Postprint of: Lipinski K. (2019) Effective configuration of a double triad planar parallel manipulator for precise positioning of heavy details during their assembling process. In: Uhl T. (eds) Advances in Mechanism and Machine Science. IFToMM WC 2019. Mechanisms and Machine Science, vol 73. Springer, Cham. https://doi.org/10.1007/978-3-030-20131-9_179

Effective configuration of a double triad planar parallel manipulator for precise positioning of heavy details during their assembling process

Krzysztof Lipinski[0000-0002-7598-4417]

Gdańsk University of Technology, ul Narutowicza 11/12, Gdańsk, 80-955, Poland
klipinsk @ pg.edu.pl

Abstract. In the paper, dynamics analysis of a parallel manipulator is presented. It is an atypical manipulator, devoted to help in assembling of heavy industrial constructions. Few atypical properties are required: small workspace; slow velocities; high loads. Initially, a short discussion about definition of the parallel manipulators is presented, as well as the sketch of the proposed structure. In parallel, some definitions, assumptions and equations are presented for the used multibody methodology. The main part of the paper presents results of the numerical tests performed in order to determinate the best work configuration of the proposed structure. It is verified, that with the initially proposed one, not all of the work requirements are satisfied, mainly about the load distribution. Obtained numerical results are confirmed by some diagrammatic simplified analysis of the load distribution. With the same diagrammatic method, alternative configurations are proposed and verified numerically, next. At the end, final configuration is presented as the one satisfying the requirements.

Keywords: Parallel Robots, Multibody System, Initial Configuration Problem.

1 General Introduction

In this paper, dynamics analysis of a parallel manipulator is considered and presented. Considered manipulator is devoted to help engineers in assembling of heavy industrial constructions. In this case, operations with hundreds of kilograms, or even tons, are required, together with positioning precision of few hundredths or few tenths of millimeter. Furthermore, in some of these processes, a given sequence of relative motions has to be performed during the processes. The paper proposed solution is the application of a dedicated planar manipulator, able to replace the hand made operations, as well as the presence of the *ad hoc* prepared provisional supporting structures. After rough positioning, performed with use of the long distance transport devices, the final positioning is executed with use of the proposed manipulator. Few atypical properties are required from the manipulator, now: big workspace is not required; high autonomy and intelligence of the manipulator is not necessary; high positioning precision is required; slow velocities are recommended, high loads are expected; any lost of the control is unacceptable (uncontrolled motion can cased considerably unrequited consequences); working point of the end-effector is undetermined (*ad hoc* selected points at the assembled element should be considered as the working points); absolute positioning is not important (relative displacements are more important).

Recalling the ISO definition of the robot [1], it is an *"automatically controlled, reprogrammable, multipurpose manipulator programmable in tree or more axes, which can be either fixed in place or mobile for use in industrial automation applications*". However, we will limit our attention to the simpler case of planar manipulators, i.e. these programmable in tree axes, only. Following recommendations proposed by the suppliers [2], all the industrial robots are classified according to their mechanical structures into 6 categories: linear robots; SCARA robots; articulated robots; parallel robots; cylindrical robots; other. When focusing on details of the presently considered structure, it can be partially interpret as the parallel robot. Detail classification is difficult, however, as precise definition have not been formulated, yet, for parallel robots.

Considered focus is sets on the parallel robots and it is not a casual selection. It is a beneficial class of robots. There is a long list of their potential merits that are listed in the literature. The most cited are: high rigidity/weight ratio [3-8]; high accuracy [3, 5, 6, 8, 10]; enlarged load capabilities [5-8, 10]; low inertias [7]; applicability of static and dynamic redundancies [8, 9], high operational speeds [5, 8, 10]. Because of these advantages, the parallel robots can be found in: medicine [6]; machine tools [3, 6]; material handling [6]; flight simulators [3, 6]; astronomy [3] and other.
The paper is divided into 5 sections. In the second one, a short discussion about the parallel manipulators is presented as well as the main idea and the sketch of the proposed construction. In section three, some definitions, assumptions and equations are presented for the used multibody methodology. The fourth section presents results of the numerical tests performed in order to determinate the best work configuration of the proposed structure. Section five presents the main conclusions and remarks.

2 Structure of the considered manipulator

Concerning the general definition of *manipulator*, in 1991, the IFToMM gave the following one: "*Device for gripping and controlled movements of objects.*" Recalling the ISO definition [1]: "*A machine, the mechanism of which usually consists of several of segments, joined or sliding relatively to one another, for the purpose of grasping and/or moving objects (pieces or tools) usually in several degrees of freedom. It may be controlled by an operator, a programmable electronic controller, or any logic system (for example cam devices, wired, etc)*". Recalled definitions are not identical, however, we can consider that any mechanical structure that is mobile and is able to move objects in several degrees of freedom can be called manipulator, whatever is the detail construction of the structure.

When focusing on *parallel manipulators*, we observe their significant presence in the scientific literature [1-10], as well as their increasing number of industrial applications. However, the precise and unique definition, as well as its classification, is not formulated, yet. E.g., they can be defined as: "*made of an end-effector with n degrees of freedom, and of a fixed base, linked together by at least two independent kinematic* *chains. Actuation takes place through n simple actuators*" [3], or "*consist of a base platform (stationary link), a mobile platform (end effector), and multiple parallel branches connecting the base and mobile platforms*" [6]. Recalling the ISO [1], "*A robot whose arms (primary axes) have three concurrent prismatic joints*". Focusing on number of the employed chains, they may not be sufficient to restrict mobility of the platform. According to it an class of *fully parallel manipulators* [3, 4] has been introduced as "*a parallel robot for which the number of chains is strictly equal to the number of d.o.f. of the end-effector*" [3]. Thus, in the planar case, three chains are required and they are attached to the three points of the moving platform. Accordingly, the platform can be interpreted as a triangle. Next, detailing composition of the supporting chains, "*If we assume that the three chains are identical … A chin is therefore made of two rigid bodies linked by a joint. Each of these rigid bodies is linked by a joint either with the base or with the end-effector. There will thus be three independent joints within the chain*" [3] or it can be said that "*the moving platform is connected to a fixed base by three links, every leg consisting of two binary links and three joints*" [4]. However a wider definition is possible, too. According to [1], term of *generalized parallel manipulator* can be formulated as "*a closed loop kinematic chain mechanism whose end-effector is linked to the base by several independent kinematic chains*". It is critical for the present paper. In its structure, the supporting chains are not identical. Moreover there is a chain that may not be considered as composed of two bodies. According to it, the presently considered structure is closer to the more general definition of the generalized parallel manipulator.

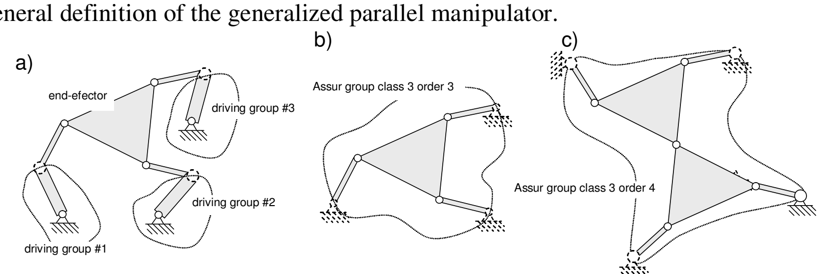

Fig. 1. Decomposition of exemplary cases: driving groups of classic 3RRR planar parallel manipulator (a); remaining Assur group (b); proposed alternative group (c)

At present, let us recall a property that is formulated in [3]: "*the mobility is zero when the actuators are locked, and that it becomes 3 when the actuator degrees of freedom are added*". Recalled property is critical for the paper, as it is similar to the fundamental property of the *structural group* (*Assur group*) used in the Assur classification of mechanisms. To express it, two kinds of objects have to be introduced:

Definition 1: *first class (driving) group* which is formed by the motionless base and by the input link, only. The joint that connects the input link with the base is a one degree of freedom joint. According to it mobility of the driving joint equals one.

Definition 2: *Assur group* composed of $\underline{n}$ bodies (links) and of *m* joints, which have to satisfy that: each of the joints is a one degree of freedom joint (a); there are *r* external joints that are used to connect the group to the motionless base; when the group is connected to this bases its mobility is zero (it is not mobile, but it is not over-constrained, too) (b); it is not possible to decompose the kinematic chine of the group into any simpler chains so, that all these chains complete the two requirements presented above (c).

According to the recalled properties of parallel manipulator, we can apply them in case of more general situation described as: considered mechanical structure is an Assure group of order *r* (i.e., a group with *r* external joints that connect the group to the motionless base) (a); the order of the group is higher then 2 (b); three of the *r* joints are liberated from the base and connected to three independent driving groups (c); as a result, one of the bodies obtains the total planar mobility (d). Next, a gripper is fixed to the fully mobile body of the obtained structure and it can be treated as a planar manipulator. Proposed methodology is in correspondence with the classic 3RRR planar parallel manipulator (Fig. 1). When the driving groups are eliminated from the structure (Fig. 1a), the obtained "remainder" (Fig. 1b) can be easily recognized as a classical Assur group (single-triad Assur group of class 3 and order 3).

In the present paper, the single triad group is replaced with a double-triad group of class 3 and order 4 (see Fig. 1c) and three driving groups are added. Resulting mechanical structure is visualized in Fig. 2. Focusing on conditions used in the classic definition of the parallel manipulator, presented sketch shows that some of them are not satisfied. However, considered structure can be classified as a generalized parallel manipulator. Its parameters are fitted to the anticipated task - selected geometrical distances visualized in Fig. 2 for the initially considered configuration. Next, user requests are added and assumed as critical for the tests: the payload mass is 500 kg; the gravity have to be carried by the driving joint #3, mainly; the driving joints #1 and #2 are weaker, not sufficiently strong and their driving torques are small, i.e., they should be responsible for horizontal motion and for rotation, mainly.

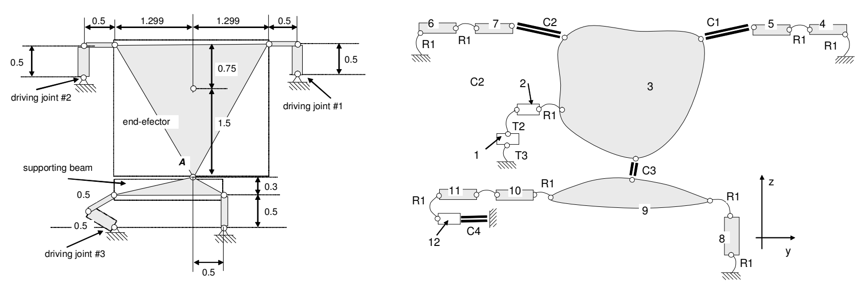

Fig. 2. Considered structure: its main geometrical parameters and the initially proposed configuration (a); structure of the used multibody system; body numbering (i); constraints numbering (= Ci), joint types (T2 - horizontal translation; T3 - vertical translation; R1 - rotation) (b)

3 Multibody model of the considered set

To model dynamics of the introduced manipulator, its reference *multibody structure* (*MBS*) [13-15] is created. In the present paper, such reference *MBS* is composed of rigid bodies, only (Fig. 3b). The *reference body* of the *MBS* is assumed as motionless. All bodies are joined via massless one-degree-of-freedom *joints* (1 dof connections of translational or rotational type). All eventual multi degree of freedom connections are modelled as chains composed of massless bodies, joints and eventually constraints). Then, a concept of *kinematical chain* (Fig. 3b) is introduced, as well as the *succession order* and *numbering* (Fig. 3b). Concerning the joint numbering, joint #*i* is the one that precedes body #*i* (it is present in the chain that connects the body #*i* and the reference). The introduced numbering technique is limited to the *tree like structures*, only. More detail description of these aspects can be found in [13-15].

Next, to obtain *dynamic equations* (*DE*) of the system, free body diagrams are prepared for each body of the considered *MBS* (Fig. 3a). All *MBS* 's joints are cut and replaced by certain joint interactions. Then, for each of the free body diagrams, *Newton/Euler equations of dynamics* are written [13-15]:

$$\vec{\omega}^i \times \left(\bar{\bar{I}}^i \cdot \vec{\omega}^i\right) + \bar{\bar{I}}^i \cdot \dot{\vec{\omega}}^i = \sum_{j=1}^{kt} \vec{t}_{j0}^{\,i} + \sum_{j=1}^{kf} \vec{r}_j^{\,i} \times \vec{f}_j^{\,i} \qquad ; \qquad m^i \cdot \ddot{\vec{x}}^i = \sum_{j=1}^{kf} \vec{f}_j^{\,i} \tag{1}$$

where: m^i – mass of i^{th} body, $\bar{\bar{I}}^i$ - its tensor of inertia, estimated about its mass centre; $\vec{f}_j^{\,i}$ - j^{th} net force set on the i^{th} body (Fig. 3a); $\vec{t}_{j0}^{\,i}$ - j^{th} net torque set on the i^{th} body (estimated about its mass centre); $\vec{r}_j^{\,i}$ - position of the j^{th} force attaching point (a vector that starts from the mass centre of the i^{th} body and directs the j^{th} force attaching point); $\vec{\omega}^i$ - vector of the angular velocity of the i^{th} body; $\vec{x}^i$ - vector of the absolute position of i^{th} body mass centre (in respect to the origin of the reference body coordinate system); kf , kt – numbers of the introduced forces and torques, respectively.

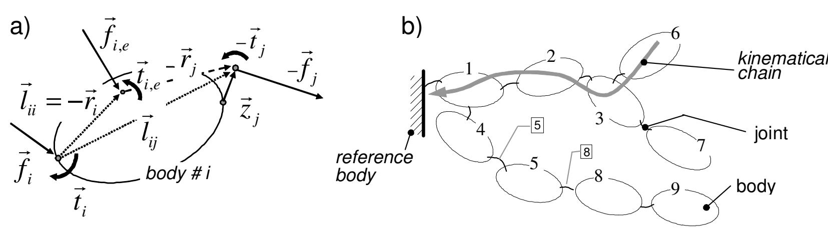

Fig. 3. Elements of the considered multibody system: forces and torques acting on i^{th} body (a); elements of the open chain multibody structure and numbering of its elements (b)

Initially, let us focus on *open chain structures*. The introduced *DE* (1) are combined with formulas of relative kinematics, where the lasts correlate kinematics of the considered body with the joints kinematics of the reference kinematic chain. Next, all the interactions cased by the successors of the considered i^{th} body are replaced with formulas obtained from their *DE* (thus the backward evaluation is necessary i.e., the one started from the terminal bodies of the chain structure). Successor free expressions are projected on mobility directions of the i^{th} joint, and components in front of the joint accelerations are extracted and collected. Resulting *DE* are written as [13-15]

$$\boldsymbol{M}(\boldsymbol{q}) \cdot \ddot{\boldsymbol{q}} + \boldsymbol{F}(\dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{f_e}, \boldsymbol{t_e}) + \boldsymbol{Q}(\dot{\boldsymbol{q}}, \boldsymbol{q}, t) = 0 \tag{2}$$

where: $\boldsymbol{M}$ - $n \times n$ mass matrix; $\boldsymbol{q}$ – $n \times 1$ matrix of joint coordinates; $\boldsymbol{F}$ - $n \times 1$ matrix of generalised forces (it collects quadratic velocity terms and all the generalised forces stem form $\vec{f}_{e,l}$, $\vec{t}_{e,l}$); $\boldsymbol{Q}$ - $n \times 1$ matrix composed of the joint torques; t - time.

As the considered *MBS* is certain *closed-loop structure*, thus a loop cutting procedure is added. The *DE*s (2) of the *reference tree structure* are extended with constraint interactions. Moreover, the *DE*s (2) are combined with algebraic *constraint equations* (*CE*) [13-15]. It leas to [16]:

$$\begin{gathered} \boldsymbol{M}(\boldsymbol{q}, t) \cdot \ddot{\boldsymbol{q}} + \boldsymbol{F}(\dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{f_e}, \boldsymbol{t_e}) + \boldsymbol{Q}(\dot{\boldsymbol{q}}, \boldsymbol{q}, t) + \boldsymbol{J}^T(\boldsymbol{q}) \cdot \boldsymbol{\lambda} = 0\,; \\ \boldsymbol{h}(\boldsymbol{q}) = 0\,; \qquad \dot{\boldsymbol{h}}(\boldsymbol{q}) = \boldsymbol{J}(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}} = 0\,; \qquad \ddot{\boldsymbol{h}}(\boldsymbol{q}) = \boldsymbol{J}(\boldsymbol{q}) \cdot \ddot{\boldsymbol{q}} + \boldsymbol{A}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = 0\,, \end{gathered} \tag{3}$$

where: $\boldsymbol{h}$ – constraints; $\boldsymbol{J}$ – Jacobian of the constraints; $\boldsymbol{\lambda}$ - Lagrange multipliers.

Composed multibody structure (Fig. 2b) consists of 12 bodies (3 of them are fictitious massless ones, denoted as not filled rectangles). Its initially independent chains are interconnected by 4 constraints denoted as Ci (C1 – C3 are the spherical body constraints; C4 is a rigid body constraint). Joints 4, 6 and 12 are motorized; the rest is torque and force free. Dimensions and inertial parameters are summarized in Tab. 1.

Table 1. Dimensions and inertial parameters of the bodies

body number	fixing position	mass	center of mass	inertia moment
3	[0.0; 0.0]	500	[0.0; 0.0]	850
4	[1.799; 0.75]	20	[-0.25; 0.0]	0.5
5	[-0.5; 0.0]	20	[-0.25; 0.0]	0.5
6	[-1.799; 0.75]	20	[0.25; 0.0]	0.5
7	[0.5; 0.0]	20	[0.25; 0.0]	0.5
8	[0.5; -2.3]	40	[0.0; 0.25]	1.01
9	[0.0; -0.5]	150	[-0.8995; 0.0]	45
10	[-1.799; 0.0]	20	[-0.25; 0.0]	0.5
11	[-0.5; 0.0]	20	[-0.25; 0.0]	0.5
12	[-0.5; 0.0]	0	[0.0; 0.0]	0

4 Numerical tests

Behavior of the system is tested during its vertical motion (see Fig. 4a,b). In the initially considered configuration the two first the driving cranks are oriented vertically, and the last is inclined at 60° (see Fig. 4a). Unfortunately, obtained dynamical behavior is far from the required. Torques at the two first driving joints (Fig. 4c,d) are relatively high. The gravity load is carried by these two joint and not by the third one (Fig. 4e), initially devoted for it. It is not required situation.

To find the origins of this phenomenon, a simplified quasi-statics analysis is proposed (Fig. **5**), where the payload's gravity is considered as the lonely load force. To determinate direction of the reaction force at point ***A*** (see Fig. 2), the three force principle of equilibrium is used (line in l_{3f} in Fig. **5**d). Next, equilibrium of the end-effector is solved. For the effector, directions of the reactions are known (i.e., the l_{f3} line and the lines collinear with the rods #5 and #7). The unknowns are their magnitudes. In the first step, we estimate position of the attaching point where the resultant force composed of the gravity force Q and the S_2 force is attached, as well as the attaching point of the resultant composed of the S_A and the S_2 forces (points C_1 and C_2 in Fig. **5**b).As these resultant forces are in equilibrium, they have to act along the l_C line (Fig. **5**b). This method and this line are called the Culmann method and the Culmann, respectively. The l_C line is used to construct the necessary polygon of forces and to evaluate magnitudes of the reactions (Fig. **5**a). As we can see, the l_C line is inclined about 40° to the gravity, thus the S_2 reaction is significant. The reaction S_1 is almost collinear with the l_C line, and thus, the S_A reaction is relatively small. With the small value of the S_A force, the S_1 and S_B forces are small (force polygon in Fig. **5**c). This result confirms the numerical results. Obtained force distribution is far from the required one. The gravity of the end-efector is transferred by the upper drives, and not by the lower one, as it is required.

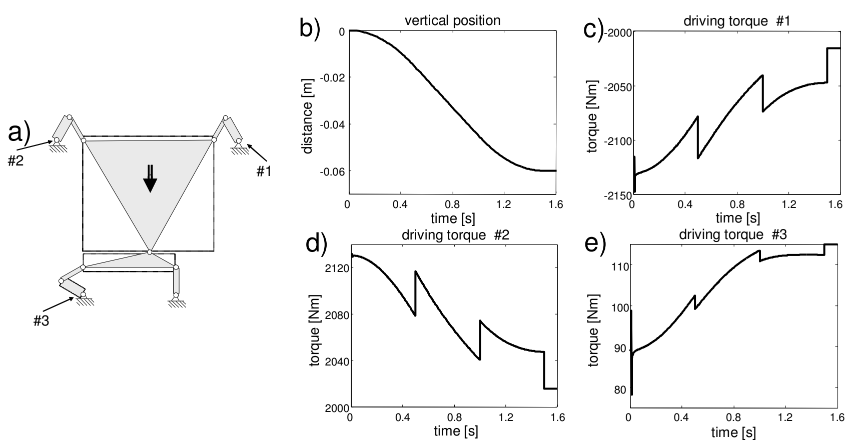

Fig. 4. Behavior of the considered structure: considered configurations (a); vertical position of its central point (b); first of the driving torques (c); second of the driving torques (d); third of the driving torques (e)

Obtained diagrammatic solution allows us to formulate certain advices for the initial configuration of the analyzed manipulator. To reduce the S_2 force, the l_C line has to be kept as vertical as possible. To reduce the S_1 line, the S_A direction has to be kept as vertical as possible, too. Formulated advices allow us to propose an alternative configuration of the system (Fig. 6). Now, the gravity of the end-efector is transferred by the lover drive, and the loads of the upper ones are relatively small.

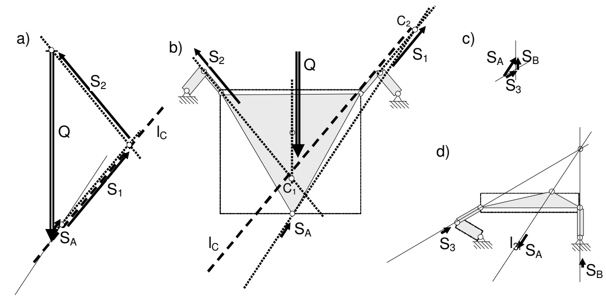

Fig. 5. Initial configuration - solution of the simplified quasi-static problem of equilibrium: l_C – Culman line; C_1 - first of the Culman points; C_2 - second of the Culman points; l_{3f} – closing line of the three forces principle; Q – payload gravity force; S_A – end-efector/second triangle interaction; S_1–S_3 – forces created by the driving torques.

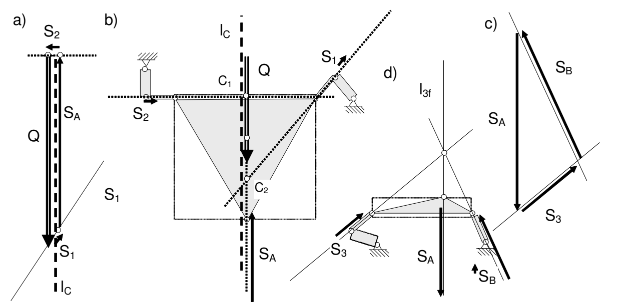

Fig. 6. Modified configuration - solution of the simplified quasi-static problem of equilibrium

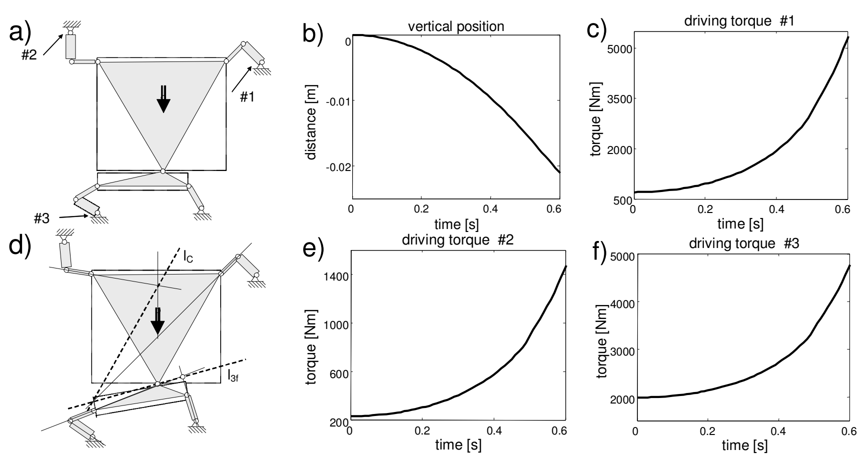

Fig. 7. Behavior of the considered structure: considered configurations (a); vertical position of its central point (b); first of the driving torques (c); second of the driving torques (d); third of the driving torques (e)

Behavior of the new system is tested numerically (Fig. 7). The test confirms the diagrammatic solution. In the initial position, the load distribution is correct. Unfortunately, this property degrades quickly, together with the end-effector displacement (Fig. 7c,e,f). The main reason is the rapid reorientation of the l_{3f} line (Fig. 7d). New advices are formulated and the final structure is visualized in Fig. 9a. Corresponding diagrammatic solution is presented in Fig 8, and the numerical confirmation in Fig. 9.

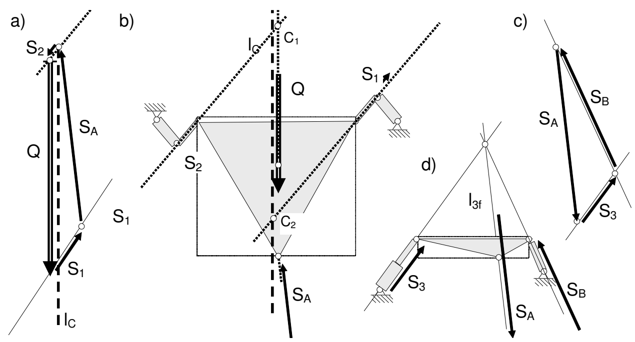

Fig. 8. Final configuration and structure - the simplified quasi-static problem of equilibrium

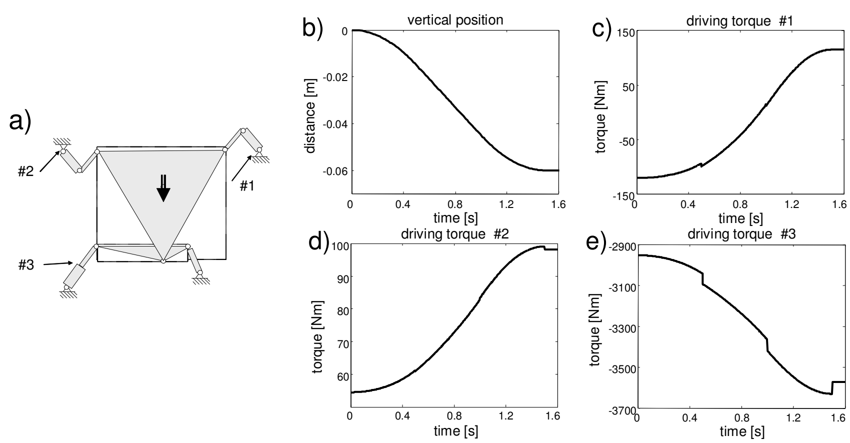

Fig. 9. Behavior of the considered structure: considered configurations (a); vertical position of its central point (b); first of the driving torques (c); second of the driving torques (d); third of the driving torques (e)

5 Conclusions

According to its internal structure, as well as its destination and function, considered structure can be treated as an example of planar parallel robot, devoted to some atypical work conditions. Numerical tests have confirmed that the initially proposed work configurations of arms leads to unwonted distribution of the gravity load between the drives. Performed diagrammatic analysis has verified it and it has indicated that the main reason of this behavior is the arm configurations. The improved configuration works better, but in the neighborhood of the initial position only. With nigger displacement, the load distribution degrades. It leads to the final proposition of the structure, more resistant on displacement dependent reconfigurations of the arms.

References

1. ISO 8373:2012 (2012)
2. IFR – International Federation of Robotics Homepage, https://ifr.org/industrial-robots, last accessed 2018/11/27.
3. Merlet, J.P.: Parallerl Robots. 2nd edn. Springer, Netherlands (2006)
4. Staicu S.: Dynamics of Parallel Robots. Springer International Publishing, Switzerland (2019)
5. Zhao, Y., Gao, F.: Dynamic Performance of the 8PSS Redundant Parallel Manipulator and Its Non-Redundant Counterpart – the 6PSS Parallel Manipulator. Mechanism and Machine Theory 44(5), 991–1008 (2009).
6. Notash, L., Huang, L.: On the design of fault tolerant parallel manipulators. Mechanism and Machine Theory 38, 85–101 (2003)
7. Kim, H.W., Lee, J.H., Suh, I-H., Yi, B-J.: Comparative Study and Experimental Verification of Singular-Free Algorithms for a 6 DOF Parallel Haptic Device. Mechatronics 15, 403–422 (2005).
8. Lipinski, K.: Redundant Actuation of 3RRR over-actuated Planar Parallel Manipulator. Acta Technica Napocensis-Series Applied Mathematics and Mechanics 52(3), 137-142 (2009).
9. Moosavian, A., Xi, F.: Statically Redundant Parallel Robots. Meccanica 51, 1623–1637 (2016).
10. Shalyukhin, K.A., Rashoyan, G.V., Aleshin, A.K., Skvortsov, S.A., Levin, S.V., Antonov, A.V.: Problems of Kinematic Analysis and Special Positions of Mechanisms of Robots with Parallel Structure. Journal of Machinery Manufacture and Reliability 47(4), 310–316 (2018).
11. Germain, C., Briot, S., Caro, S., Wegner Ph.: Natural Frequency Computation of Parallel Robots. Journal of Computational and Nonlinear Dynamics 10(2), (2015).
12. Cammarata, A., Calio, I., D'Urso, D., Greco, A., Lacagnina, M., Fichera G.: Dynamic stiffness model of spherical parallel robots. Journal of Sound and Vibration 384, 312–324 (2016).
13. Fisette, P., Lipinski, K. and Samin, J.C.: Modelling for the Simulation Control and Optimization of Multibody System. Advances in Multib. Sys. and Mechat., Gratz(Austria), 139-174 (1999).
14. Fisette, P. and Samin, J.C.: Symbolic Modeling of Multibody System. Kluwer Acad. Publ., Netherlands (2003).
15. Lipiński, K.: Multibody Systems with Unilateral Constraints in Application to Modelling of Complex Mechanical Systems. Seria Monografie 123, Wydaw.P.G., Gdańsk (2012) (in Polish).
16. Haug, E.J. Yen, J.: Generalized Coord. Partitioning Meth. for Num. Integ., Real-Time Integ. Meth. for Mech. Sys. Simul.. NATO ASI Series 69, 97-114 (1990).