DEPENDENCE OF HARMONIC GENERATION ON PRIMARY WAVE PRESSURE DISTRIBUTION

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The aim of this paper is theoretical analysis of harmonic generation during finite amplitude wave propagation. The paper presents mathematical model and some results of numerical calculations. The mathematical model was built on the basis of the KZK equation. To solve the problem the Fourier series expansion and finite-difference method were applied. The harmonic pressure amplitudes as a function of axial distance and their transverse distributions at fixed distances from the source were examined. The pressure amplitude distributions at horizontal section were investigated, too. The calculations were carried out for different pressure distributions on the circular source and different values of medium parameters.

Keywords: nonlinear acoustics, wave propagation, numerical methods in physics.

1. Introduction

The wave distortion is observed during finite amplitude wave propagation in water. The waveform change is equivalent with spectrum change. It means that new harmonic components appear during wave propagation.

The Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation is often used during theoretical investigations of the finite amplitude wave propagation problem. This equation describes the acoustic pressure changes in nonlinear and dissipative medium along sound beam. The exact analytical solution of this equation is not known till now, and consequently it is necessary to solved it using approximate methods. The method of successive approximations to find this equation solution can be used when the nonlinear effects are not very big [1]. Generally the KZK equation must be solved using numerical methods [2].

Assuming axial symmetry of the problem it is comfortably to solve the problem in cylindrical coordinates. Correct modeling of the pressure distribution on the primary

wave source and correct choice of values of source and medium parameters are very important during theoretical investigations.

The aim of this paper is numerical analysis of the finite amplitude wave propagation in water. The changes of the harmonic pressure amplitudes along sound beam were investigated. Calculations were done for different pressure distributions on the circular source. The Fourier series expansion and finite-difference method were used to solve the problem.

2. Mathematical model

We assume that circular piston with a fixed radius a which is placed in plane yOz radiates single frequency wave. This finite amplitude wave is propagated in the x direction, i.e. this axis corresponds with sound beam axis.

Mathematical model is built on the basis of the KZK equation:

$$\frac{\partial}{\partial \tau}\left(\frac{\partial p'}{\partial x} - \frac{\varepsilon}{\rho_0 c_0^3} p' \frac{\partial p'}{\partial \tau} - \frac{b}{2\rho_0 c_0^3} \frac{\partial^2 p'}{\partial \tau^2}\right) = \frac{c_0}{2}\left(\frac{\partial^2 p'}{\partial r^2} + \frac{1}{r}\frac{\partial p'}{\partial r}\right), \qquad (1)$$

where $p' = p - p_0$ denotes an acoustic pressure, variable $\tau = t - x/c_0$ is the time in the coordinate system fixed in the zero phase of the propagating wave, ρ_0 – medium density at rest, c_0 – speed of sound, b – dissipation coefficient of the medium, ε – nonlinear coefficient, $r = (y^2 + z^2)^{1/2}$.

To complete the problem the boundary conditions are defined:

$$p'(x = 0, r, \tau) = \begin{cases} A(r)\sin\omega\tau & \text{for } r \leq a, \\ 0 & \text{for } r > a \end{cases}$$

and

$$\left.\frac{\partial p'}{\partial r}\right|_{r=0} = 0, \qquad \left.\frac{\partial p'}{\partial r}\right|_{r=R_{\max}} = 0,$$

where $A(r) = p_0 f(r)$ denotes primary wave amplitude and angular frequency is defined by $\omega = 2\pi f$. Moreover we assume that function p' is a periodic function of the coordinate τ.

Solution of Eq. (1) is looked for inside hypothetical cylinder, i.e. in domain $D = \{(x, r) : x \in [0, X_{\max}], r \in [0, R_{\max}]\}$ where $X_{\max}$ denotes the biggest investigated distance from the source and $R_{\max}$ is cylinder radius.

The solution of the KZK equation is looked for in form:

$$p'(x, r, \tau) = 0.5 p_0 \sum_{n=1}^{N} \left(A_n(x, r)e^{in\omega\tau} + c.c.\right). \qquad (2)$$

Substituting (2) into Eq. (1) after calculations we obtain partial differential equations for harmonic components A_n ($n = 1, 2, \ldots, N$). Including boundary conditions we can calculate these amplitudes. Presented in this paper results of computer calculations were done for $N = 3$ harmonic components.

To solve the KZK equation numerically the non-dimensional coordinates are defined [3]. The finite-difference method was used to solve the problem numerically. Harmonic pressure amplitudes along sound beam are the result of computer calculations. Pressure distribution along sound beam can be calculated substituting harmonic components into (2).

3. Numerical investigations

The harmonic pressure amplitude distributions along sound beam were investigated numerically. All presented in this paper results of computer calculations were done assuming that wave which frequency $f = 1$ MHz is propagated in water where density $\rho_0 = 1000$ kg/m^3, speed of sound $c_0 = 1500$ m/s, nonlinear coefficient $\varepsilon = 3.5$. The pressure p_0 and dissipation coefficient b were changed.

The even and polynomial pressure distributions on the finite amplitude wave source are often considered in mathematical models [1, 2]. Normalized on-axis first, second and third harmonic pressure amplitudes as a function of distance from the source obtained for different pressure distributions on the source are presented in Fig. 1. Calculations were done for pressure $p_0 = 150$ kPa and dissipation coefficient $b = 0$. The results obtained for even pressure distribution are presented in Fig. 1a. Similar results of computer calculations obtained for polynomial pressure distribution are shown in Fig. 1c. The results presented in Fig. 1b were done for pressure distribution which is combination of even and polynomial distribution.

Figure 2 shows transverse distribution of first, second and third harmonic pressure amplitude for polynomial pressure distribution on the source. Dashed line presents normalized pressure amplitude of primary wave. Solid lines show distribution of harmonic pressure amplitudes at distance $x = 0.7$ m from the source.

Influence of pressure distribution on the primary wave source on the pressure distribution along sound beam presents next figure. The results of computer calculation obtained for two different distributions are shown in Fig. 3. Calculations were done for even pressure distribution on the source and distribution which is combination of even and polynomial one respectively. Transverse distributions of first, second and third harmonic pressure amplitude at distance $x = 0.7$ m from the source are shown in left figures. The first harmonic pressure amplitude distribution at horizontal section obtained for suitable primary wave pressure distributions are shown in right figures. To compare the harmonic pressure amplitudes obtained for different primary wave pressure distributions all calculations presented in Figs. 2 and 3 were done for the same values of physical and numerical parameters.

The results of numerical calculations presented till now were done for dissipation coefficient $b = 0$. It means that wave is propagated in non-dissipative medium. An example of computer calculation for $b = 0.04$ and different values of pressure p_0 is shown in Fig. 4. This figure presents on-axis first and second harmonic pressure amplitude as a function of distance from the source for even pressure distribution. Solid lines present the results obtained for pressure $p_0 = 150$ kPa and dashed line shows similar results obtained for pressure $p_0 = 300$ kPa.
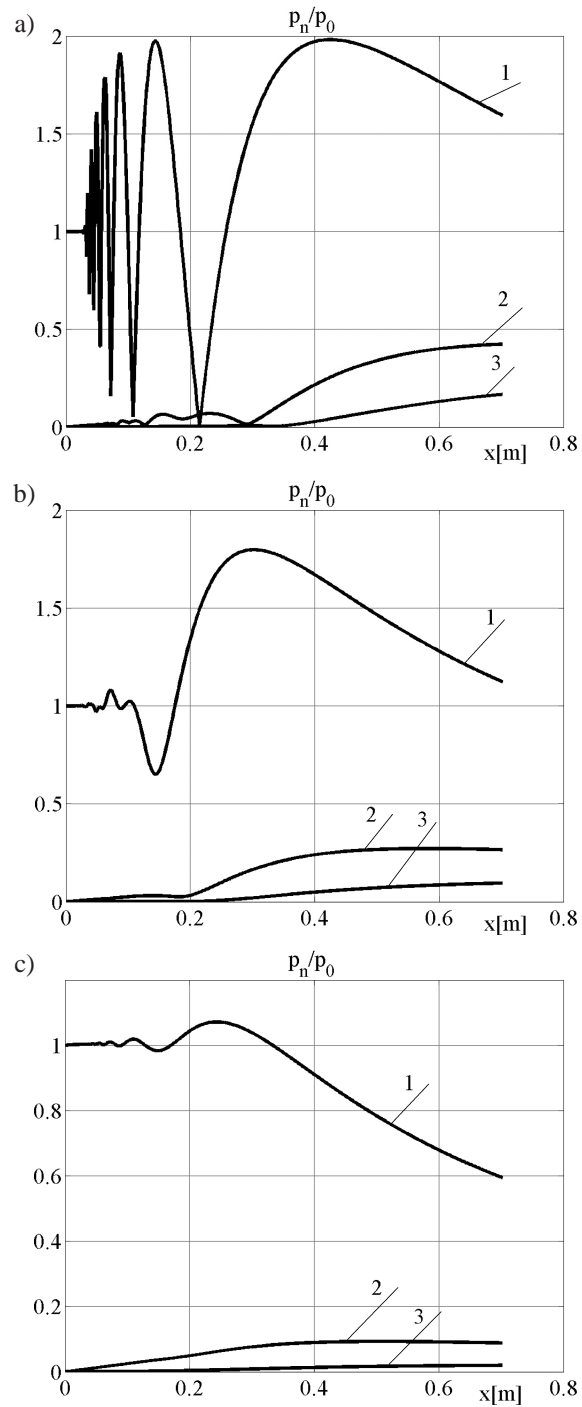
Fig. 1. On-axis first, second and third harmonic pressure amplitude as a function of distance from the source for different pressure distributions on the source.
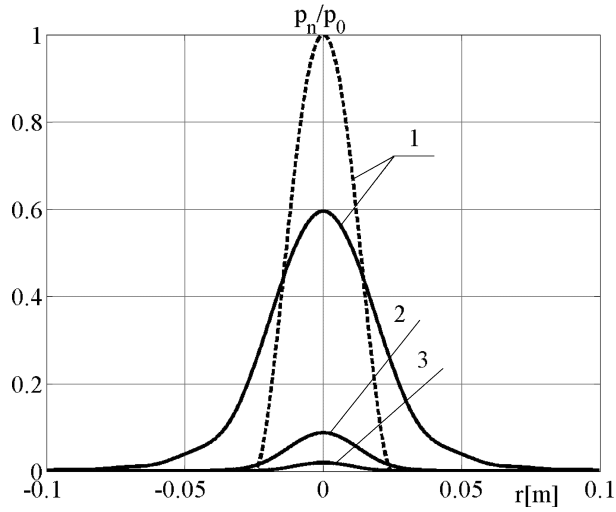
Fig. 2. Transverse distribution of first, second and third harmonic pressure amplitude for polynomial pressure distribution on the source.
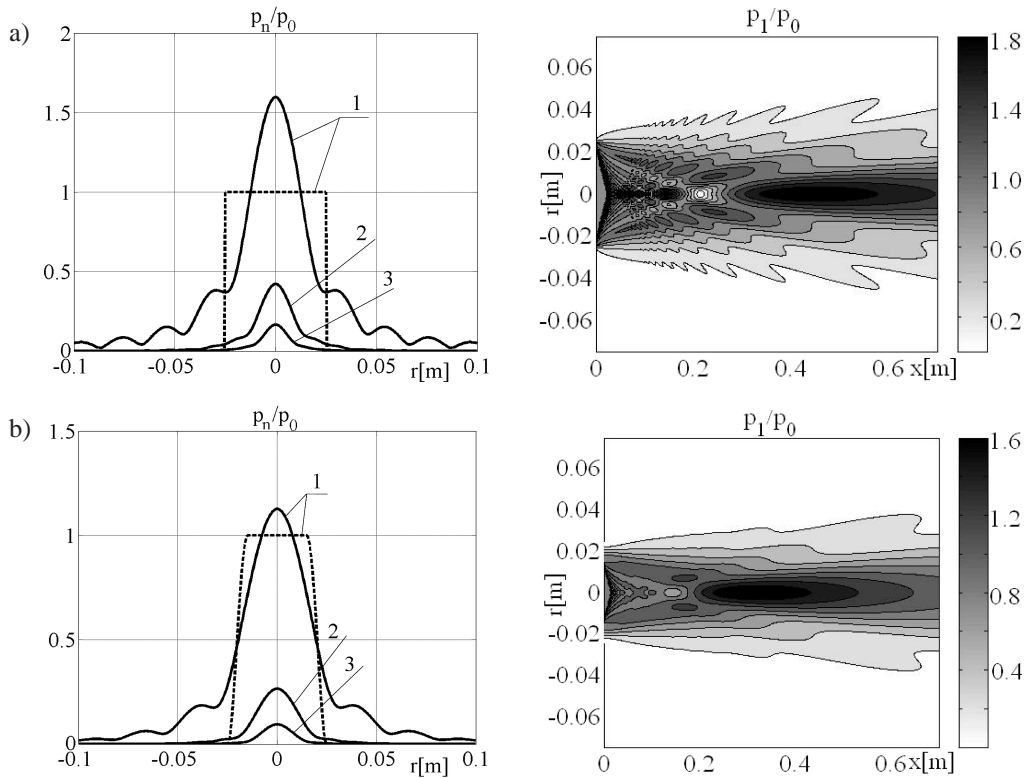

Fig. 3. Transverse distribution of first, second and third harmonic pressure amplitude and normalized first harmonic pressure amplitude distribution at horizontal section.
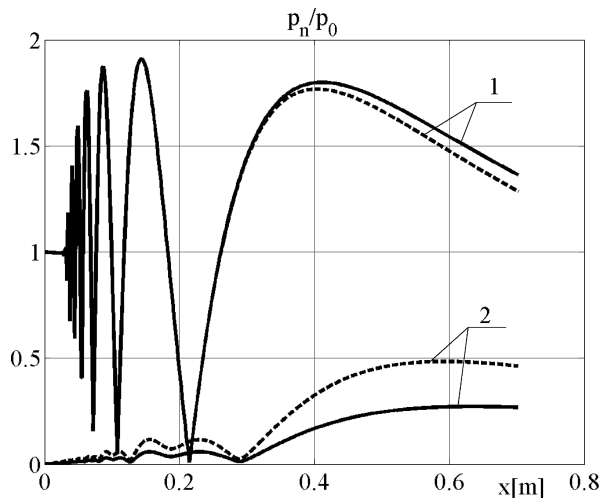
Fig. 4. On-axis first and second harmonic pressure amplitude as a function of distance from the source for even pressure distribution on the source.

4. Conclusions

The analysis of the results of numerical calculation shows that correct choice of the physical parameters like the correct description of pressure distribution on the primary wave source are very important during theoretical investigations.

All presented in this paper results were done for three harmonic components (parameter $N = 3$ in Eq. (2)). Of course, it is possible to calculate more harmonic components but exact analysis of this problem yields that it is not necessary for considering in this paper physical parameters and investigated distances from the source. It is important to remember that correct choice of values of numerical parameters is also very important during numerical investigations [3].

Numerical investigations were carried out using own computer program which was worked out on the basis on obtained mathematical model. This program allows to analyze finite amplitude wave propagation problem for circular source. Proposed method can be used to investigate the wave propagation for different values of source and medium parameters. Mathematical and numerical models can be modify for sources without axial symmetry.

References

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