diesel or other combustion engines.

Application of a semi-Markov process as a three-state model of transitions of the mentioned reliability states of a main engine, at defined time, results from that the random variable $T_{(ij)}$ denoting duration of state $s_{(i)}$, provided that the successive state is s_j, and the variable random T_i denoting duration of engine state $s_{(i)}$ (e.g. i = 1, 2, 3), regardless of which state is successive, have arbitrary distributions from the set $R_+ = [0, +\infty)$. While using an appropriate *SDG* that enables development of a complete diagnosis (instantaneous diagnosis, prognosis and genesis) with defined reliability, it is reasonable during studies on reliability of this kind of engines, to apply a Markov process, because in such a case where additionally assurance of a high safety level to a sea-going ship is demanded, the random variables $T_{(ij)}$ and $T_{(i)}$ can be assumed to have exponential distributions.

The presented models are of significant practical meaning due to the ease of defining the estimators of transition probabilities $p_{(ij)}$ and the ease of estimating the expected values $E(T_{(j)})$.

It can be expected that the proposed models may also be useful in studies on reliability of other machines.

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