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DETERMINATION OF VERTICAL DISPLACEMENTS IN RELATIVE MONITORING NETWORKS

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The problem of determining displacements of objects is an important and current issue, in particular in terms of operational safety. This is a requirement that covers geodetic, periodic control measurements in order to determine horizontal and vertical displacements. The paper is focused on the analysis of vertical displacements. Geodetic measurements and their interpretation allow to reduce the risk of possible structural catastrophes. The major research topic of the majority of available papers is displacement determination of individual controlled points, in a situation where there are identified as fixed reference points. There are cases making identification of such points difficult or impossible to use in displacement analysis. This paper addresses a rare case of determining vertical displacements in unstable reference systems. Due to the fact that most of the existing and known literature methods do not always bring satisfactory results, the paper propose a new method of vertical displacement determination in the absence of reference points in the local coordinate system. Practical considerations on simulated data show that the presented method performs the task correctly.

Keywords: vertical displacements, unstable reference system, a lack of reference points

1. THEORETICAL CONSIDERATIONS

Changes in the groundwater level, improperly made foundations, deep excavations, heavy load on the ground, hidden material defects and structural operation are just a few factors that can cause displacement of a ground and objects located on it. Possible consequences of ground movement are horizontal and vertical displacements. Therefore, an important issue related to the safe operation of

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an object is to determine the value of these displacements. The significance of the problem of displacement determination is indicated by a large number of papers, eg.: [1], [2], [3], [7], [9], [10], [22], [23], [24], [25], [32]. Our paper focuses on vertical displacements. There exist literature references exclusively addressing the issue, eg.: [5], [8], [12], [13], [15], [33].

Displacement analysis distinguishes, stable and unstable reference systems. Most of the papers focus on the situation when vertical displacements are determined based on stable reference points. There may be situations when identification of such points is difficult or it is not possible to use them in the analysis of displacements (eg they have been completely destroyed). In the literature on the subject we can also find a division of geodetic networks assumed for the purpose of determining displacements: relative - composed of controlled points located on the tested object and absolute (absolute) - composed of reference points located outside the range of the tested object and used to determine absolute displacements of objects controled points [4], [22]. In the paper, detailed theoretical and empirical analysis were carried out for the issue of the lack of stability of the reference system, rare in available literature.

Literature references include numerous methods used to determine the displacements of points. The list includes: the IWST method (Itterative Weighted Similarity Transformation) [6], [7], [9], [22], [26], free adjustment [18], [19], [21], [31], robust free adjustement [20], DiSTFA method [16], [17], R – estimation [11], [12], M-split estimation [28], [29], [30] and other. While regarding relative networks, the majority of existing methods should be applied with care, carefully used, because results may lead to wrong conclusions. The problem considered in this paper, shows the methods work well if no more than 50% of the control network points are displaced. While more than 50% of controlled points are displaced, and stable reference points are not identified, i.e. the analysis is conducted due to relative networks, the methods mentioned are not always correct (eg. [14]). The conclusion is also confirmed by a presented practical example, the application display of one of the above mentioned methods. In geodetic practice it is not always possible to determine reference points, allowing to determine displacements of controlled points, the existing methods are no always satisfactory here. Hence the paper includes an algorithm to determine vertical displacements of individual points in the absence of reference points. Thus all points contributing the measurement and estimation process are considered potentially displaced. Therefore displacement determination of controlled points is carried out in relative networks.

2. THE CONCEPT OF A NEW METHOD

The theoretical considerations led to create a new concept to determine vertical displacements of individual controlled points in the absence of stable reference points or a difficulty in identification. The proposed concept is implemented in six stages, described below [14].

In the first stage, a point is chosen as the origin of the local coordinate system (OCS). Based on the detailed numerical analysis carried out for the presented algorithm, one of the external controlled points should be considered as OCS. In this system coordinates (X, Y) of all controlled points are determined. Additionally, this system is the base to further determine the displacements of all points. A right-handed coordinate system was adopted in the analysis.

The second stage of the algorithm assumes the use of the concept presented in [27], hereinafter referred to as the ZW method, in order to obtain two vectors: a rotation vector $\mathbf{Y}^j = [\varepsilon_X^j \quad \varepsilon_Y^j]^T$ and a vector of spacing of the momentary surface of the object from the momentary optimal plane $\hat{\mathbf{r}}^j = [\hat{r}_1^j, \quad \dots \quad, \hat{r}_i^j]^T$, both vectors are the basis for further analysis. In order to understand this algorithm step the most important assumptions of the concept presented in [27] are presented below. This concept was dedicated to determining vertical displacements of the foundation slab of the designed Nuclear Power Plant in Żarnowiec.

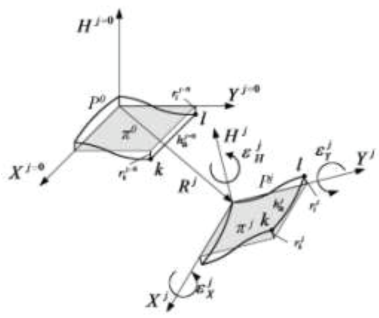

Fig. 1. ZW concept

The designations adopted in Fig. 1 are: k and l – controlled points, r_k^j, r_l^j – spacing of the momentary surface of the object from the momentary optimal plane at points k and l in time instants t^j(j=0, 1, …,), eg. $t^{j=0}$ - time moment of measurement in epoch j=0, h_{lk}^j – the result height difference measurement in epoch t^j, ε_X^j, ε_Y^j, ε_H^j – rotation angles around the X, Y and H axes, respectively, in epoch t^j, $\mathbf{R}^j = [R_X^j \quad R_Y^j \quad R_H^j]^T$ – translation vector at the time moment t^j.

The author of the concept assumed that the position parameters significantly affecting the object operational safety are rotation angles around the X and Y axes: ε_X^j, ε_Y^j, respectively. However, the rotation angle around the H axis ε_H^j is negligible. The position vector of an object represented by the angles of rotation at the time instant j - t^j takes the form $\mathbf{Y}^j = [\varepsilon_X^j \quad \varepsilon_Y^j]^T$. Considering the location of controlled points of an object, attention should also be paid to the spacing of the instantaneous surface of the object from the momentary optimal plane r^j, determined due to all controlled points at the time j. A set of such spacings create the object shape vector $\mathbf{r}^j = [r_k^j \quad \Lambda \quad r_l^j]^T$. Summing up, the location and shape of the object in a time instant are defined by two vectors: $\mathbf{Y}^j$ and $\mathbf{r}^j$. Considering two controlled points (in the following considerations marked k and l) and given the heights of these points H_k and H_l, the height difference between them is $h_{lk} = H_k - H_l$. Due to the fact that $h_{lk} = h_{lk}^{obs} + v_{lk}$, where v_{lk} is a correcion to height difference, hence (2.1):

(2.1)
$$v_{lk} = H_k - H_l - h_{lk}^{obs}$$

The heights of individual points obtained in the measuring epochs should be transformed into an input system. Z. Wiśniewski adopted the transformation model in the form (2.2):

(2.2)
$$\begin{bmatrix} X_i^0 \\ Y_i^0 \\ H_i^0 \end{bmatrix} = \begin{bmatrix} R_X^j \\ R_Y^j \\ R_H^j \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & \varepsilon_H^j & -\varepsilon_Y^j \\ -\varepsilon_H^j & 1 & \varepsilon_X^j \\ \varepsilon_Y^j & -\varepsilon_X^j & 1 \end{bmatrix}}_{\mathbf{T}^j} \begin{bmatrix} X_i^j \\ Y_i^j \\ H_i^j \end{bmatrix}$$

where:

$\mathbf{T}^j$ - a matrix including the rotation angles at the moment t^j

$\mathbf{R}^j = [R_X^j \quad R_Y^j \quad R_H^j]^T$ – translation vector at the moment t^j.

Regarding on the height of points, we obtain (2.3):

(2.3)
$$\begin{aligned} H_k &= R_H^j + X_k^j \varepsilon_Y^j - Y_k^j \varepsilon_X^j + H_k^j \\ H_l &= R_H^j + X_l^j \varepsilon_Y^j - Y_l^j \varepsilon_X^j + H_l^j \end{aligned}$$

hence the equation (2.1) takes the form (2.4):

(2.4) $$v_{lk} = H_k^j - H_l^j - (Y_k^j - Y_l^j)\varepsilon_X^j + (X_k^j - X_l^j)\varepsilon_Y^j - h_{lk}^{obs(j)}$$

The author of the concept assumed that for small values of rotation angles it holds:

$$\begin{array}{l} X_i^j = X_i^{j=0} = X_i \\ Y_i^j = Y_i^{j=0} = Y_i \end{array} \quad \Longrightarrow \quad \begin{array}{l} (X_k^j - X_l^j) \cong (X_k - X_l) \\ (Y_k^j - Y_l^j) \cong (Y_k - Y_l) \end{array}$$

so the heights of controlled points H_l^j and H_k^j can be replaced by spaces r_l^j and r_k^j (2.5):

(2.5) $$v_{lk} = r_k^j - r_l^j - (Y_k^j - Y_l^j)\varepsilon_X^j + (X_k^j - X_l^j)\varepsilon_Y^j - h_{lk}^{obs(j)}$$

in matrix notation (2.6):

(2.6) $$\mathbf{V} = \mathbf{A}\mathbf{r}^j + \mathbf{B}\mathbf{Y}^j - \mathbf{h}$$

where: $\mathbf{A}, \mathbf{B}$ – matrices of coefficients and $\mathbf{h}$ – vector of measurement results.

Due to the fact that the matrix $\mathbf{A}$ is a matrix of an incomplete row (due to the lack of reference points) Z. Wiśniewski conducted a detailed analysis employing a free adjustment theory.

The ZW method corresponds to the following adjustment task (2.7):

(2.7) $$\left.\begin{array}{l} \mathbf{V} = \mathbf{A}\mathbf{r}^j + \mathbf{B}\mathbf{Y}^j - \mathbf{h} \\ \Phi(\mathbf{r}^j, \mathbf{Y}^j) = \mathbf{V}^T\mathbf{P}\mathbf{V} = \min \\ \Psi(\mathbf{r}^j, \mathbf{Y}^j) = (\mathbf{r}^j)^T \mathbf{P}_\mathbf{r}\mathbf{r}^j = \min \end{array}\right\}$$

where:

$\mathbf{P}$ – matrix of weights ($\mathbf{P} = \mathbf{Q}_\mathbf{h}^{-1}$, $\mathbf{Q}_\mathbf{h}$ - matrix of cofactors of measurement results), $\mathbf{P}_\mathbf{r}$ – matrix of spaces $\mathbf{r}^j$ weights. Due to the fact that levelling network defect equal to 1 matrix $\mathbf{A} = [\mathbf{A}_1 \quad \mathbf{A}_0]$ and $\mathbf{r}^j = \left[\mathbf{r}_1^j \quad \mathbf{r}_0^j\right]^T$.

Assuming: $\mathbf{N}_1 = \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1, \mathbf{N}_0 = \mathbf{A}_1^T\mathbf{P}\mathbf{A}_0, \mathbf{N}_\mathrm{B} = \mathbf{A}_1^T\mathbf{P}\mathbf{B}, \mathbf{L}_h = \mathbf{A}_1^T\mathbf{P}\mathbf{h}$ and $\mathbf{N} = \left[\mathbf{N}_1 \quad \mathrm{M}\ \mathbf{N}_0\right]$ in the solution the normal equation is applied (2.8):

(2.8) $$\mathbf{N}\mathbf{r}^j + \mathbf{N}_\mathrm{B}\mathbf{Y}^j - \mathbf{L}_h = \mathbf{0}$$

Solution of the adjustment task, leads to (2.9), (2.10) and (2.11):

- estimator of the vector of rotation angles

(2.9) $$\hat{\mathbf{Y}}^j = \{\mathbf{N}_\mathrm{B}^\mathrm{T}(\mathbf{N}\mathbf{P}_\mathbf{r}^{-1}\mathbf{N}^T)^{-1}\mathbf{N}_\mathrm{B}\}^{-1}\mathbf{N}_\mathrm{B}^\mathrm{T}(\mathbf{N}\mathbf{P}_\mathbf{r}^{-1}\mathbf{N}^T)^{-1}\mathbf{L}_h$$

- estimator of the vector of distances

(2.10) $$\hat{\mathbf{r}}^j = \mathbf{P}_\mathbf{r}^{-1}\mathbf{N}^T\hat{\boldsymbol{\kappa}}^j$$

where $\hat{\boldsymbol{\kappa}}^j = -(\mathbf{N}\mathbf{P}_\mathbf{r}^{-1}\mathbf{N}^T)^{-1}(\mathbf{N}_\mathrm{B}\hat{\mathbf{Y}}^j - \mathbf{L}_\mathbf{h})$

- estimator of the vector of corrections

(2.11) $$\hat{\mathbf{V}} = \mathbf{A}\hat{\mathbf{r}}^{j} + \mathbf{B}\hat{\mathbf{Y}}^{j} - \mathbf{h}$$

The object safety cannot be considered on the basis of the abovementioned parameters only. Differences between these parameters computed at two measurement epochs play an important role in safety assessment. It is assumed that the sole rotation angle changes can be applied to consider displacements of the tested object.

<u>In the third stage</u>, the differences of the object rotation angles are determined, relevant statistical significance analysis is performed. The differences of angles of rotation at two measuring epochs (notation P means the initial measurement, A – actual measurement) are determined (2.12):

(2.12) $$\begin{aligned} \Delta\hat{\varepsilon}_X &= \hat{\varepsilon}_X^{\mathrm{A}} - \hat{\varepsilon}_X^{\mathrm{P}} \\ \Delta\hat{\varepsilon}_Y &= \hat{\varepsilon}_Y^{\mathrm{A}} - \hat{\varepsilon}_Y^{\mathrm{P}} \end{aligned}$$

They serve as a basis for preliminary assessment of the potential vertical displacement of the object. In order to assess the significance of the obtained differences in rotation angles, the mean errors $m_{\Delta\hat{\varepsilon}_X}$ and $m_{\Delta\hat{\varepsilon}_Y}$ are determined. In order to assess the mean errors $m_{\Delta\hat{\varepsilon}_X}$ and $m_{\Delta\hat{\varepsilon}_Y}$, the covariance matrix for the differences of rotation angles $\mathbf{C}_{\Delta\hat{\mathbf{Y}}}$ is taken here $\mathbf{C}_{\Delta\hat{\mathbf{Y}}} = \mathbf{C}_{\hat{\mathbf{Y}}^{\mathrm{P}}} + \mathbf{C}_{\hat{\mathbf{Y}}^{\mathrm{A}}}$ and $\mathbf{C}_{\hat{\mathbf{Y}}^{\mathrm{P}}}$, $\mathbf{C}_{\hat{\mathbf{Y}}^{\mathrm{A}}}$ - the covariance matrix of adjusted angles of rotation in the initial measurement epoch ($\mathbf{C}_{\hat{\mathbf{Y}}^{\mathrm{P}}}$) and in the actual measurement epoch ($\mathbf{C}_{\hat{\mathbf{Y}}^{\mathrm{A}}}$) respectively, in the form (2.13):

(2.13) $$\mathbf{C}_{\hat{\mathbf{Y}}} = m_0^2\{\mathbf{N}_{\mathrm{B}}^{\mathrm{T}}(\mathbf{N}\mathbf{P}_{\mathbf{r}}^{-1}\mathbf{N}^{T})^{-1}\mathbf{N}_{\mathrm{B}}\}^{-1}\mathbf{N}_{\mathrm{B}}^{\mathrm{T}}(\mathbf{N}\mathbf{P}_{\mathbf{r}}^{-1}\mathbf{N}^{T})^{-1}\mathbf{A}_1^{T}\mathbf{P}^{T}\mathbf{A}_1(\mathbf{N}\mathbf{P}_{\mathbf{r}}^{-1}\mathbf{N}^{T})^{-1}\mathbf{N}_{\mathrm{B}}\{\mathbf{N}_{\mathrm{B}}^{\mathrm{T}}(\mathbf{N}\mathbf{P}_{\mathbf{r}}^{-1}\mathbf{N}^{T})^{-1}\mathbf{N}_{\mathrm{B}}\}^{-1}$$

where: $m_0^2 = \dfrac{\hat{\mathbf{V}}^{T}\mathbf{P}\hat{\mathbf{V}}}{\mathrm{n-m+d}}$ and n – number of observations, m – number of parameters, d – network defect.

<u>In the fourth stage</u>, the height of controlled points in the local coordinate system should be determined. It should be checked if the space difference $\left|\hat{r}_{i=OCS}^{\mathrm{A}} - \hat{r}_{i=OCS}^{\mathrm{P}}\right|$ computed at the point taken as the origin of the coordinate system did not take its highest value in the set of differences $\left|\hat{r}_i^{\mathrm{A}} - \hat{r}_i^{\mathrm{P}}\right|$ computed for all network controlled points. If this condition holds, the value representing a shift $\hat{s}_i$ of each controlled point due to each measuring epoch should take the form (2.14):

(2.14) $$\hat{s}_i = X_i^{j}\hat{\varepsilon}_Y^{j} - Y_i^{j}\hat{\varepsilon}_X^{j}$$

Regarding computed values $\hat{s}_i$ and the previously determined spaces $\hat{r}_i$, the heights of controlled points in the local coordinate system should be determined by the following relations (2.15):

(2.15)
$$\hat{H}_i^{\mathrm{P}} = \hat{s}_i^{\mathrm{P}} + \hat{r}_i^{\mathrm{P}}$$
$$\hat{H}_i^{\mathrm{A}} = \hat{s}_i^{\mathrm{A}} + \hat{r}_i^{\mathrm{A}}$$

In the fifth step, based on the determined height of the controlled points, vertical displacements of these points should be computed from the relation (2.16):

(2.16)
$$\hat{d}_i = (\hat{H}_i^{\mathrm{A}} - \hat{H}_{i=OCS}^{\mathrm{A}}) - (\hat{H}_i^{\mathrm{P}} - \hat{H}_{i=OCS}^{\mathrm{P}})$$

Two differences here: $\hat{H}_i^{\mathrm{P}} - \hat{H}_{i=OCS}^{\mathrm{P}}$ and $\hat{H}_i^{\mathrm{A}} - \hat{H}_{i=OCS}^{\mathrm{A}}$, where $\hat{H}_{i=OCS}^{\mathrm{P}}, \hat{H}_{i=OCS}^{\mathrm{A}}$ are the heights determined due to the point representing the origin of the coordinate system – OCS). They allow to determine the height of individual controlled points with reference to the origin of the coordinate system in the initial and actual epochs. Due to the above relations, the OCS displacement cannot be neglected in the analysis, the vertical displacement of this point is determined: $\Delta\hat{H}_{i=OCS} = \hat{H}_{i=OCS}^{\mathrm{A}} - \hat{H}_{i=OCS}^{\mathrm{P}}$.

In the sixth step, significance of the deteremined displacements should be assessed. Here the F-test is used. The global F-test allows to assess stability of all tested points (2.17):

(2.17)
$$T = \frac{\hat{\mathbf{d}}^T \mathbf{Q}_{\hat{\mathbf{d}}}^{-1} \hat{\mathbf{d}}}{u\hat{\sigma}_0^2} \le F_\alpha(u, f)$$

$\hat{\mathbf{d}}$ – estimator of vector of points displacements, $\mathbf{Q}_{\hat{\mathbf{d}}}$ – matrix of cofactors of estimator vector of displacement, $\hat{\sigma}_0^2$ – global estimator of the coefficient of variance due to two measuring epochs $\hat{\sigma}_0^2 = \dfrac{f_{\mathrm{P}}(\hat{\sigma}_{0\mathrm{P}}^2) + f_{\mathrm{A}}(\hat{\sigma}_{0\mathrm{A}}^2)}{f_{\mathrm{P}} + f_{\mathrm{A}}}$, u – row of matrices $\mathbf{Q}_{\hat{\mathbf{d}}_i}$, f – the number of degrees of freedom, α – assumed significance level, F_α – the critical value of the F - Snedecor distribution according to the assumed significance level α.

Due to each of (i-th) controlled point local tests are performed (2.18):

(2.18)
$$T_i = \frac{\hat{\mathbf{d}}_i^T \mathbf{Q}_{\hat{\mathbf{d}}_i}^{-1} \hat{\mathbf{d}}_i}{u_i\hat{\sigma}_0^2} \le F_{\alpha_i}(u_i, f)$$

These tests allow to examine displacement significance of each controlled point.

3. PRACTICAL CONSIDERATIONS

The concept of free adjustment is applied in the proposed method, thus practical tests were carried out in two variants:

- Variant I: application of free adjustment in the study of vertical displacements.
- Variant II: application of the proposed method in the study of vertical displacements.

The analysis was carried out using the simulated levelling network presented in Fig. 2. This network consists of 9 controlled points and 20 simulated height differences. The coordinates of the controlled points are shown in Table 1. The simulated values of displacements are presented in table 2. Gaussian distribution of measurement errors was assumed and generated.

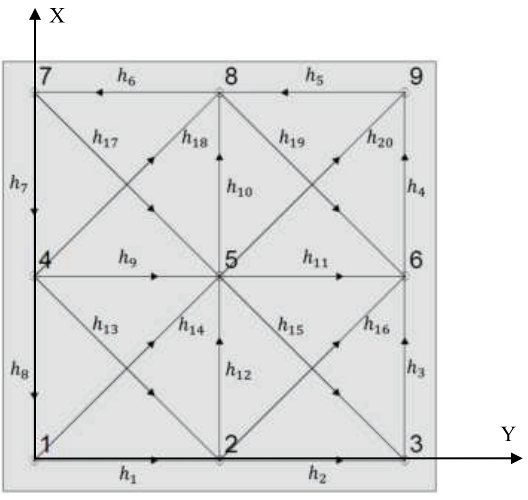

Fig. 2. Simulated levelling network

Table 1. Coordinates of controlled points

Point number	X [m]	Y [m]
1	0	0
2	0	20
3	0	40
4	20	0
5	20	20
6	20	40
7	40	0
8	40	20
9	40	40

The analysis was carried out in three options:

- option 1: subsidence of point no. 3 by 5mm

- option 2: subsidence of points no. 3, 6 and 9 by 5mm

- option 3: subsidence of points no. 3, 5, 6, 7, 8, 9. by 5mm

In order to verify a correctness of the method, displacement of one and three points was assumed, what constituted 11% and 33%, respectively, of all controlled points. A case when more than 50% of the network points were displaced (6 points- 67% of points controlled) was also considered. The vertical displacement value of 5mm adopted in the analysis enables the numerical computations presented in [13] to be continued.

Table 2. Simulated measurement results

The number of height difference	Simulated values of height differences [m]			
	initial epoch „P”	actual epoch „A”		
		Option 1	Option 2	Option 3
h_1	0,0014	0,0014	0,0014	0,0014
h_2	0,0009	-0,0041	-0,0041	-0,0041
h_3	0,0012	0,0062	0,0012	0,0012
h_4	0,0011	0,0011	0,0011	0,0011
h_5	0,0008	0,0008	0,0058	0,0008
h_6	0,0007	0,0007	0,0007	0,0007
h_7	-0,0007	-0,0007	-0,0007	0,0043
h_8	-0,0008	-0,0008	-0,0008	-0,0008
h_9	-0,0011	-0,0011	-0,0011	-0,0061
h_{10}	-0,0012	-0,0012	-0,0012	-0,0012
h_{11}	-0,0009	-0,0009	-0,0059	-0,0009
h_{12}	-0,0014	-0,0014	-0,0014	-0,0064
h_{13}	-0,0012	-0,0012	-0,0012	-0,0012
h_{14}	0,0012	0,0012	0,0012	-0,0038
h_{15}	0,0006	-0,0044	-0,0044	0,0006
h_{16}	-0,0006	-0,0006	-0,0056	-0,0056
h_{17}	0,0013	0,0013	0,0013	0,0013
h_{18}	-0,0013	-0,0013	-0,0013	-0,0063
h_{19}	0,0010	0,0010	-0,0040	0,0010
h_{20}	-0,0010	-0,0010	-0,0060	-0,0010

<u>Variant I</u>

The estimated vertical displacements in the course of conducted computations are presented in Table 3.

The idea of free adjustment is often employed to determine displacements, its elements are also used in the proposed concept. Decision was made to compare its results with the outcomes of the proposed method. The free adjustment methodology assumes no stable points. A functional model takes the form $\mathbf{V} = \mathbf{A}d\mathbf{X} + \mathbf{L}$, where $\mathbf{A}$ – matrix of known coefficients, $d\mathbf{X}$ – vector of increments to parameters, $\mathbf{X}$ – vector of adjusted heights ($\mathbf{X} = \mathbf{X}^0 + d\mathbf{X}$) and $\mathbf{X}^0$ - vector of approximate heights of points, while $\mathbf{L} = \mathbf{L}^0 - \mathbf{L}^{ob}$, where: $\mathbf{L}^{ob}$ – vector of observations (height differences), $\mathbf{L}^0$ – vector of approximate values of observations, $\mathbf{V}$ - vector of corrections to observations. The solution is investigated fulfilling the condition $\mathbf{V}^T\mathbf{P}\,\mathbf{V} = \min$ (where $\mathbf{P}$ is a weight matrix) and at the same time minimizing the square form $d\mathbf{X}^T\mathbf{P}_{\mathbf{X}}d\mathbf{X}$ ($\mathbf{P}_{\mathbf{X}}$ – matrix of weights of points coordinates). Applying the generalized reverse of Moore Penrose $\mathbf{A}^+$ the solution takes the form (3.1):

$$d\hat{\mathbf{X}} = -\mathbf{A}^+\mathbf{L} \tag{3.1}$$

where: $\mathbf{A}^+ = \mathbf{P}_{\mathbf{X}}^{-1}\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{P}_{\mathbf{X}}^{-1}\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{A}_1^T\mathbf{P}$, $\mathbf{B} = [\mathbf{A}_1^T\mathbf{P}\,\mathbf{A}_1 \;\; \mathrm{M} \;\; \mathbf{A}_1^T\mathbf{P}\,\mathbf{A}_2]$, $\mathbf{A} = [\mathbf{A}_1 \;\; \mathrm{M} \;\; \mathbf{A}_2]$ and the number of columns in the submatrix $\mathbf{A}_2$ is equal to the determined defect of the geodetic network.

The accuracy analysis of the estimation results based on the F-tests previously addressed in the paper. The covariance matrix of the estimation results takes the form (3.2):

$$\mathbf{C}_{d\mathbf{X}} = \mathbf{C}_{\mathbf{X}} = \hat{\sigma}_0^2\mathbf{A}^+\mathbf{P}^{-1}(\mathbf{A}^+)^T \tag{3.2}$$

where: $\hat{\sigma}_0^2$ - estimator of the coefficient of variance.

In order to determine vertical displacements of the points, computations should be carried out on the basis of the relations stated above due to each measurement epoch. It leads to the displacement vector $\hat{\mathbf{d}} = \hat{\mathbf{X}}^{\mathrm{A}} - \hat{\mathbf{X}}^{\mathrm{P}}$ and the matrix $\mathbf{C}_{\hat{\mathbf{d}}} = \mathbf{C}_{\hat{\mathbf{X}}^{\mathrm{A}}} + \mathbf{C}_{\hat{\mathbf{X}}^{\mathrm{P}}}$ necessary to assess the accuracy.

The computational results of the free adjustment approach are presented below. Table 3 and figure 3 present obtained vertical displacements of individual controlled points due to all computational options.

Table 3. Estimated displacements of controlled points with the use of free adjustment

Point number	Vertical displacements $\hat{d}_i$ of controlled points [mm]		
	Option 1	Option 2	Option 3
1	0,6	1,7	3,3
2	0,6	1,7	3,3
3	-4,4	-3,3	-1,7
4	0,6	1,7	3,3
5	0,6	1,7	-1,7
6	0,6	-3,3	-1,7
7	0,6	1,7	-1,7
8	0,6	1,7	-1,7
9	0,6	-3,3	-1,7

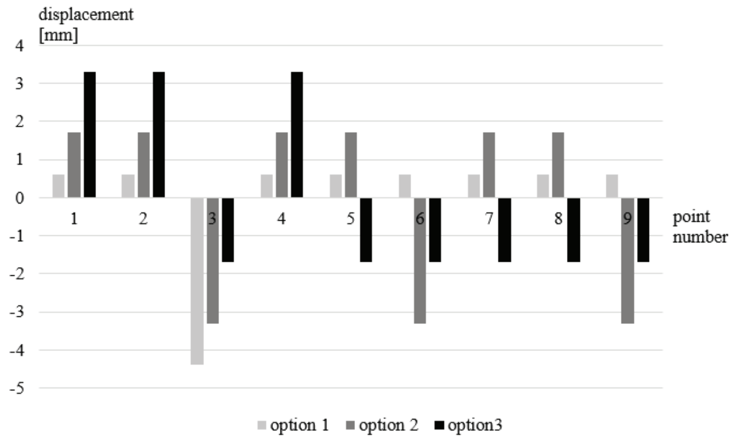

Fig 3. Estimated values of vertical displacements - variant I

Next significance was assessed of the obtained vertical displacements of controlled points. The results are presented in Table 4 (the assumed significance level $\alpha = 0.05$).

Table 4. Significance assessment of vertical displacements

Point number	Significance assessment of the displacement		
	Option 1	Option 2	Option 3
1	NO	NO	**YES**
2	NO	NO	**YES**
3	**YES**	**YES**	NO
4	NO	NO	**YES**
5	NO	NO	NO
6	NO	**YES**	NO
7	NO	NO	NO
8	NO	NO	NO
9	NO	**YES**	NO

Regarding the computational results presented in table 3 and in table 4, we note that correct estimation results were obtained due to the first two computational options. The results of the third option (here subsidence of more than 50% of controlled points was assumed) prove the obtained results contrary to those assumed previously. The displaced points considered in analysis are actually undisplaced. Thus the analysis confirms the prior theoretical considerations. The presented analysis shows that the free adjustment should be applied to determine displacements with a relevant care, there are cases of careless use, leading to incorrect conclusions.

Variant II

The computational results following the concept to determine vertical displacements, are presented in the paper. The computations were carried out based on the proposed algorithm. Tables 5 - 9 show the results of the next algorithm steps. In the following considerations the controlled point no. 1 was stated the origin of the local coordinate system. Based on the algorithm, the significance of the object displacement is investigated due to individual computational options.

Table 5. Assessment of possibility of displacement of the object

Computation options	Differences of rotation angles $[rad]$				Suspicion about the existence of object displacement YES/NO
	$\Delta\hat{\varepsilon}_X$	$m_{\Delta\hat{\varepsilon}_X}$	$\Delta\hat{\varepsilon}_Y$	$m_{\Delta\hat{\varepsilon}_Y}$	
Option 1	0,00004167	0,00000002	0,00004167	0,00000002	YES
Option 2	0,00012500	0,00000002	0,00000000	0,00000002	YES
Option 3	0,00008333	0,00000002	-0,00008333	0,00000002	YES

All the worked examples proved right the previous predictions on the object displacements. In the next step, the estimators $\hat{r}_i^{\mathrm{P}}$ and $\hat{r}_i^{\mathrm{A}}$ necessary for further analysis, presented in table 6, are determined.

Table 6. Values of $\hat{r}_i^{\mathrm{P}}$ and $\hat{r}_i^{\mathrm{A}}$ obtained for individual controlled points

Point number	$\hat{r}_i^{\mathrm{P}}$ and $\hat{r}_i^{A}$ for controlled points [mm]			
	Epoch P $\hat{r}_i^{\mathrm{P}}$	Epoch A Option 1	Epoch A Option 2	Epoch A Option 3
1	-0,86	-0,31	-1,70	-0,86
2	0,23	1,62	1,90	1,90
3	0,21	-2,57	-0,62	-1,46
4	0,60	0,32	-0,23	2,27
5	0,28	0,83	1,94	-1,39
6	-0,04	1,35	-0,87	-0,04
7	0,23	-0,89	-0,61	-1,44
8	-0,44	-0,72	1,22	-0,44
9	-0,20	0,35	-1,04	1,46

According to the next algorithm step, the heights of controlled points in the initial and actual epochs are determined in the local coordinate system (table 7).

Table 7. Values $\hat{H}_i^{\mathrm{P}}$ and $\hat{H}_i^{\mathrm{A}}$ due to individual controlled points

Point number	Epoch P $\hat{H}_i^{\mathrm{P}}$ [mm]	$\hat{H}_i^{\mathrm{A}}$ determined at individual controlled points [mm]		
		Epoch A Option 1	Epoch A Option 2	Epoch A Option 3
1	-0,86	-0,31	-1,70	-0,86
2	0,19	0,75	-0,64	0,19
3	0,13	-4,31	-5,70	-4,87
4	0,49	1,05	-0,34	0,49
5	0,13	0,68	-0,70	-4,87
6	-0,23	0,33	-6,06	-5,23
7	0,01	0,56	-0,83	-4,99
8	-0,70	-0,14	-1,53	-5,70
9	-0,50	0,06	-6,33	-5,50

Regarding the presented relation (16), displacements of individual controlled points were determined due to all tested computational options. The results are shown in table 8 and figure 4.

Table 8. Values of displacements of individual controlled points

Point number	displacements $\hat{d}_i$ of individual controlled points [mm]		
	Epoch A Option 1	Epoch A Option 2	Epoch A Option 3
1	0,56	-0,83	0,00
2	0,00	0,00	0,00
3	-5,00	-5,00	-5,00
4	0,00	0,00	0,00
5	0,00	0,00	-5,00
6	0,00	-5,00	-5,00
7	0,00	0,00	-5,00
8	0,00	0,00	-5,00
9	0,00	-5,00	-5,00

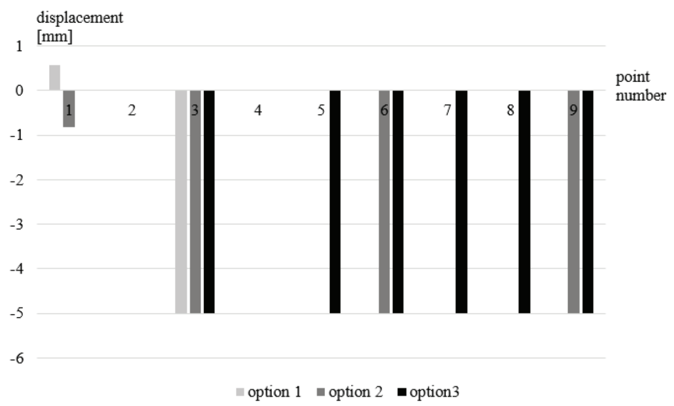

Fig. 4. Estimated values of vertical displacements - variant II

Significance of the obtained displacements was assessed with a significance level $\alpha = 0.05$ (Table 9).

Table 9. Significance assessment of displacements

Point number	Significance of displacements (YES – significant, NO – insignificant)		
	Epoch A Option 1	Epoch A Option 2	Epoch A Option 3
1	NO	NO	NO
2	NO	NO	NO
3	**YES**	**YES**	**YES**
4	NO	NO	NO
5	NO	NO	**YES**
6	NO	**YES**	**YES**
7	NO	NO	**YES**
8	NO	NO	**YES**
9	NO	**YES**	**YES**

The above results show that the use of the proposed concept allows to correctly recognize the displaced points due to all computation options. The obtained results are consistent with earlier assumptions. The proposed method allowed to correctly determine the displaced points in computation options adopted for numerical analysis. Particularly important are the results obtained for option 3 - in a situation where more than 50% of points of the network were displaced. The analysis also showed displacement values of the point assumed as the origin of the coordinate system, while the significance analysis showed that these displacements are insignificant.

4. Conclusions

Due to the importance of the problem of displacement determination, pointing out the aspect of security, this issue is a widespread major research topic. Most of the papers addresses the case of displacements determined based on a stable reference points. This paper considers the case these points do not exist. Literature shows various methods to determine vertical displacements of objects (or controlled points of the object), while in the absence of stable reference points the methods may act improperly, especially in the cases of more than 50% of the points displaced. Therefore, it was decided to propose a new method for determining vertical displacements of controlled points, in order to consider this restriction. Practice shows that the proposed method works properly even in the case of more than half the amount of the controlled points were displaced. The analysis proves that the method may be useful while relative networks are considered and no stable reference points are identified. The considerations included in the paper also allow to state that the proposed method works properly.

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LIST OF FIGURES AND TABLES:

WYZNACZANIE PRZEMIESZCZEŃ PIONOWYCH WE WZGLĘDNYCH SIECIACH MONITORINGU

Słowa kluczowe: przemieszczenia pionowe, niestabilny układ odniesienia, brak punktów referencyjnych

STRESZCZENIE:

Problematyka wyznaczania przemieszczeń obiektów jest ważnym zagadnieniem, w szczególności w aspekcie dotyczącym bezpieczeństwa użytkowania obiektu. Zmiany poziomu zwierciadła wody, głębokie wykopy, obciążenie gruntu ciężką budowlą, zużycie konstrukcji na skutek jej eksploatacji to tylko kilka czynników mogących powodować ruchy gruntu i posadowionych na nim obiektów. Konsekwencją ruchów gruntu mogą być przemieszczenia poziome i pionowe. Zatem istotne jest określenie wielkości tych przemieszczeń, na co pozwalają okresowe pomiary kontrolne. W analizie przemieszczeń wyróżnić można układy odniesienia stabilne oraz niestabilne. Praca ta skupia się na wyznaczaniu przemieszczeń pionowych w niestabilnych układach odniesienia, zatem porusza problem, który rzadko prezentowany jest w literaturze przedmiotu. Z uwagi na to, że nie zawsze mamy możliwość wykorzystania stabilnych

punktów odniesienia (np. uległy uszkodzeniu) lub ich identyfikacja jest utrudniona, w pracy zaproponowano nową metodę wyznaczania przemieszczeń pionowych w lokalnym układzie współrzędnych, która uwzględnia brak punktów referencyjnych. W literaturze przedmiotu można także spotkać podział sieci geodezyjnych zakładanych do celów wyznaczania przemieszczeń na: względne – złożone z punktów kontrolowanych umieszczonych na badanym obiekcie oraz bezwzględne (absolutne) – złożone z punktów odniesienia ulokowanych poza zasięgiem badanego obiektu oraz wykorzystywanych do wyznaczania bezwzględnych przemieszczeń punktów kontrolowanych obiektu. Przeprowadzone rozważania pozwoliły stwierdzić, że rozpatrując sieci względne należy ostrożnie wykorzystywać większość istniejących metod, gdyż otrzymane rezultaty mogą prowadzić do błędnych wniosków. Zaproponowana w niniejszej pracy koncepcja wyznaczania przemieszczeń pojedynczych punktów kontrolowanych realizowana jest w 6 etapach:

- etap I - Przyjęcie lokalnego układu współrzędnych
- etap II - Wyrównanie obserwacji metodą zaproponowaną w pracy [27]
- etap III - Wyznaczenie różnic kątów obrotu obiektu oraz analiza statystyczna ich istotności
- etap IV - Wyznaczenie wysokości punktów kontrolowanych w lokalnym układzie współrzędnych
- etap V - Wyznaczenie przemieszczeń pionowych punktów kontrolowanych.
- etap VI - Ocena istotności wyznaczonych przemieszczeń

Etapy III-VI stanowią rozwinięcie metody zaproponowanej w pracy [27] przez co możliwe jest wyznaczanie przemieszczeń pionowych poszczególnych punktów kontrolowanych w sytuacji braku stabilnych punktów odniesienia.

Rozważania praktyczne przeprowadzono dla symulowanej sieci niwelacyjnej, która składała się z 9 punktów kontrolowanych oraz symulowanych wartości 20 przewyższeń pomiędzy nimi. Analizy przeprowadzone zostały w dwóch wariantach:

- Wariant I: wykorzystanie wyrównania swobodnego w badaniu przemieszczeń pionowych.
- Wariant II: wykorzystanie proponowanej koncepcji do badania przemieszczeń pionowych.

Co więcej analizy przeprowadzono dla trzech opcji obliczeń: opcja 1 – osiadanie jednego punktu; opcja 2 – osiadanie trzech punktów, opcja 3 – osiadanie 6 punktów.

Jak pokazały rezultaty obliczeń dla wariantu 1, prawidłowe rezultaty estymacji uzyskano dla dwóch pierwszych opcji obliczeń. Dla trzeciej opcji, tj. kiedy założone zostało osiadanie powyżej 50% punktów kontrolowanych nie uzyskano zadawalających rezultatów zgodnych z wcześniejszymi założeniami.

Rezultaty obliczeń dla wariantu 2, gdzie wykorzystano proponowana w pracy koncepcję, pokazały, że metoda prawidłowo rozpoznała punkty przemieszczone dla wszystkich opcji obliczeń. Otrzymane rezultaty były zgodne z wcześniejszymi założeniami.

Zatem przedstawione w pracy analizy pozwalają stwierdzić, że proponowana metoda może być w szczególności przydatna w sytuacji, kiedy rozpatrujemy sieci względne i nie mamy zidentyfikowanych jako stabilne punktów odniesienia. Przedstawione rozważania praktyczne pozwalają także stwierdzić, że zaproponowana metoda działa w sposób prawidłowy.

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