Effect of the working liquid compressibility on the picture of volumetric and mechanical losses in a high pressure displacement pump used in a hydrostatic drive

Part I

Energy losses in a drive system, volumetric losses in a pump

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ABSTRACT

Working liquid compressibility may considerably change the values and proportions of coefficients of volumetric and mechanical energy losses in the displacement pump used in a hydrostatic drive system. This effect can be particularly seen in the operation under high pressure and also in the system, where aeration of the working liquid can occur. In the Part I a diagram is presented, proposed by the author, of power increase in a hydrostatic drive system (hydraulic motor, pump) opposite to the direction of power flow, replacing the Sankey diagram of power decrease in the direction of power flow. Mathematical model is presented of volumetric losses in the pump and its laboratory verification.

Keywords: hydrostatic drive system; pressure displacement pump; energy losses; energy efficiency; energy saving; liquid compressibility; Sankey diagram;

1. INTRODUCTION

The evaluation of energy efficiency of hydrostatic drive system elements (pump, hydraulic motor, throttling control assembly, conduits) is only justified when performed during their operation in the system. The hydraulic motor speed control structure determines in an essential way the level of power of energy losses in the system elements, losses related to the instantaneous values of the hydraulic motor speed $\overline{\omega}_M$ and load $\overline{M}_M$ coefficients as well as to the working liquid viscosity ratio ν/ν_n [1÷19].

Evaluation of the energy efficiency of a displacement machine (pump, hydraulic motor), performed independently of the drive system cannot be used for evaluation of energy efficiency of the system where the machine is operating.

It is necessary to evaluate the energy (mechanical, volumetric and pressure) losses in the pump and hydraulic motor as a function of the machine parameters which directly effect the losses. It is also necessary to determine the k_i coefficients of those losses. The knowledge of k_i coefficients allows to develop the mathematical models of losses taking into account also the system structure. The mathematical models should also allow to evaluate losses and energy efficiency of the elements and the system as a whole as a function of the hydraulic motor speed $\overline{\omega}_M$ and load $\overline{M}_M$ coefficients as well as a function of the working liquid viscosity ratio ν/ν_n.

The knowledge of k_i coefficients of losses in the elements allows also to determine the necessary mathematical models describing the maximum values $\overline{\omega}_{Mmax} = f(\overline{M}_M)$ and $\overline{M}_{Mmax} = f(\overline{\omega}_M)$ of the hydraulic motor operating field $(0 \le \overline{\omega}_M \langle \overline{\omega}_{Mmax}, 0 \le \overline{M}_M \langle \overline{M}_{Mmax})$ in a system with a given structure, the field that changes also with the change of the working liquid viscosity ratio ν/ν_n.

Compressibility of the working liquid (hydraulic oil) in the working chambers of the investigated pump decreases the volumetric intensity Q_P of flow in the discharge conduit compared with the intensity of low pressure flow in the pump inlet conduit. Therefore it reduces the theoretical volume q_{Pt} (V_P) and the geometrical volume q_{Pgv} (V_{Pgv}) of the pump operating under high pressure. The liquid compressibility increases at the same time the value of coefficient k1 of the volumetric losses in the pump.

Simultaneously, the liquid compressibility, reducing the theoretical volume q_{Pt} (V_P) and the geometrical volume q_{Pgv} (V_{Pgv}) of the pump working chambers, decreases the value, determined in the investigation, of the coefficient $k_{4.2}$ of the increase $\Delta M_{Pm|\Delta pPi}$, b_P, ν_n of torque of mechanical losses in the „working chambers - shaft" assembly. The increase of torque is a function of the torque MPi indicated in the working chambers.

The coefficient $k_{lc|p_n}$ of working liquid compressibility has been introduced in the analysis.

The $k_{lc|p_n}$ coefficient is defined at the increase Δp_{Pi} of pressure in the pump working chambers equal to the system nominal pressure $p_n - \Delta p_{Pi} = p_n$.

The working liquid compressibility coefficient $k_{lc|p_n}$ determines the degree of decrease, as an effect of compressibility (without taking leakage into account), of the active volume of the working liquid displaced by the pump in one shaft revolution, compared with the active volume equal to the theoretical capacity $q_{Pt}(V_{Pt})$ or geometrical capacity $q_{Pgv}(V_{Pgv})$ per one shaft revolution determined at the increase Δp_{Pi} of pressure in the working chambers equal to zero $-\Delta p_{Pi} = 0$:

$$k_{lc|p_n} = \frac{q_{Pt} - q_{Pt}|_{\Delta p_{Pi} = p_n}}{q_{Pt}}$$

$$k_{lc|p_n} = \frac{q_{Pgv} - q_{Pgv}|_{\Delta p_{Pi} = p_n}}{q_{Pgv}} = \frac{b_P q_{Pt} - b_P q_{Pt}|_{\Delta p_{Pi} = p_n}}{b_P q_{Pt}} \quad (1.1)$$

It is possible to evaluate the effect of coefficient $k_{lc|p_n}$ of liquid compressibility on the value of the coefficient $k_{4.2}$ of the increase of torque of mechanical losses in the pump „working chambers - shaft" assembly and then to evaluate the effect of that coefficient on the value of coefficient k_1 of volumetric losses in the pump working chambers. This is possible during investigation of the pump with variable capacity q_{Pgv} ($b_P q_{Pt}$) per one shaft revolution.

The working liquid compressibility has no effect on the evaluation of coefficient k_8 of volumetric losses in the working chambers of the hydraulic rotational motor and also on the evaluation of coefficient $k_{7.2}$ of the increase of torque of mechanical losses in the motor „shaft - working chambers" assembly.

The hydraulic motor working chambers are fed with compressed working liquid (with the pressure resulting, among others, from the torque M_M on the motor shaft). The calculated value of the torque M_{Mi} indicated in the motor working chambers is not influenced by the change of the theoretical volume q_{Mt} (V_M) or geometrical volume q_{Mgv} (V_{Mgv}) of the motor working chambers.

2. DIRECTION OF POWER INCREASE IN THE DRIVE SYSTEM

The Sankey diagram is the fundamental cause of incorrect evaluation of losses in a drive system.

The Sankey diagram connected with a drive system suggests an evaluation defining the output power of the system as a difference between the system input power and sum of the powers of losses occurring in the system. Therefore the energy losses in the system are evaluated as a function of parameters describing the input power.

In accordance with the proposed by the author (Fig. 2.1) diagram of power increase in a power transmission system opposite to the direction of power flow, **the system input power is a sum of the system output power and powers of losses occurring in the system. The energy losses in the system are evaluated as a function of parameters describing the system output power**.

Full picture of the energy losses in a hydrostatic drive system is a picture of power of energy losses in the system elements. Shaft power of the system feeding pump is equal to the sum of hydraulic motor shaft (or piston rod) power and powers of individual losses in the power flowing from the pump shaft to the hydraulic motor shaft (or piston rod).

<u>**The quantity of power increases, in order to overcome the power of energy losses, in the opposite direction to the direction of power flow**</u>. Therefore, the image of power of energy losses in the system should be constructed in the direction

from the hydraulic rotational motor shaft or from linear motor piston rod towards the system feeding pump shaft.

Power of the hydraulic motor, pump and other system element losses (mechanical, volumetric, pressure losses) should be determined as functions of the parameters independent of those losses and influencing directly those losses.

Power of energy losses in the system elements and also power of those elements must be precisely defined.

Fig. 2.1 presents (as an example of the system with the hydraulic motor speed series throttling control assembly fed by a constant capacity pump cooperating with an overflow valve) the diagram, proposed by the author, of the direction of power increase from the shaft or piston rod of a hydraulic motor to the pump shaft, power increasing as an effect of the imposed powers of energy losses in the hydrostatic drive and control elements.

The useful power $P_{Mu} = M_M \omega_M$ determined on the rotational hydraulic motor shaft or $P_{Mu} = F_M v_M$ determined on the linear hydraulic motor piston rod, changing in the work field ($0 \leq M_M < M_{Mmax}$ and $0 \leq \omega_M < \omega_{Mmax}$) or ($0 \leq F_M < F_{Mmax}$ and $0 \leq v_M < v_{Mmax}$) of the motor driven device, is a result of instantaneous requirements of the device toward motor both as regards load M_M (F_M) and speed ω_M (v_M). Therefore, the instantaneous useful power P_{Mu} is independent of the structure of hydraulic system (used for changing the motor speed ω_M (v_M)) and of the power ΔP of losses in the system elements. However, this does not apply to the upper limits of the device parameter ranges, i.e. maximum load M_{Mmax} (F_{Mmax}) and maximum speed ω_{Mmax} (v_{Mmax}).

The upper limits of work field, determined by the $M_{Mmax} = f(\omega_M)$ and $\omega_{Mmax} = f(M_M)$ or $F_{Mmax} = f(v_M)$ and $v_{Mmax} = f(F_M)$ lines, of the hydraulic motor driving the device, depend on the theoretical capacity Q_{Pt} of the pump in the system, on the nominal pressure p_n level of the system operation (the product $Q_{Pt} p_n$ is the reference power), on the motor speed control structure used, and also on the sum ΔP of powers of losses in the system elements. The upper limits of the motor work field differ in systems with different motor speed control structures.
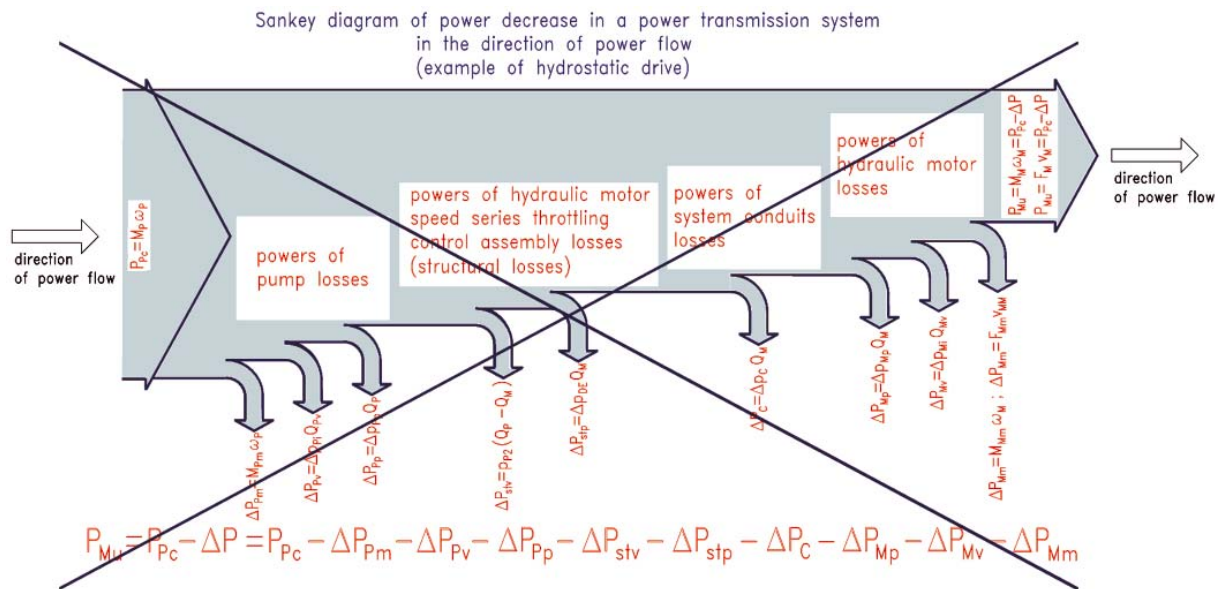
On the other hand, the instantaneous power $P_{Pc} = M_P \omega_P$ required (consumed) by the pump from the drive (electric or internal combustion) motor, determined on the pump shaft, is a sum of the instantaneous useful power P_{Mu} of the hydraulic motor and instantaneous sum ΔP of powers of losses in the hydraulic system elements:

$$P_{Pc} = P_{Mu} + \Delta P \quad (2.1)$$

Power P_{Pc} on the pump shaft is equal to the sum ΔP of power of energy losses in the system elements ($P_{Pc} = \Delta P$) during operation with the unloaded motor (when $M_M = 0$ or $F_M = 0$) or with the stopped motor (when $\omega_M = 0$ or $v_M = 0$). The useful power P_{Mu} of the motor and energy efficiency η of the system are then equal to zero ($P_{Mu} = 0$, $\eta = 0$). The information on the level of power P_{Pc} absorbed then by the pump and entirely lost in the system is particularly important at the system operation with unloaded motor (when $M_M = 0$ or $F_M = 0$). Power P_{Pc} of the pump operating with hydraulic motor stopped (when $\omega_M = 0$ or $v_M = 0$) may be minimized by simultaneous cutting of the pump discharge conduit from the hydraulic motor and connecting this conduit with the tank, i.e. unloading the pump. However, this is only possible in an individual system, when the pump feeds only one hydraulic motor.

3. DIAGRAM OF POWER INCREASE IN A HYDRAULIC MOTOR

Fig. 3.1 illustrates the diagram, proposed by the author, of power increase in a rotational hydraulic motor opposite to the direction of power flow, replacing the Sankey diagram of power decrease in the direction of power flow.

Sankey diagram of power decrease in a power transmission system
in the direction of power flow
(example of hydrostatic drive)

$$P_{Mu} = P_{Pc} - \Delta P = P_{Pc} - \Delta P_{Pm} - \Delta P_{Pv} - \Delta P_{Pp} - \Delta P_{stv} - \Delta P_{stp} - \Delta P_C - \Delta P_{Mp} - \Delta P_{Mv} - \Delta P_{Mm}$$

diagram, according to Z.Paszota, of power increase in a power transmission system
opposite to the direction of power flow
(example of hydrostatic drive)

$$P_{Pc} = P_{Mu} + \Delta P = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp} + \Delta P_C + \Delta P_{stp} + \Delta P_{stv} + \Delta P_{Pp} + \Delta P_{Pv} + \Delta P_{Pm}$$
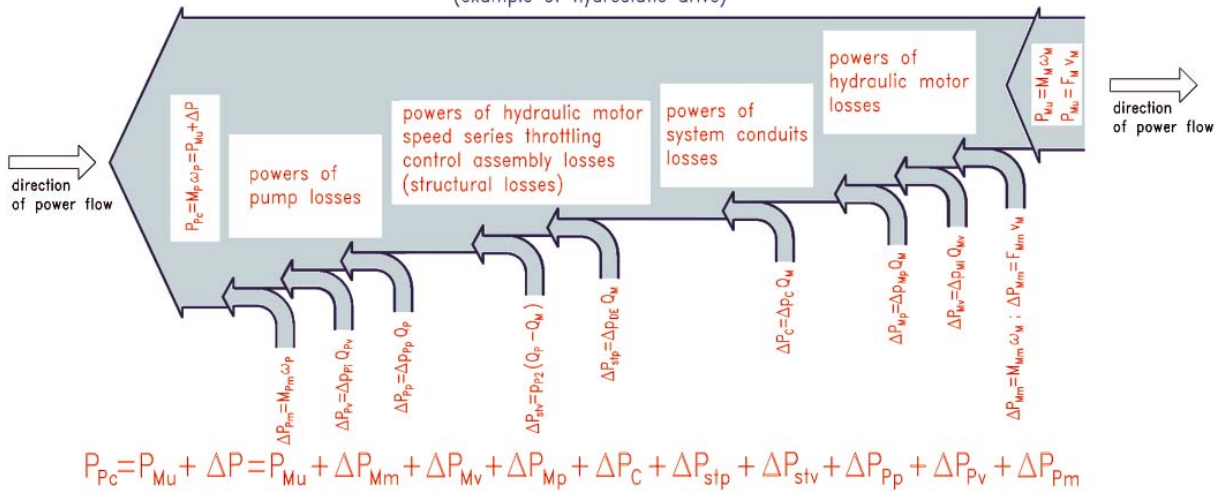
Fig. 2.1. Diagram, proposed by the author, of power increase in a hydrostatic drive system opposite to the direction of power flow, replacing the Sankey diagram of power decrease in the direction of power flow.

Power increases from the motor useful power P_{Mu}, required by the driven machine (device) on the rotational motor shaft or on the linear motor piston rod, to the power P_{Pc} consumed and required by the pump shaft from the motor (electric or internal combustion) driving the pump. The increase of power is a sum ΔP of powers of losses in the system elements (example of a system with the hydraulic motor speed series throttling control fed by a constant capacity pump cooperating with an overflow valve).

Powers of the losses are functions of the output parameters: M_M, ω_M of the rotational hydraulic motor shaft or F_M, v_M of the linear hydraulic motor piston rod and diversified functions of the working fluid viscosity v.

$P_{Mu} = M_M \omega_M$ – motor useful power on the rotational motor shaft required by the driven machine (device); $P_{Mu} = F_M v_M$ – motor useful power on the linear motor piston rod, $\Delta P_{Mm} = M_{Mm} \omega_M$ – power of mechanical losses in the rotational motor, $\Delta P_{Mm} = F_{Mm} v_M$ – power of mechanical losses in the linear motor, $\Delta P_{Mv} = \Delta p_{Mi} Q_{Mv}$ – power of volumetric losses in the motor, $\Delta P_{Mp} = \Delta p_{Mp} Q_M$ – power of pressure losses in the motor, $\Delta P_C = \Delta p_C Q_M$ – power of pressure losses in the system conduits, $\Delta P_{stp} = \Delta p_{DE} Q_M$ – power of structural pressure losses in the throttling control assembly (in servo-valve, directional proportional valve, set throttling valve or set two-way flow regulator), $\Delta P_{stv} = p_{SP} (Q_P - Q_M)$ – power of structural volumetric losses in the throttling control assembly (in the overflow valve), $\Delta P_{Pp} = \Delta p_{Pp} Q_P$ – power of pressure losses in the pump, $\Delta P_{Pv} = \Delta p_{Pi} Q_{Pv}$ – power of volumetric losses in the pump, $\Delta P_{Pm} = M_{Pm} \omega_P$ – power of mechanical losses in the pump, $P_{Pc} = M_P \omega_P$ – power consumed on the pump shaft, required by the pump shaft from the motor driving the pump

The power P_{Mc} consumed by the hydraulic motor is a sum of motor shaft useful power P_{Mu} and powers of three different energy losses in the motor. The losses occur in series (one after another), increasing the power in the opposite direction to the direction of power flow. In effect, the power in the motor increases from the shaft useful power P_{Mu} to the working fluid power P_{Mc} consumed by the motor:

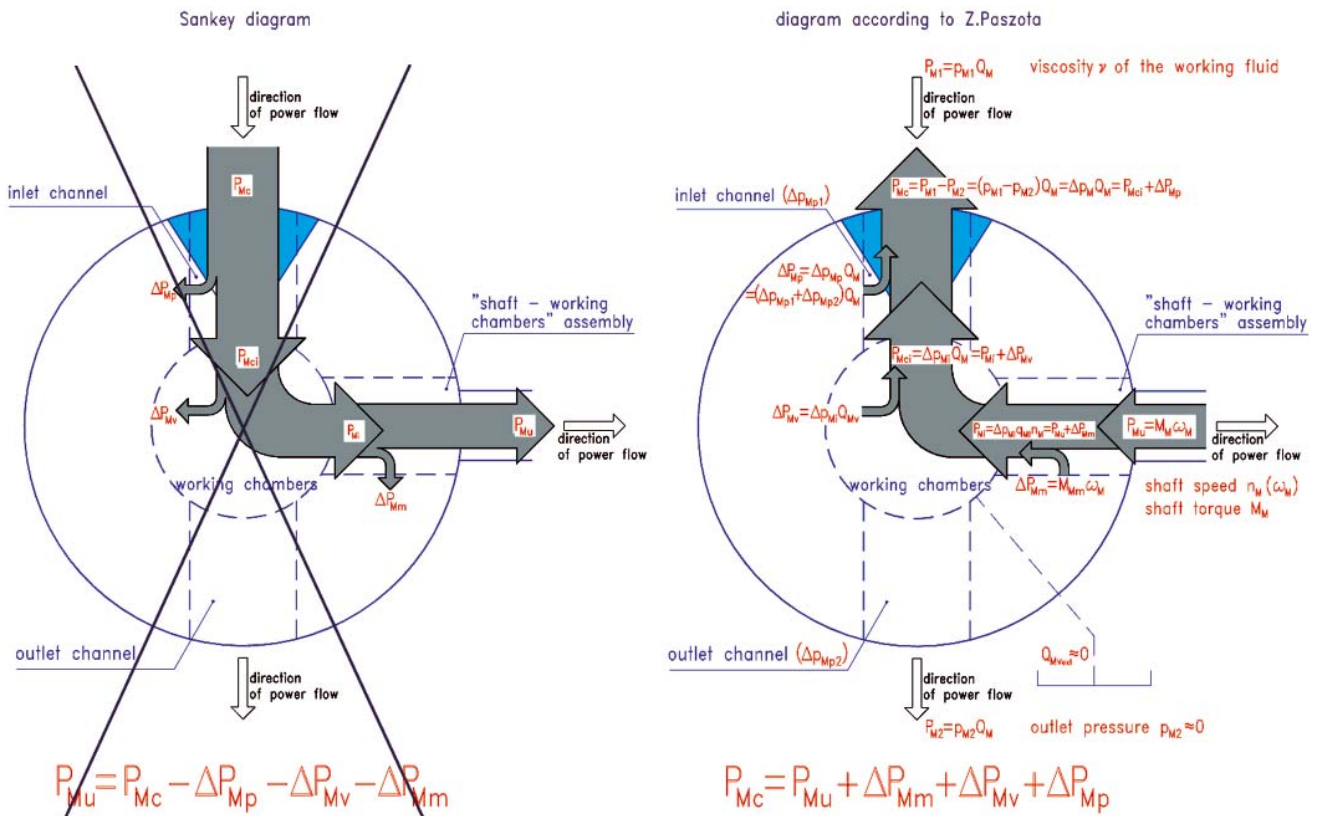$$P_{Mc} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp} \qquad (3.1)$$

Mechanical losses (and power ΔP_{Mm} of mechanical losses) occur in the „shaft – working chambers" assembly.

Volumetric losses (and power ΔP_{Mv} of volumetric losses) occur in the working chambers.

Pressure losses (and power ΔP_{Mp} of pressure losses) occur in the motor channels.

4. DIAGRAM OF POWER INCREASE IN A PUMP

Fig. 4.1 presents the diagram, proposed by the author, of power increase in a pump opposite to the direction of power flow, replacing the Sankey diagram of power decrease in the direction of power flow.

Sankey diagram

diagram according to Z.Paszota

$$P_{Mu} = P_{Mc} - \Delta P_{Mp} - \Delta P_{Mv} - \Delta P_{Mm}$$

$$P_{Mc} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp}$$

Fig. 3.1. *Diagram of power increase in a rotational displacement hydraulic motor opposite to the direction of power flow, replacing the Sankey diagram of power decrease in the direction of power flow Power increases from the motor useful power P_{Mu} required on the motor shaft by the driven machine (device) to power P_{Mc} consumed and required by the motor from the working fluid.*
The increase of power is an effect of the powers of losses in the motor: power ΔP_{Mm} of mechanical losses in the „shaft- working chambers" assembly, power ΔP_{Mv} of volumetric losses in the working chambers and power ΔP_{Mp} of pressure losses in the motor channels.
Powers ΔP_{Mm}, ΔP_{Mv} and ΔP_{Mp} of the losses are functions of the output parameters of the motor assembly where the losses occur and diversified functions of the working fluid viscosity v: power ΔP_{Mm} of mechanical losses is a function of torque M_M and shaft speed n_M (Δ_M) required from the motor by the driven machine (device) and a function of the working fluid viscosity Δ, power ΔP_{Mv} of volumetric losses is a function of the decrease Δp_{Mi} of pressure indicated in working chambers (torque M_{Mi} indicated in the chambers) and of the shaft rotational speed n_M as well as a function of the working fluid viscosity v, power ΔP_{Mp} of pressure losses is a function of motor capacity Q_M and of the working fluid viscosity v.
Power P_{Mi} indicated in the working chambers: $P_{Mi} = P_{Mu} + \Delta P_{Mm}$, power P_{Mci} of the working fluid consumed in the working chambers: $P_{Mci} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv}$, power P_{Mc} of the working fluid consumed by the motor: $P_{Mc} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp}$.
The diagram replaces the Sankey diagram of distribution of power in a hydraulic motor causing incorrect loss evaluation during the hydraulic motor energy investigations

The power P_{Pc} consumed by the pump shaft is a sum of pump useful power P_{Pu} and powers of three different losses in the pump. The losses occur in series (one after another), increasing the power in the opposite direction to the direction of power flow. In effect, the power in the pump increases from the working fluid useful power P_{Pu} to the pump shaft consumed power P_{Pc}:

$$P_{Pc} = P_{Pu} + \Delta P_{Pp} + \Delta P_{Pv} + \Delta P_{Pm} \qquad (4.1)$$

Pressure losses (and power ΔP_{Pp} of pressure losses) occur in the pump channels.

Volumetric losses (and power ΔP_{Pv} of volumetric losses) occur in the working chambers.

Mechanical losses (and power ΔP_{Pm} of mechanical losses) occur in the „working chambers - shaft" assembly.

5. MATHEMATICAL MODELS OF THE DISPLACEMENT MACHINE LOSSES

Assessment of the capability of energy savings in the hydrostatic drive system operation requires the system losses to be defined.

The simulation determination of the system energy efficiency may be used for the purpose in the system design and operation process [24]. The following factors in the simulation model should be taken into account:
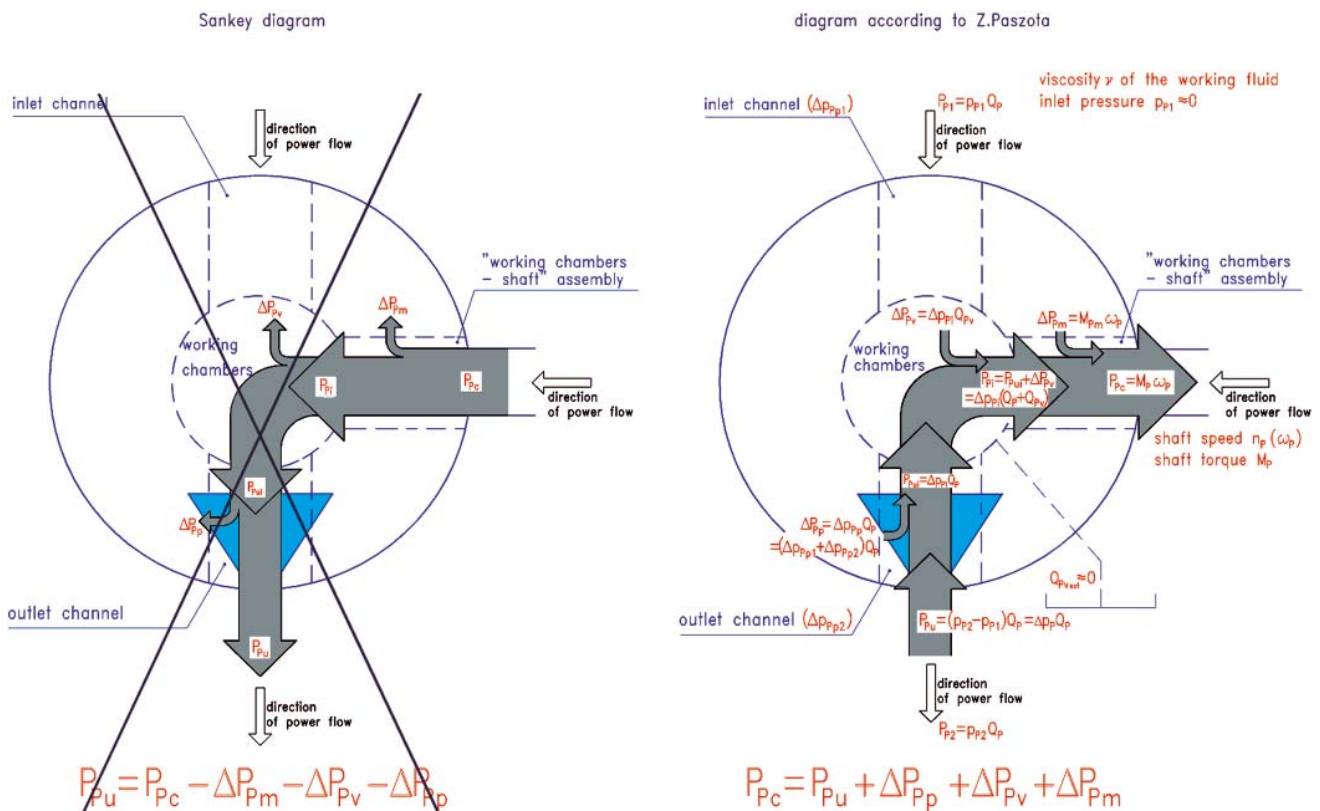
– the hydraulic motor speed control system structure,
– energy losses in the system elements,
– the pump driving motor speed decrease,
– the system control element characteristics,
– load and speed of the controlled hydraulic motor,
– hydraulic oil viscosity, changing in the system operation process due to change of the oil temperature.

In order to make the transmission system efficiency determination method easily applicable, it is necessary to:
1) use the computer programs for the mathematical models, allowing to analyse the hydraulic system efficiency as a function of the decisive parameters (hydraulic motor speed coefficient $\overline{\omega}_M$ and load coefficient $\overline{M}_M$, ratio v/v_n of the hydraulic oil viscosity v to the reference viscosity v_n),
2) determine the values of energy loss coefficients k_i for the pump, rotational hydraulic motor or hydraulic cylinder. Those coefficients should be clearly defined and precisely determined for a given displacement machine.

The mathematical model of the displacement machine losses allowing to fulfil the conditions given in points 1 and 2 above should take into account:
a) the form and simplicity of the description, deciding of the possible use of that description in the system efficiency model, with maintaining the system efficiency precise assessment,

Sankey diagram

diagram according to Z.Paszota

$$P_{Pu} = P_{Pc} - \Delta P_{Pm} - \Delta P_{Pv} - \Delta P_{Pp}$$

$$P_{Pc} = P_{Pu} + \Delta P_{Pp} + \Delta P_{Pv} + \Delta P_{Pm}$$

Fig. 4.1. *Diagram of power increase in a displacement pump opposite to the direction of power flow replacing the Sankey diagram of power decrease in the direction of power flow Power increases from the motor useful power P_{Pu}, required by the working fluid in the discharge conduit of the pump, to power P_{Pc} consumed and required by the pump shaft from (electric or internal combustion) motor driving the pump.*
The increase of power is an effect of the powers of losses in the pump: power ΔP_{Pp} of pressure losses in the pump channels, power Δp_{Pv} of volumetric losses in the working chambers and power ΔP_{Pm} of mechanical losses in the „working chambers - shaft" assembly.
Powers ΔP_{Pp}, ΔP_{Pv} and ΔP_{Pm} of the losses are functions of the output parameters of the pump assembly where the losses occur and diversified functions of the working fluid viscosity v: power ΔP_{Pp} of pressure losses is a function of pump capacity Q_P and of the working fluid viscosity v, power ΔP_{Pv} of volumetric losses is a function of the increase Δp_{Pi} of pressure indicated in working chambers and of the shaft rotational speed n_P as well as a function of the working fluid viscosity v, power ΔP_{Pm} of mechanical losses is a function of the torque M_{Pi} indicated in the chambers, of the pump shaft speed n_P (ω_P) and a function of the working fluid viscosity v.
Useful power P_{Pui} of the working fluid in the working chambers: $P_{Pui} = P_{Pu} + \Delta P_{Pp}$, power P_{Pi} indicated in the working chambers:
$P_{Pi} = P_{Pu} + \Delta P_{Pp} + \Delta P_{Pv}$, power P_{Pc} consumed on the pump shaft: $P_{Pc} = P_{Pu} + \Delta P_{Pp} + \Delta P_{Pv} + \Delta P_{Pm}$.
The diagram replaces the Sankey diagram of distribution of power in a pump, causing incorrect loss evaluation during the pump energy investigations

b) description of the displacement machine mechanical, volumetric and pressure losses, allowing to evaluate the impact of hydraulic oil kinematic viscosity changing with oil temperature,

c) separate treatment of the mechanical, volumetric and pressure losses in the machine.

It is necessary to perform the laboratory and simulation investigations in the displacement machine real operating conditions. The investigations should allow to verify the proposed models of:

– machine mechanical losses,
– machine volumetric losses,
– machine pressure losses,

in the full range of working pressure up to nominal pressure p_n, in the wide range of pump capacity up to theoretical capacity Q_{Pt} and in wide range of the hydraulic oil kinematic viscosity v, and also to **determine the k_i coefficients of specific losses**.

6. MODEL OF VOLUMETRIC LOSSES IN THE PUMP WORKING CHAMBERS

Volumetric losses in the pump are connected first of all with the working liquid leaks through slots between displacement elements and the working chamber walls, distributor (if it exists) elements and **are also effect of the liquid compressibility**, change of the pump working chambers volume and change of the slot height due to changes of pressure and temperature.

The model of volumetric losses presented in [24, 25 and 26] meets the requirements given in chapter 5. The author assumes the conditions and simplifications of the impact of certain factors on those losses and that impact is reflected in the coefficient k_1 of volumetric losses in the pump (equation (6.3)) and in power exponents describing the dependence of volumetric losses on Δp_{Pi} and v (equation (6.4)).

The theoretical working volume q_{Pt} (V_P) of constant or variable capacity pump (theoretical capacity q_{Pt} for one pump shaft revolution – at the pump capacity coefficient $b_P = 1$) – the geometrical difference between the maximum and minimum volume of working chambers – is a characteristic value of a pump. **It is determined at the pressure value $p_{P1i} = 0$ in the pump working chambers during their filling and at the increase $\Delta p_{Pi} = 0$ of the indicated pressure in the working chambers**.

Under the pressure and temperature, the geometric working volume q_{Pg} of the pump changes slightly compared with q_{Pt}. **It is assumed** (in order to simplify the description of volumetric and mechanical losses in the pump) **that the theoretical pump working volume q_{Pt} is constant and equal to the geometrical working volume q_{Pg} determined at the working liquid**

temperature corresponding to the recommended kinematic viscosity $\nu_n = 35\,mm^2s^{-1}$:

$$q_{Pt} = q_{Pg}\Big|_{\substack{p_{P1i}=0 \\ \Delta p_{Pi}=0 \\ \nu_n}} \quad (6.1)$$

and the change of geometric working volume q_{Pg} during the system operation will be taken into account in the values of loss coefficients k_i in the pump.

The theoretical capacity Q_{Pt} of a constant capacity pump is given by the formula:

$$Q_{Pt} = q_{Pt}n_{P0} \quad (6.2)$$

where:

n_{P0} – the rotational shaft speed of an unloaded pump ($\Delta p_{Pi} = 0$).

Coefficient k_1 of the volumetric losses Q_{Pv}, determined during one shaft revolution of a constant or variable capacity pump, at the pressure increase Δp_{Pi} equal to the hydraulic system nominal pressure $\Delta p_{Pi} = p_n$ and at the viscosity ν_n, the losses related to the pump theoretical working volume $q_{Pt}(V_P)$, is described by the formula:

$$k_1 = \cfrac{Q_{Pv}\Big|_{\substack{q_{Pt} \\ \Delta p_{Pi}=p_n \\ \nu_n}}}{n_P\Big|_{\substack{q_{Pt} \\ \Delta p_{Pi}=p_n \\ \nu_n}}} \cdot \frac{1}{q_{Pt}} \quad (6.3)$$

Using the volumetric loss coefficient k_1, the following mathematical model for the intensity Q_{Pv} of volumetric losses in the pump is proposed:

$$Q_{Pv} = k_1 q_{Pt} n_P \Big|_{\substack{q_{Pt} \\ \Delta p_{Pi}=p_n \\ \nu_n}} \left(\frac{\Delta p_{Pi}}{p_n}\right)^1 \left(\frac{\nu}{\nu_n}\right)^{-0.8} \quad (6.4)$$

The use of coefficient k_1 of volumetric losses for description of the relation of intensity Q_{Pv} to the indicated increase Δp_{Pi} of pressure in the pump working chambers allows to describe that relation by an exponential function **with the exponent not necessarily equal to 1**.

The expression describing the variable capacity pump capacity takes the form:

$$Q_P = b_P q_{Pt} n_P - k_1 q_{Pt} n_P \Big|_{\substack{q_{Pt} \\ \Delta p_{Pi}=p_n \\ \nu_n}} \left(\frac{\Delta p_{Pi}}{p_n}\right)^1 \left(\frac{\nu}{\nu_n}\right)^{-0.8} \quad (6.5)$$

The expressions (6.4) and (6.5) assume, that **change of the capacity coefficient b_P in the variable capacity pump (change of the pump capacity) does not influence the intensity of pump volumetric losses Q_{Pv}.**

In expressions (6.4) and (6.5), the value 1 of exponent describing the impact of the $\Delta p_{Pi}/p_n$ ratio and also the value -0.8 of exponent describing the impact of the ν/ν_n ratio on the intensity Q_{Pv} of pump volumetric losses should take into account all the factors influencing the volumetric losses (character of the flow in slots, change of the slots cross-section with pressure and temperature, **liquid compressibility**, change of the liquid viscosity in slots etc.).

The value 1 of exponent describing the impact of the $\Delta p_{Pi}/p_n$ ratio and also the value -0.8 of exponent describing the impact of the ν/ν_n ratio on the intensity Q_{Pv} of pump volumetric losses must be verified experimentally for each pump type.

Compressibility of the working liquid has an essential effect on the evaluation of the intensity Q_{Pv} of volumetric losses in the pump (equation (6.4)) and on the evaluation of coefficient k_1 of volumetric losses in the pump (equation (6.5)). The greater the liquid compressibility (mainly due to its aeration with the air not dissolved in the liquid) the greater the value of coefficient k_1 determined in the pump investigation. Therefore, **the liquid compressibility effects the evaluation of tightness of the pump structural solution**. It may be particularly evident in the displacement pump operating at high values ($32 \div 40$ MPa) of indicated pressure increase Δp_{Pi} in the pump working chambers.

VERIFICATION OF THE MATHEMATICAL MODEL OF VOLUMETRIC LOSSES IN THE PUMP WORKING CHAMBERS

Laboratory investigations of an axial piston variable displacement pump of bent axis design (HYDROMATIK A7V.58.DR.1.R.P.F.00 type – Fig. 7.1) were carried out by Jan Koralewski [20, 22] on a test stand (Fig. 7.2) in the Chair of Hydraulics and Pneumatics of the Gdansk University of Technology, Mechanical Engineering Faculty.
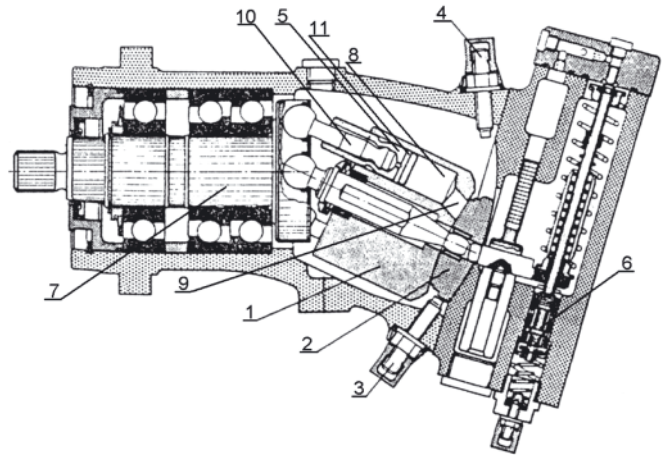

Fig. 7.1. *Axial piston variable displacement pump of bent axis design (HYDROMATIK 7V.58.DR.1.R.P.F.00 type) [23].* ***1)*** *cylinder block,* ***2)*** *collector (distributor), 3,* ***4)*** *blockage of pump setting,* ***5)*** *piston,* ***6)*** *assembly of capacity zeroing at maximum set pressure,* ***7)*** *shaft,* ***8)*** *working chamber,* ***9)*** *hole in the cylinder block connecting working chamber (8) with collector (distributor) (2),* ***10)*** *connector with ball-and-socket joints-element of the „working chamber - shaft" assembly,* ***11)*** *circumferential groove in the piston*

In order to verify the mathematical model described by formula (6.4), it was replaced by a mathematical expression taking into account the relations, obtained during the investigations, of the intensity Q_{Pv} of pump volumetric losses to the indicated increase Δp_{Pi} of pressure in the pump working chambers (to the $\Delta p_{Pi}/p_n$ ratio) and also to the oil viscosity ratio ν/ν_n [20, 22]:

$$Q_{Pv} = k_1 q_{Pt} n_P \Big|_{\substack{q_{Pt} \\ \nu_n \\ \Delta p_{Pi}=p_n}} \left(\frac{\Delta p_{Pi}}{p_n}\right)^{a_{pv}} \left(\frac{\nu}{\nu_n}\right)^{a_{vv}} \quad (7.1)$$

The obtained values (Fig. 7.3) of **the coefficient $k_1 = 0.065$** of intensity Q_{Pv} of volumetric losses, exponent $a_{pv} = 0.97$ of the relation of intensity Q_{Pv} of the volumetric losses to the ratio $\Delta p_{Pi}/p_n$ of the pressure increase in the working chambers, exponent $a_{vv} = -0.30$ of the relation of intensity Q_{Pv} of the volumetric losses to the ν/ν_n ratio of oil viscosity, have made it possible to present the mathematical model of the intensity Q_{Pv} of pump volumetric losses in the form:
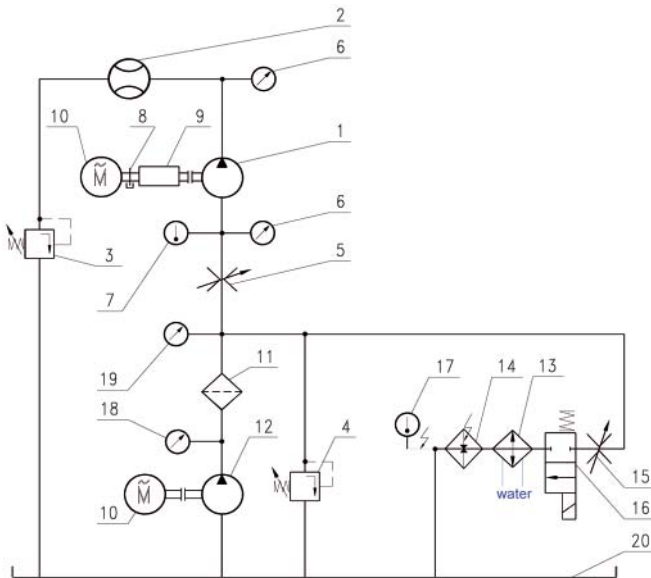
Fig. 7.2. *Test stand for investigating of the volumetric and mechanical losses in the pump (HYDROMATIK A7V.58.DR.1.R.P.F.00 type) [23].*
1) investigated pump, 2) piston flow- meter, 3,4) overflow valve,
5,15) throttling valve, 6) pressure sensor, 7) temperature sensor,
8) rotational speed sensor, 9) torque sensor, 10) asynchronic motor,
11) filter, 12) feeding screw pump, 13) cooler, 14) electric heater,
16) distributor, 17) thermostat, 18) manometer,
19) vacuum meter, 20) tank

$$Q_{Pv} = 0.065 q_{Pt} n_P \left|_{\substack{q_{Pt} \\ v_n \\ \Delta p_{Pi}=p_n}} \left(\frac{\Delta p_{Pi}}{p_n}\right)^{0.97} \left(\frac{v}{v_n}\right)^{-0.30} \right. \quad (7)$$

Model (7.2) describes precisely the intensity Q_{Pv} of volumetric losses in the nominal conditions of the pump operation, i.e. at the pump capacity coefficient $b_P = 1$, the pressure increase ratio $\Delta p_{Pi}/p_n = 1$ ($p_n = 32MPa$) and the oil viscosity ratio $v/v_n = 1$ ($v_n = 35mm^2 s^{-1}$) . At the same time, this mathematical model is a simulation formula describing the change of intensity Q_{Pv} of volumetric losses with the change of the pressure increase ratio $\Delta p_{Pi}/p_n$ and the oil viscosity ratio v/v_n (the change of pump capacity coefficient b_P has practically no impact on the change of intensity Q_{Pv} of volumetric losses).

However, **the value of coefficient $k_1 = 0.065$** (equations (6.3), (7.1), (7.2)) of the intensity Q_{Pv} of volumetric losses, determined during one shaft revolution of the pump, at the pressure increase Δp_{Pi} equal to the system nominal pressure p_n – $\Delta p_{Pi} = p_n$ and at the viscosity v_n (the losses related to the pump theoretical working volume q_{Pt}) **is influenced by the working liquid compressibility coefficient k_{lc}.**

The liquid compressibility coefficient k_{lc} increases the value of coefficient k_1 of volumetric losses in the pump, „loading" the pump structure with non-existent excessive leakage mainly from the working chambers under the high discharge channel pressure to the working chambers connected with the low pressure inlet channel.

In the conditions of aerated hydraulic oil and high working pressure in the pump discharge channel, the value of liquid compressibility coefficient k_{lc} can significantly increase the value of volumetric loss coefficient k_1 and at the same time significantly decrease the value of coefficient $k_{4.2}$ of the increase $\Delta M_{Pm|\Delta p_{Pi}}$, b_P, v_n of the torque of mechanical losses in the pump.

Part II of the paper (titled: „Effect of the working liquid compressibility on the picture of volumetric losses in a high pressure displacement pump used in a hydrostatic drive, Part II
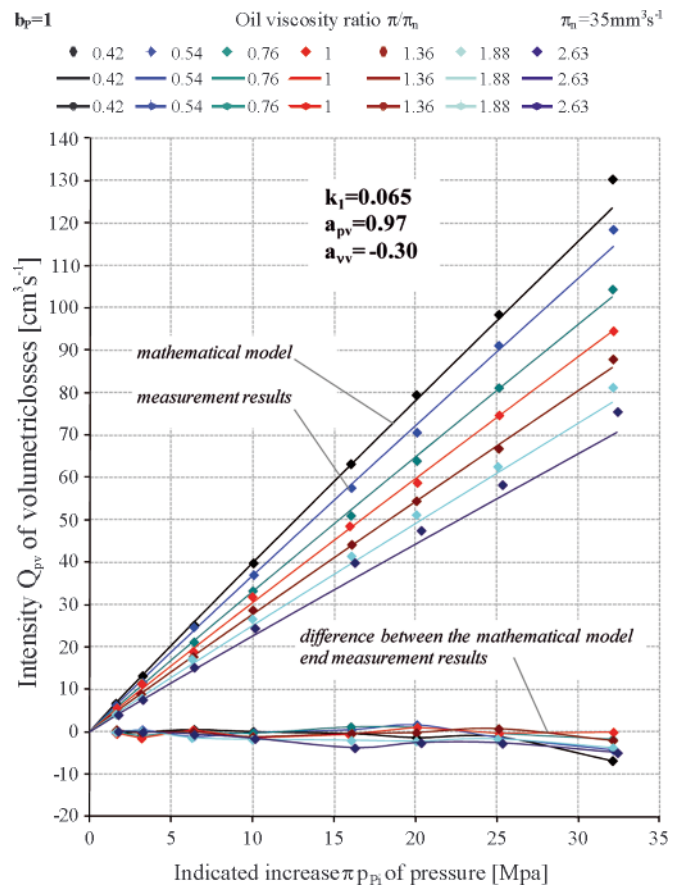

Fig. 7.3. *Comparison of the intensity Q_{Pv} of volumetric losses described by the mathematical model (6) with the laboratory investigation results and the absolute difference between the mathematical model values and the laboratory investigation values; assumed were: the coefficient $k_1 = 0.065$ of volumetric losses, exponent $a_{pv} = 0.97$, exponent $a_{vv} = - 0,30$; example for the pump capacity coefficient $b_P = 1$ (pump HYDROMATIK A7V.58. DR.1.R.P.F.00 type) [20, 22]*

– Mechanical losses in a pump) to be published in the next issue of this journal will present a possibility of determining a value of the $k_{lc|p_n}$ coefficient and its effect on the $k_{4.2}$ and k_1 coefficient.

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