DETERMINATION OF RECTIFICATION CORRECTIONS FOR SEMI GANTRY CRANE RAIL AXES IN THE LOCAL 3D COORDINATE SYSTEM

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Abstract

Electronic tacheometers are currently the standard instruments used in geodetic work, including also geodetic engineering measurements. The main advantage connected with this equipment is among others high accuracy of the measurement and thus high accuracy of the final determinations represented for example by the points' coordinates. One of many applications of the tacheometers is the measurement of crane rail axes. This measurement is based on polar method and it allows to get the spatial coordinates of points in 3D local system. The standard technology of measurement of crane rail axes and development of its calculations' results is well-known and widely presented in the subject literature. At the same time new methods of observations results evaluation are developing.

Some new proposals for the development of measurement results were already presented in (Kamiński, 2013).

This paper is a generalisation of the paper quoted above. The authors developed the concept which was presented there by a proposal for determining rectification corrections for semi gantry crane rail axes. To carried out the task, the parametric method with conditions on parameters was used. Moreover the practical tests on simulated measurement results were conducted. The results obtained from alignment confirmed the theoretical assumptions.

Despite the fact that analyses were carried out only on the simulated data, it is already possible to say that presented method for determination of rectification corrections for crane rail axes can be used for development of the observations from real measurement.

Keywords: rectification corrections for crane rail axes, semi gantry crane, local 3D system

1. Introduction

The measurement of crane rail axes using electronic tacheometers based on polar method allows to obtain the coordinates in 3D local system. At present electronic tacheometers provide the high measurements accuracies. This is the reason why they have become commonly used equipment in the geodetic works. Many papers related to the measurement of crane rail axes are connected with rail axes' control or installation. Furthermore, some authors analysed the geometric conditions necessary for correct crane work. Those problems are presented, among others, by (Aleš Marjetič et al. 2012, Tomáš Křemen et al. 2008, Grala, Kopiejewski 2003) and by (Peter Kyrinovič et al.,2010),respectively.

In this paper the authors presented the method of determination of rectification corrections of semi gantry crane rail axes. It is related to local 3D coordinates system. It should be mentioned that similar problem was already presented in (Kamiński, 2013).

The measurement methodology is not a part of this paper however it is described for example in (Gocał, 2010).

The new method for determination of rectification corrections was not only briefly described in the following sections but also its practical applications using simulated measurement results were presented. The proposed method, which was used in the computations was the parametric method with conditions on parameters. The obtained results allow to claim that proposed method can be used not only for simulated data but also for the real data.

2. The parametric method with conditions on parameters

As mentioned previously, similar problem was presented in (Kaminski, 2013). There was also a description how to get the three equations which allow to solve the undertaken problem. For more details on parametric method with conditions on parameters and its application to determination of adjustment corrections for crane rail axes, reference can be made also to this paper and to (Baran, 1999) and (Wiśniewski, 2005). This paper provides only basic information about this method.

The problem which is discussed in this paper can be formulated by the following system of equations:

$$\left. \begin{array}{l} AX + L = v \\ BX + \Omega = 0 \end{array} \right\}$$

where:

A, B – the known coefficients matrices, X – the vector of estimated parameters, v – the vector of corrections, L – the vector of measurements results, Ω – the vector of free terms.

To solve the optimisation problem the Lagrangian method is used:

$$v^T P v - 2k^T (BX + \Omega) = min$$

where P is the weight matrix, k – is called a Lagrangian multiplier.

The computations enabling to solve the described issue can be presented in the subsequent stages:

1. $k = -\{B(A^T P A)^{-1} B^T\}^{-1}\{\Omega - B(A^T P A)^{-1} A^T P L\}$
2. $X = -(A^T P A)^{-1}(A^T P L - B^T k)$
3. $v = AX + L$

There is also the simplified method whereby the problem which was presented at the beginning of this section can also be solved. This solution enables to use the conditional equations as additional equations of corrections with high weights' values, hence:

$$BX + \Omega = v_\Omega \text{ with weights } P_\infty$$

where $P_\infty = diag(\eta_1, \dots, \eta_r)$ and η - any number, r - number of conditions.

Condition for the correctness of the calculation is the adoption of such high weights. It causes that the corrections which were obtain from the solution are equal to zero. By that means that the computations are carried out using the parametric method. The system of the parametric method equations of corrections is presented below.

$$\tilde{A}X + \tilde{L} = \tilde{v}$$

where:

$$\tilde{A} = \begin{bmatrix} A \\ B \end{bmatrix}, \tilde{L} = \begin{bmatrix} L \\ \Omega \end{bmatrix}, \tilde{v} = \begin{bmatrix} v \\ v_\Omega \end{bmatrix}$$

with weights:

$$\tilde{P} = \begin{bmatrix} P & 0 \\ 0 & P_\infty \end{bmatrix}$$

If the solution by the parametric method is used then we obtain:

$$X = -\left(\tilde{A}^T \tilde{P} \tilde{A}\right)^{-1} \tilde{A}^T \tilde{P} \tilde{L}$$

This allows to write the vector $\tilde{v}$:

$$\tilde{v} = -\tilde{A}\left(\tilde{A}^T \tilde{P} \tilde{A}\right)^{-1} \tilde{A}^T \tilde{P} \tilde{L} + \tilde{L}$$

For the estimation results the statistical interpretation should be carried out. It is related to the creation, among others, a cofactor matrix of parameters and a cofactor matrix of the vector of corrections.

The cofactor matrix (Q_X) can be written as follows:

$$Q_X = (\tilde{A}^T \tilde{P} \tilde{A})^{-1}$$

and the cofactor matrix of the vector of corrections Q_v is taken the form of:

$$Q_v = (\tilde{P} - \tilde{A}(\tilde{A}^T \tilde{P} \tilde{A})^{-1} \tilde{A}^T)$$

3. Observation equations

Using electronic tacheometer, geodetic measurements give the coordinates x, y, z for controlled point. This paper presents the semi gantry crane. It is worth noting that both rails are not situated at the same height like it is in case of the gantry crane. It is assumed that the right rail is located higher than the left one. Assuming that crane

rails are parallel there is a possibility to write the equations like those below in the table 1. In this table the straight lines equations as well as corrections' equations are presented.

The following marks are used in Table 1 and 2:

v_{y_i}, v_{z_i}- the corrections to observations

a, b, c, H – the estimated parameters

x_i, y_i, z_i - the space coordinates of the controlled points

$i = 1, 2, ..., n$ - point number

n - number of the controlled points

Table 1. The observations' equations and the corrections' equations

Left rail	Right rail
$y = ax + b$	$y = ax + b + c$
$v_{y_i} = ax_i + b - y_i$	$v_{y_i} = ax_i + b + c - y_i$

In this paper beyond the coordinates x, y, the observations of z coordinate are also concerned. The observations' equations of points' height and the corrections equations were created for the left and right rail, like it was in the case of x, y coordinates. Considerations are conducted for the semi gantry crane so the height difference between parallel crane rails should be taken into account.

Table 2. The observations' equations and corrections' equations of points' height

Left rail	Right rail
$z = z^w$	$z = z^w + H$
$v_{z_i} = z^w - z_i$	$v_{z_i} = z^w + H - z_i$

4. Example of practical application

For the proposed method analysing the simulated results were used. It allows to known their practical properties. It was assumed that all controlled points (1,2,…10) were located on the axes of crane rails (Fig.1). The points from 1 to 5 were placed on the left rail and the points from 6 to 10 on the right rail. The space coordinates in a local coordinates system x, y, z are presented in the Table 3.

Table 3. 3D coordinates of the controlled points

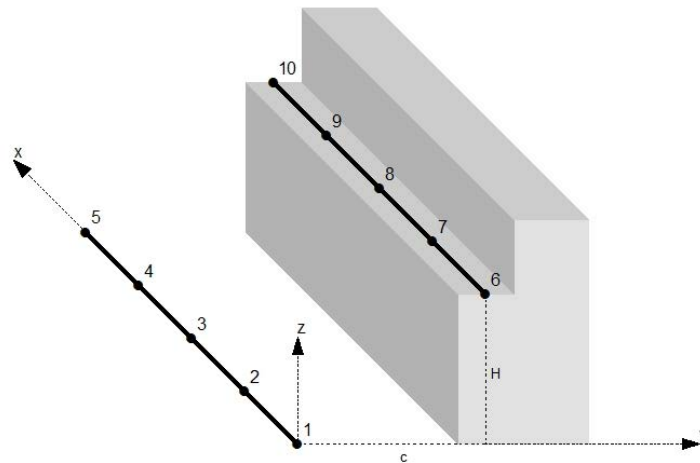| Point no.	x [mm]	y [mm]	z [mm]
1	0	0	0
2	4002	4	-10
3	8016	12	20
4	11999	-10	-15
5	15985	-6	15
6	18	10002	8010
7	3992	10011	7985
8	8005	9994	8015
9	12003	10002	7990
10	16001	9989	8020

Fig. 1. The distribution of the controlled points (3D system)

Further considerations were divided into two variants. In both cases the simplified method of performing computations was used. The first variant was based on the use of appropriate equations of corrections, which were formulated for all controlled points. Moreover, this variant was associated with two conditions. The second variant was based on the adoption of an additional condition to the constraints used in first variant.

Variant 1

Based on appropriate equations of corrections, which were formulated for controlled points, matrix A, X and L was created (shown below)

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
4002 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
8016 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
11999 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
15985 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
18 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
3992 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
8005 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
12003 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
16001 & 1 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\qquad
X = \begin{bmatrix} a \\ b \\ c \\ H \end{bmatrix}
\qquad
L = \begin{bmatrix}
0 \\
0 \\
-4 \\
10 \\
-12 \\
-20 \\
10 \\
15 \\
6 \\
-15 \\
-10002 \\
-8010 \\
-10011 \\
-7985 \\
-9994 \\
-8015 \\
-10002 \\
-7990 \\
-9989 \\
-8020
\end{bmatrix}
$$

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The two conditions were also adopted:
1. associated with the distance (c) between the axes of both rails, what can be written as follows: c = 10 000mm,
2. connected with the height difference of the axes of both rails in the form of H = 8 000mm

Those conditions can be formulated as follows:

$$c - 10\ 000 = 0$$
$$H - 8\ 000 = 0$$

It can be also written in matrix notation:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ H \end{bmatrix} + \begin{bmatrix} -10\ 000 \\ -8\ 000 \end{bmatrix} = \begin{bmatrix} v_\Omega \\ v_\Omega \end{bmatrix}$$

$$B \qquad X + \qquad \Omega \quad = \quad v_\Omega$$

The vectors X and $\tilde{v}$ were determined with the use of the relationship which was described earlier.

Variant 2

The two conditions described in variant 1 were supplemented with additional condition. The height of the left semi gantry crane rail was z = 10mm. The conditions from Variant 2 are described below:

$$c - 10\ 000 = 0$$
$$H - 8\ 000 = 0$$
$$z^w - 10 = 0$$

or, in matrix notation:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ H \\ z^w \end{bmatrix} + \begin{bmatrix} -10\ 000 \\ -8\ 000 \\ -10 \end{bmatrix} = \begin{bmatrix} v_\Omega \\ v_\Omega \\ v_\Omega \end{bmatrix}$$

$$B \qquad X + \qquad \Omega \quad = \quad v_\Omega$$

In this variant vectors X and $\tilde{v}$ were determined using the simplified method of performing computations.

The Table 4 compiles the results of computations of first and second variant.

Table 4. Results of computations

Parameters and corrections		Variant 1 [mm]	Variant 2 [mm]
X parameter	a	0	0
	b	6	6
	c	10000	10000
	H	8000	8000
	z^w	-	10
Corrections	v_{y_1}	6	6
	v_{z_1}	0	10
	v_{y_2}	-1	-1
	v_{z_2}	10	20
	v_{y_3}	-12	-12
	v_{z_3}	-20	-10
	v_{y_4}	7	7
	v_{z_4}	15	25
	v_{y_5}	0	0
	v_{z_5}	-15	-5
	v_{y_6}	4	4
	v_{z_6}	-10	0
	v_{y_7}	-8	-8
	v_{z_7}	15	25
	v_{y_8}	6	6
	v_{z_8}	-15	-5
	v_{y_9}	-5	-5
	v_{z_9}	10	20
	$v_{y_{10}}$	5	5
	$v_{z_{10}}$	-20	-10
	v_c	0	0
	v_H	0	0
	v_{z^w}	-	0

The results of computations were also visualized at the figures 2a, 2b and 2c. The figures show not only vertical but also horizontal adjustment corrections.
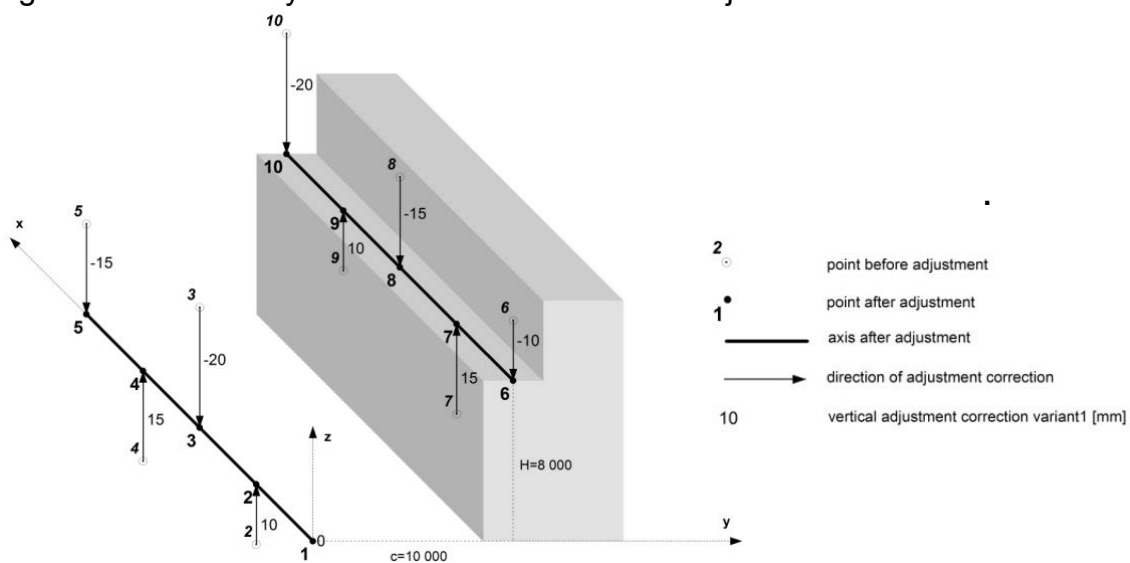

Fig. 2a Vertical adjustment corrections in the xyz system [variant 1]
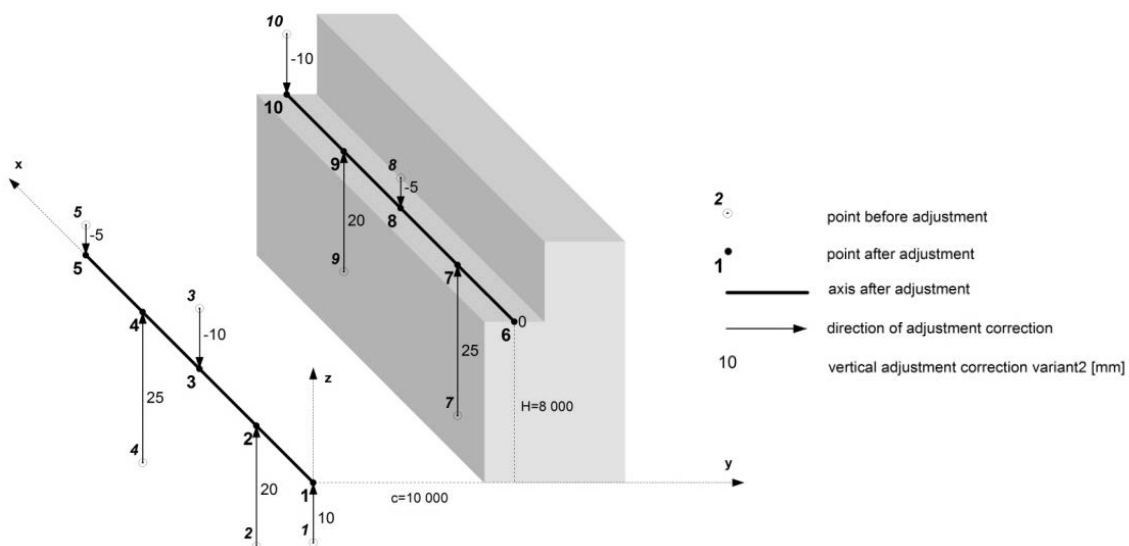
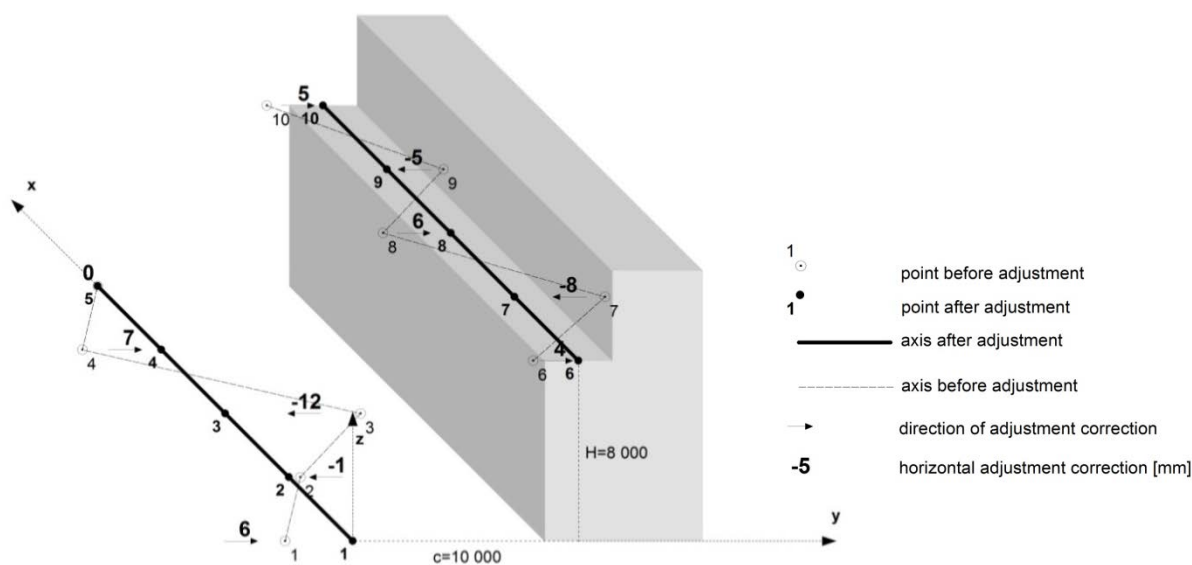**Fig. 2b.** Vertical adjustment corrections in the xyz system [variant 2]

Fig. 2c. Horizontal adjustment corrections in the xyz system

5. Summary

The new method for determination of adjustment corrections for the crane rail axes which was presented in (Kamiński, 2013) was developed in this paper. The use of this method allowed to determine rectification corrections for semi gantry crane rail axes. The example, which was based on simulated data, illustrated the use of described method for this type of crane. Despite the fact that the computations were performed on simulated data it can be claimed that this method can be also use for real data. The proposed method allows to determine the rectification corrections in the local 3D coordinate system. Taking into account that the use of the proposed method was presented for the gantry crane in (Kamiński, 2013) and for the semi gantry crane in this paper, it can be ascertained that this method can be used also for other types of cranes.

References

Baran L. W., (1999), *Theoretical basis for analysis of geodetic survey results*, Polish Scientific Publishers PWN SA, Warsaw.

Gocał J., (2010), *Engineering and industrial geodesy. Part III*, AGH University of Science and Technology Press, Cracow.

Grala M., & Kopiejewski G., (2003), *Engineering Geodesy. Selected Branches*, UWM Press, Olsztyn.

Kamiński W., (2013), *New method for determination of adjustment corrections for crane rail axes*, Reports on Geodesy vol. 94/2013, 47-55, doi: 10.2478/rgg-2013-0006

Křemen T., Koska B., Pospíšil J., Kyrinovič P.,Haličková J., & Kopáčik A., (2008), *Checking of crane rails by terrestrial laser scanning technology*, 13th FIG, 4th IAG, Lnec, Lisbon 12-15 May

Kyrinovič P., & Kopáčik A., (2010), *Automated measurement system for crane rail geometry determination*, 27 th International Symposium on Automation and Robotics in Construction (ISARC 2010), 294 - 305

Marjetič A., Kregar K., Ambrožič T., & Kogoj D., (2012), *An Alternative Approach to Control Measurements of Crane Rails*, Sensors, 12, 5906-5918

Wiśniewski Z., (2005), *Adjustment calculus in geodesy (with examples)*, UWM Press, Olsztyn.

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