

Elastic electron scattering in neon in the 110° – 180° scattering angle range

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Differential cross sections for elastic electron scattering in neon have been measured in the angular range of backward scattering from 110° – 180° at incident energies of 7, 10, and 15 eV. These measurements combined the use of a magnetic angle changer and an electrostatic electron spectrometer. The differential cross sections measured in the above scattering angle range, together with results obtained previously in the range below 110° , have been integrated to obtain integral elastic and momentum transfer cross sections. Detailed comparison is presented of the measured differential and integral cross sections with the results of various theoretical calculations.

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I. INTRODUCTION

The ability to calculate accurate elastic scattering cross sections in the noble gases represents a fundamental test of our understanding of the dynamics of electron-atom interactions. It provides the basis upon which we can proceed to open-shell atoms and to molecular systems. The large amount of theoretical effort applied to these calculations attests to their importance. Producing sufficiently accurate experimental data to test the various theories also presents challenges to the experimentalist. For detailed comparison with theory, measurements over the full range of angular scattering 0° – 180° are required. Until recently, however, the majority of the experimental measurements were confined to the angular range below about 130° and none could be made in the backward hemisphere over a continuous angular range up to 180° . This scattering angle region became accessible with the recent development of the magnetic angle changing technique [1–3]. So far, in the noble gases this technique has been used to measure differential cross sections for elastic scattering in argon [4] and krypton [5,6].

In this work we present the first measurements of the differential cross sections for elastic scattering in neon over the backward hemisphere from 110° to 180° and for incident energies of 7, 10, and 15 eV, that is below the first excitation threshold. The magnetic angle-changing technique [3] has been applied and the absolute values of the cross sections have been obtained using the relative flow technique. Scattering of electrons by neon atoms has particular importance because neon is the simplest noble gas atom after helium. Experimentally the need for accurate absolute differential cross sections in neon has been emphasized by the suggestions [15,16] that it can be used as a secondary standard, after helium, in the determination of the differential cross sections of other gases.

The first measurements of the elastic differential cross sections for neon atoms in the low-energy region (<20 eV) were carried out in the 1930s [7–9]. There was then a gap of more than thirty yrs before the next measurements in this energy range were performed [10,11]. More recent measurements of the absolute differential cross sections have been made by Williams [12] (who presented derived phase shifts

in the range from 0.58 eV to 20 eV), Brewer *et al.* [13], Register and Trajmar [14], Shi and Burrow [15], and Gulley *et al.* [16]. None of these studies, however, measured these cross sections in the scattering angle range above 145° .

Theoretical studies of elastic electron scattering by neon atoms below 20 eV has been more extensive than the experimental studies. The general aim of these theoretical works was to develop a description of the polarization and exchange interactions between the target atom and the incident electron. Saha used a multiconfiguration self-consistent-field method [17] that included dynamic polarization and electron correlation effects. The polarized-orbital method has been used by Thompson [18] and Dasgupta and Bhatia [19] to calculate differential, total, and momentum transfer cross sections and by Garbaty and Labahn [20] to obtain the total cross section. McEachran and Stauffer [21,22] have also made polarized-orbital calculations to obtain the differential and integral cross sections. These latter calculations involved an adiabatic exchange approximation that included the dipole polarization potential and treated exchange exactly. Fon and Berrington [23] performed *R*-matrix calculations for elastic scattering that took into account static dipole polarizability of the ground state by coupling the Hartree-Fock wave function of neon with a 1P pseudostate. Most recently Zatsarinny and Bartschat [24] used the *B*-spline *R*-matrix (BSR) approach to calculate elastic scattering from the ground state, taking into account the relativistic effects by including terms of the Breit-Pauli Hamiltonian in the inner region. However, by using term dependent, nonorthogonal sets of one-electron orbitals for each state, the emphasis in this approach was put on obtaining highly accurate results for near-threshold excitation of the lowest few excited states [25]. Reid and Wadehra [26] proposed a new correlation-polarization model that was parameter free and calculated differential cross sections in the energy range below 15 eV. Model polarization interaction potentials had been exploited previously in the calculations of Thirumalai and Truhlar [27] and Nakanishi and Schrader [28]. O'Connell and Lane [29] have proposed a nonadjustable model potential based on free-electron gas theory that includes both electron exchange and correlation. These authors and also Yuan [30] used this model to calculate the total cross section in neon. A new procedure for

including the local exchange-correlation potential has been introduced by Fritsche *et al.* [31] who used it in relativistic calculations. Gianturco and Rodriguez-Ruiz [32] have given an extended account of polarization-correlation forces. They used density-functional theory to describe the short-range interaction and a polarization adiabatic approach for the long-range interaction. Among other theoretical studies of electron scattering by neon atoms have been the works of Berg [33], Kemper *et al.* [34], Haberland *et al.* [35]. Berg used a relativistic approach to investigate the differential cross sections using the exchange-correlation $X\alpha$ potential. Kemper *et al.* used a relativistic Hartree-Fock two-channel approach that incorporated dynamic polarization, nonlocal exchange and inelastic scattering. Haberland *et al.* applied Kohn-Sham density functional theory, together with an exchange-correlation potential, derived from the defined correlation factors, to obtain differential cross sections.

II. EXPERIMENT

The apparatus consists of an electrostatic electron spectrometer and a magnetic angle changer. This device provides a localized magnetic field at the interaction region that deflects the incident and scattered electrons. This allows the elastic differential cross section of the target gas to be measured over the full backward scattering hemisphere. The apparatus and the experimental procedures used in the present work have been described in detail previously [3].

The electron spectrometer consists of an electron monochromator which is fixed in position and an electron energy analyzer that can be rotated through the angular range -10° to $+120^\circ$ with respect to the incident electron beam direction. The electron monochromator employs a hemispherical electrostatic deflector of 50 mm mean radius. Electrons leaving the deflector are focused onto the target region by two triple-aperture lenses. The monochromator delivers typically 3 nA of incident electron beam current at the interaction region with an energy spread (FWHM) of typically 70 meV over the energy range of interest (7 to 15 eV). The incident electron current is measured using a deep Faraday cup placed beyond the interaction region. The gas beam is formed by a single capillary of 0.3 mm internal diameter and length 10 mm. Scattered electrons from the target region are focused by a single three-cylinder lens onto the entrance aperture of a hemispherical electrostatic deflector of 20 mm mean radius. Electrons transmitted by the analyzer are detected by a channel electron multiplier. The incident electron energy was calibrated by comparing the observed position of the 2S resonance in helium with the accepted value of 19.366 eV [36] (see also [37] for a recent measurement of the energy of the helium 2S resonance). The uncertainty in the incident energy scale is estimated to be ± 30 meV.

The magnetic angle changer has been described in detail by Linert *et al.* [3,38]. Briefly, it consists of two pairs of conical solenoids of cylindrical symmetry. These produce the localized and shaped magnetic field that is perpendicular to the scattering plane at the interaction region. There is a gap between the solenoids to allow passage of the electrons and the scattering plane lies at the midpoint of this gap. The

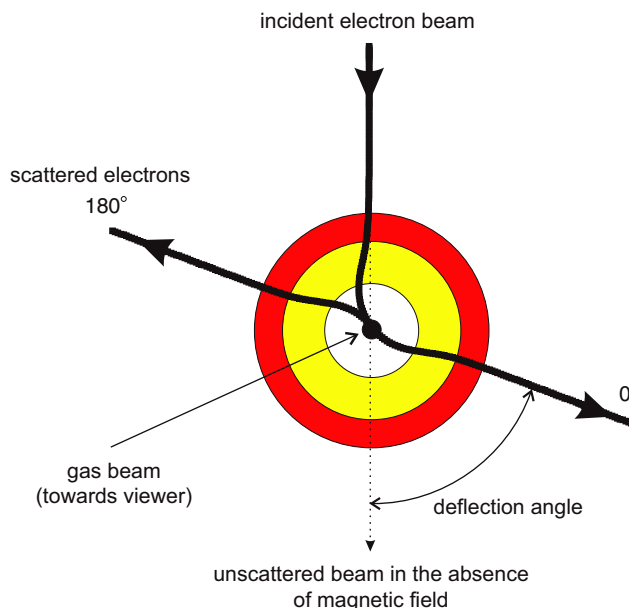


FIG. 1. (Color online) Cross section through the magnetic angle changer with inner and outer solenoids, shown by rings surrounding the scattering region. The electron trajectories computed for the incident electron beam and scattered electrons of 10 eV and the inner coil's current of 0.83 A are shown in the figure.

operation of the magnetic angle changer is illustrated in Fig. 1. The incident electrons are deflected by the magnetic field but still pass through the center of the interaction region. The magnetic field also deflects the elastically scattered electrons by the same angle. The total angular deflection of the electrons of 70° combined with the angular range of the mechanical rotation of the analyzer (-10° to $+120^\circ$) gives an accessible scattering angle range of $+60^\circ$ to $+190^\circ$. This gives access to the whole backward scattering hemisphere. The angular scale in the present measurements was calibrated by observing the minimum in the elastic differential cross section of argon that occurs in the region of 120° and the uncertainty in the measurements is estimated to be $\pm 2^\circ$. The angular resolution of the spectrometer is estimated to be 4° .

Absolute values of the differential cross sections have been determined at scattering angles from 110° to 180° in steps of 10° using the relative flow technique. This technique has been described in detail by Khakoo and Trajmar [39] and Nickel *et al.* [40]. To obtain the cross section at a given scattering angle, the yields of elastically scattered electrons were measured in neon and helium and the helium elastic differential cross section of Nesbet [41] was then used to normalize the neon cross sections. The pressure of neon behind the capillary was maintained within the range 5.8–6.7 Pa and the ratio of the helium to neon driving pressures was 1:0.72 to fulfill the requirement of equal mean free path lengths for both gases in the beam-forming capillary [40]. The gas flow rates were measured at each angle by recording the pressure increase, with a baratron, in a volume behind the leak valve when the gas flow into the collision region was cut off. In order to maintain highly stable performance of the spectrometer, both the neon and helium gases

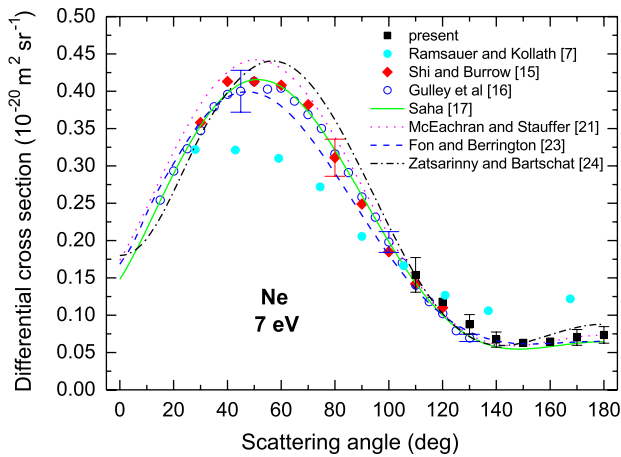


FIG. 2. (Color online) Differential cross sections for electron elastic scattering in neon at the energy of 7 eV: ■—present results. In the figure are shown experimental results of Ramsauer and Kollath [7], Shi and Burrow [15], and Gulley *et al.* [16] and theoretical results of Saha [17], McEachran and Stauffer [21], Fon and Berrington [23], and Zatsarinny and Bartschat [24].

were always present in the vacuum chamber. This was achieved by admitting one of the gases to the interaction region via the capillary and the other directly to the vacuum chamber via a side valve. Any background contributions to the measured electron yields were accounted for by bypassing the capillary and admitting both the neon and helium gases directly into the vacuum chamber through the side valve. These contributions are of special significance for measurements in neon where the elastic cross sections are relatively small.

The uncertainties in the presented values of the elastic cross sections arise from the statistical uncertainties in the determination of the scattered electron intensities, the relative flow rates of neon and helium, the incident electron beam current, and the uncertainty in the elastic cross section for helium. From the statistical distribution of the measurements it is estimated that the overall uncertainty in the presented differential cross sections is 15%.

III. RESULTS AND DISCUSSION

A. Differential cross sections

The absolute differential cross sections obtained in the present work in neon at energies of 7, 10, and 15 eV are presented in Figs. 2–4, respectively. These cover the angular scattering range 110°–180°. The numerical values of the cross sections are listed in Table I. The figures also show previous experimental cross sections and results from several theoretical calculations. The previous experimental work relates to the angular range below 140°, although the much earlier work of Ramsauer and Kollath [7] employed a technique that could reach angles up to 167.5°. The theoretical calculations cover the complete angular range, 0°–180°.

There is some overlap in the angular range 110°–130° between the present and the more recent experimental studies of Shi and Burrow [15] and Gulley *et al.* [16] at 7 eV, and

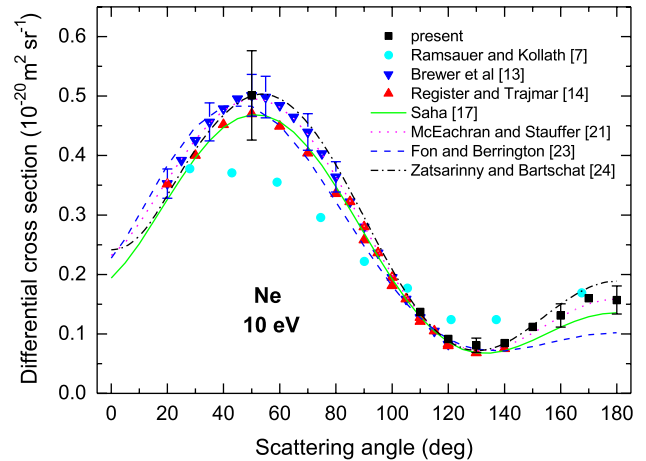


FIG. 3. (Color online) Differential cross sections for electron elastic scattering in neon at the energy of 10 eV: ■—present results. In the figure are shown experimental results of Ramsauer and Kollath [7], Brewer *et al.* [13], and Register and Trajmar [14] and theoretical results of Saha [17], McEachran and Stauffer [21], Fon and Berrington [23], and Zatsarinny and Bartschat [24].

Brewer *et al.* [13] and Register and Trajmar [14] at 10 and 15 eV. In this region of overlap, there is good agreement between all the recent experimental data including those of the present work. The earlier results of Ramsauer and Kollath [7] tend to be higher. Over the angular range 110°–130°, the theoretical calculations are also in very good agreement with each other and also in agreement with the experimental data.

In the angular range below 110° the calculated differential cross sections tend to deviate from each other by a small amount (≈10%). The available experimental cross sections are in very good mutual agreement at 7 and 15 eV (Figs. 2 and 4). However, at 10 eV (Fig. 3) the results of Brewer *et al.* [13] and Register and Trajmar [14] show differences

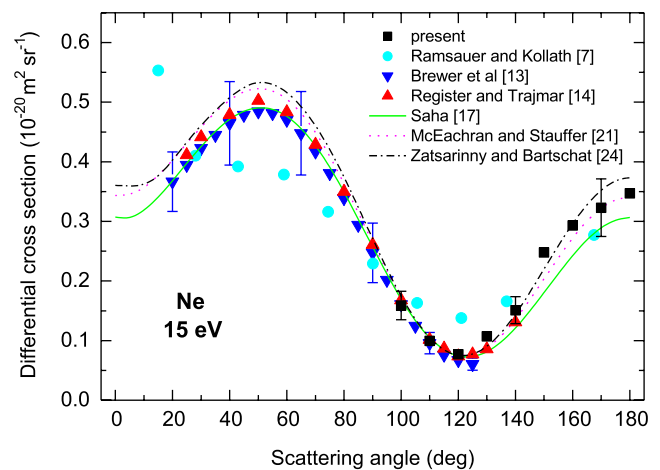


FIG. 4. (Color online) Differential cross sections for electron elastic scattering in neon at the energy of 15 eV: ■—present results. In the figure are shown experimental results of Ramsauer and Kollath [7], Brewer *et al.* [13], and Register and Trajmar [14] and theoretical results of Saha [17], McEachran and Stauffer [21], and Zatsarinny and Bartschat [24].

TABLE I. Differential cross sections, in units of $10^{-20} \text{ m}^2 \text{ sr}^{-1}$, for elastic electron scattering in neon at incident energies of 7, 10, and 15 eV.

Scattering angle (deg)	Electron energy		
	7 eV	10 eV	15 eV
50		0.501	
100			0.159
110	0.154	0.137	0.0996
120	0.118	0.0915	0.0772
130	0.0878	0.0808	0.107
140	0.0674	0.0847	0.151
150	0.0631	0.112	0.248
160	0.0646	0.131	0.293
170	0.0702	0.160	0.323
180	0.0736	0.157	0.347

which at the angle of 50° increase to 10%. To resolve this discrepancy we measured the differential cross section at 50° and obtained a value which is in excellent agreement with that of Brewer *et al.* [13] (see Table I).

For the angular range above 130° our measured differential cross sections rise to a maximum at 180° at all three energies. Over this angular range, at an incident electron energy of 7 eV, the three theoretical calculations of Saha [17], McEachran and Stauffer [21], and Fon and Berrington [23] are in very good agreement with each other and with the present results while those of Zatsarinny and Bartschat [24] are higher than our results by about 20%. The much earlier measurements of Ramsauer and Kollath [7] lie much higher. The situation above 130° is very different at 10 eV. There are significant differences between the results of calculations as can be seen in Fig. 3. The best agreement with the present measurements comes from the calculations of McEachran and Stauffer [21] (and Dasgupta and Bhatia [19] which coincide with each other). The results of Saha [17] and in particular of Fon and Berrington [23] are considerably lower than the present cross section while the results of Zatsarinny and Bartschat [24] are again about 20% higher than the experimental cross section. At the energy of 15 eV there is again good agreement between the theoretical calculations and the present results. The best agreement is with the theoretical calculations of McEachran and Stauffer [21]. However the calculations of Saha [17] and Zatsarinny and Bartschat [24] also agree with the present results within the quoted uncertainty. Interestingly, the calculations of Saha [17] are in better agreement with the experimental results below about 80° .

B. Integral and momentum transfer cross sections

The integral elastic cross section σ_i and the momentum transfer cross section σ_m have been deduced at energies of 7, 10, and 15 eV from integration of the differential cross sections over the total angular range 0° – 180° . For the integration in the angular range 110° – 180° the present cross sec-

TABLE II. Integral cross section σ_i and momentum transfer cross section σ_m , in units of 10^{-20} m^2 , for elastic electron scattering in neon at incident energies of 7, 10, and 15 eV.

Energy (eV)	σ_i	σ_m
7	3.03 ± 0.25	2.21 ± 0.22
10	3.51 ± 0.22	2.49 ± 0.21
15	3.55 ± 0.24	2.74 ± 0.26

tions were used. For the angular range 20° – 110° the cross sections of Gulley *et al.* [16] at 7 eV and that of Brewer *et al.* [13] at 10 and 15 eV were used. For the angular range 0° – 20° the results of Gulley *et al.* and Brewer *et al.* were extrapolated down to 0° . The deduced integral cross sections and their uncertainties are listed in Table II. These uncertainties arise from the uncertainties in the present measurements and those in the cross sections in the angular range 20° – 110° . The uncertainty in the present measurements is 15% while the uncertainties in the measurements in [16,13] are 7% and 5%, respectively. The uncertainties arising from the extrapolated cross sections in the 0° – 20° range contribute less than 1% to the total uncertainty. The deduced integral cross sections are compared with recent measurements of the total cross sections [42–49] and determination from differential cross sections [16] in Fig. 5 and with the experimental momentum transfer cross section [50] in Fig. 6. (For earlier works on total cross sections see references in [48].)

As can be seen from Fig. 5 the present integral cross sections at the three values of electron energy 7, 10, and 15 eV, respectively are in very good agreement with all the previous experimental results. This in particular applies to our 10 eV value which again supports the results of the differential cross sections of Brewer *et al.* [13] in favor of those

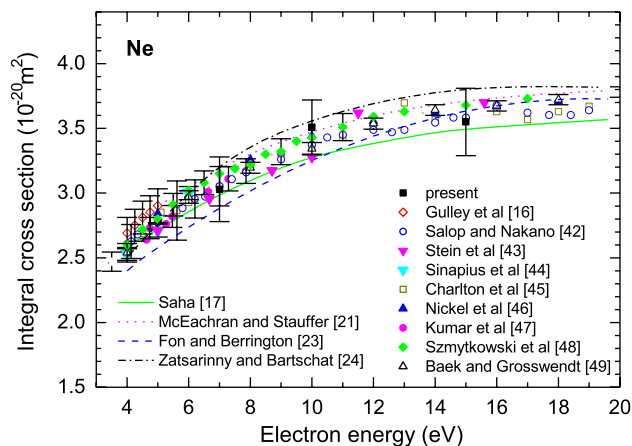


FIG. 5. (Color online) Integral cross sections for elastic electron scattering in neon: ■—present results. In the figure are shown experimental results of Gulley *et al.* [16], Salop and Nakano [42], Stein *et al.* [43], Sinapius *et al.* [44], Charlton *et al.* [45], Nickel *et al.* [46], Kumar *et al.* [47], Szmytkowski *et al.* [48], and Baek and Grosswendt [49] and theoretical results of Saha [17], McEachran and Stauffer [21], Fon and Berrington [23], and Zatsarinny and Bartschat [24].

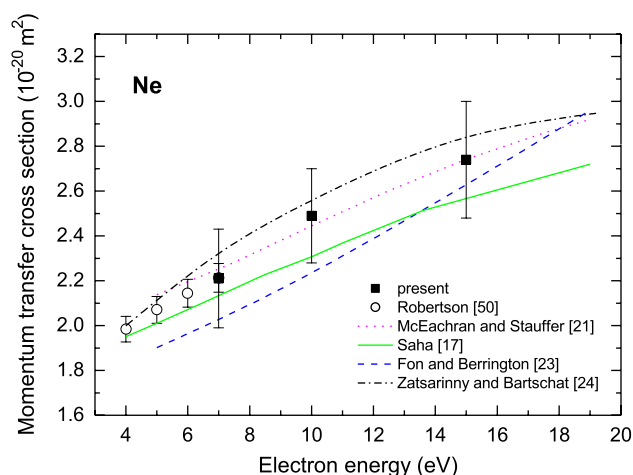


FIG. 6. (Color online) Momentum transfer cross sections in neon: ■—present results. In the figure are shown experimental results of Robertson [50] and theoretical results of Saha [17], McEachran and Stauffer [21], Fon and Berrington [23], and Zatsarinny and Bartschat [24].

of Register and Trajmar [14]. The theoretical calculations presented in Fig. 5 are generally in good agreement with the experimental total cross sections. However the results of Saha [17] are 3–5 % lower than most of the experimental results while the results of Zatsarinny and Bartschat [24] are higher by the same amount. The results of Fon and Berrington [23] are lower by about 7% at lower energies.

Figure 6 shows the comparison of the present values of the momentum transfer cross sections with the experimental results of Robertson [50]. As far as the authors are aware this is the only available experimental measurement of the momentum transfer cross section of neon in this energy range. The value of our cross section at 7 eV is within 3% of the value reported by Robertson. Figure 6 also shows the results of various theoretical calculations [17,21,23,24]. The best agreement with theory is obtained with the calculations of McEachran and Stauffer [21] (and Dasgupta and Bhatia [19]).

IV. CONCLUSIONS

The present work has considerably extended the range of measurement of the elastic differential cross section of neon in the backward scattering hemisphere. This cross section has been measured over the continuous range of scattering angle from 110°–180°. This has been done for the incident electron energies of 7, 10, and 15 eV, respectively. These differential cross sections are observed to reach a maximum at 180°. The present work overlaps previous experimental works in the angular range 110°–130° and in this range there is very good agreement with all the recent experimental and theoretical values of the differential cross section. At 7 and 15 eV the present differential cross sections are also in agreement with the recent theoretical calculations above 130°. This is not the case, however, at 10 eV. At this energy only the calculations of McEachran and Stauffer [21] are in agreement with the present results. The calculations of Saha [17] are lower by about 15% and those of Fon and Berrington [23] are lower still, by about 35%.

The cross sections integrated over the total angular range 0°–180° have been deduced from the present results and those of Gulley *et al.* [16] and Brewer *et al.* [13]. These differential cross sections have been integrated to obtain values giving the integral and the momentum transfer cross sections at energies of 7, 10, and 15 eV. The deduced integral cross sections are in very good agreement with existing values of this cross section. The deduced value of the momentum transfer cross section at 7 eV is in very good agreement with the only known measurement of this cross section.

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- [1] M. Zubek, N. Gulley, G. C. King, and F. H. Read, *J. Phys. B* **29**, L239 (1996).
- [2] F. H. Read and J. M. Channing, *Rev. Sci. Instrum.* **67**, 2372 (1996).
- [3] I. Linert, G. C. King, and M. Zubek, *J. Phys. B* **37**, 4681 (2004).
- [4] B. Mielewska, I. Linert, G. C. King, and M. Zubek, *Phys. Rev. A* **69**, 062716 (2004).
- [5] H. Cho, R. J. Gulley, and S. J. Buckman, *J. Korean Phys. Soc.* **42**, 71 (2003).
- [6] H. Cho, R. P. McEachran, H. Tanaka, and S. J. Buckman, *J. Phys. B* **37**, 4639 (2004).
- [7] C. Ramsauer and Kollath, *Ann. Phys.* **12**, 529 (1932).
- [8] E. C. Bullard and H. S. Massey, *Proc. R. Soc. London* **133**, 637 (1931).
- [9] A. L. Hughes and J. H. McMillen, *Phys. Rev.* **43**, 875 (1933).
- [10] J. Mehr, *Z. Phys.* **198**, 345 (1967).
- [11] D. Andrick, in *Adv. At. Mol. Phys.* Vol. 9, edited by D. R. Bates, I. Estermann (Academic Press, New York, 1973), p. 207.
- [12] J. F. Williams, *J. Phys. B* **12**, 265 (1979).
- [13] D. F. C. Brewer, W. R. Newell, S. F. W. Harper, and A. C. H. Smith, *J. Phys. B* **14**, L749 (1981).
- [14] D. F. Register and S. Trajmar, *Phys. Rev. A* **29**, 1785 (1984).
- [15] X. Shi and P. D. Burrow, *J. Phys. B* **25**, 4273 (1992).
- [16] R. J. Gulley, D. T. Alle, M. J. Brennan, M. J. Brunger, and S. J. Buckman, *J. Phys. B* **27**, 2593 (1994).
- [17] H. P. Saha, *Phys. Rev. A* **39**, 5048 (1989).
- [18] D. G. Thompson, *Proc. R. Soc. London* **294**, 160 (1966).
- [19] A. Dasgupta and A. K. Bhatia, *Phys. Rev. A* **30**, 1241 (1984).
- [20] E. A. Garbaty and R. W. LaBahn, *Phys. Rev. A* **4**, 1425 (1971).

- [21] R. P. McEachran and A. D. Stauffer, *J. Phys. B* **16**, 4023 (1983).
- [22] R. P. McEachran and A. D. Stauffer, *Phys. Lett.* **107A**, 397 (1985).
- [23] W. C. Fon and K. A. Berrington, *J. Phys. B* **14**, 323 (1981).
- [24] O. Zatsarinny and K. Bartschat, *J. Phys. B* **37**, 2173 (2004); and private communication (2006).
- [25] M. Allan, K. Franz, H. Hotop, O. Zatsarinny, and K. Bartschat, *J. Phys. B* **39**, L139 (2006).
- [26] D. D. Reid and J. M. Wadehra, *Phys. Rev. A* **50**, 4859 (1994).
- [27] D. Thirumalai and D. G. Truhlar, *Phys. Rev. A* **26**, 793 (1982).
- [28] H. Nakanishi and D. M. Schrader, *Phys. Rev. A* **34**, 1823 (1986).
- [29] J. K. O'Connell and N. F. Lane, *Phys. Rev. A* **27**, 1893 (1983).
- [30] J. Yuan, *J. Phys. B* **21**, 3753 (1988).
- [31] L. Fritsche, J. Noffke, and H. Gollisch, *J. Phys. B* **17**, 1637 (1984).
- [32] F. A. Gianturco and J. A. Rodriguez-Ruiz, *Phys. Rev. A* **47**, 1075 (1993).
- [33] H. P. Berg, *J. Phys. B* **15**, 3769 (1982).
- [34] F. Kemper, F. Rosicky, and R. Feder, *J. Phys. B* **17**, 3763 (1984).
- [35] R. Haberland, L. Fritsche, and J. Noffke, *Phys. Rev. A* **33**, 2305 (1986).
- [36] J. N. H. Brunt, G. C. King, and F. H. Read, *J. Phys. B* **10**, 1289 (1977).
- [37] A. Gopalan, J. Bommels, S. Gotte, A. Landwehr, K. Franz, M.-W. Ruf, H. Hotop, and K. Bartschat, *Eur. Phys. J. D* **22**, 17 (2003).
- [38] I. Linert, G. C. King, and M. Zubek, *J. Electron Spectrosc. Relat. Phenom.* **134**, 1 (2003).
- [39] M. A. Khakoo and S. Trajmar, *Phys. Rev. A* **34**, 138 (1986).
- [40] J. C. Nickel, C. Mott, I. Kanik, and D. C. McCollum, *J. Phys. B* **21**, 1867 (1988).
- [41] R. K. Nesbet, *Phys. Rev. A* **20**, 58 (1979).
- [42] A. Salop and H. H. Nakano, *Phys. Rev. A* **2**, 127 (1970).
- [43] T. S. Stein, W. E. Kauppila, V. Pol, J. H. Smart, and G. Jesion, *Phys. Rev. A* **17**, 1600 (1978).
- [44] G. Sinapius, W. Raith, and W. G. Wilson, *J. Phys. B* **13**, 4079 (1980).
- [45] M. Charlton, G. Laricchia, T. C. Griffith, G. L. Wright, and G. R. Heyland, *J. Phys. B* **17**, 4945 (1984).
- [46] J. C. Nickel, K. Imre, D. F. Register, and S. Trajmar, *J. Phys. B* **18**, 125 (1985).
- [47] V. Kumar, E. Krishnakumar, and K. P. Subramanian, *J. Phys. B* **20**, 2899 (1987).
- [48] C. Szmytkowski, K. Maciąg, and G. Karwasz, *Phys. Scr.* **54**, 271 (1996).
- [49] W. Y. Beak and B. Grosswendt, *J. Phys. B* **36**, 731 (2003).
- [50] A. G. Robertson, *J. Phys. B* **5**, 648 (1972).