

Evaluation of a Small Inland Ferry's Energy Requirements from the Acceleration Stage of Towing Tank Model Tests

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ABSTRACT

Computing the power required to meet a ship's operational needs is one of the most important tasks in naval design. The power required to propel a vessel is directly related to the resistance the hull experiences as it moves through the water. The conventional method of determining a ship's resistance involves towing tank tests of ship models at a fixed speed; however, for short-range vessels, where constant speed is not the primary mode of operation, a dynamic model is needed. This paper demonstrates a way in which different operational motion profile models can be retrieved from the acceleration stage of towing tank tests. We show that the data from the acceleration stage, often overlooked in towing tank tests, allow us to derive the gliding equations of motion. A dynamic model of a small inland ferry on the Motława River in the city of Gdańsk is developed, which enables optimisation of the required power based on different operation profiles.

Keywords: Inland ferry, Green shipping, Hybrid, Power requirement, Acceleration stage towing tests, Added mass

INTRODUCTION

The trend towards designing increasingly energy-efficient ships has led to an increase in the variety of proposed propulsion and power structures. 'Smart' and advanced control strategies are required to improve the reliability and system performance of these structures, while in general, conventional human control strategies are still currently in use. In recent years, electric propulsion has undergone significant development in the marine sector [1]. Each type of propulsion system requires a different strategy for controlling the movement process. In research carried out at the Maritime Institute of the University of Delft, it was determined that the use of units with hybrid or electric propulsion, where

advanced algorithms are used to implement the strategy for controlling the movement process, can reduce fuel consumption and emissions by up to 10–35%, while also reducing the generated noise, improving the manoeuvrability of the vessel, and increasing the comfort of movement by reducing the forces acting on passengers [2].

Hybrid propulsion is mainly used for ships with a variable operating profile and vessels in coastal areas, where emissions regulations are strict [3]. Solutions to protect the environment are sought not only in Europe but also worldwide, with a particular focus on the industrial sector [4]. Energy consumption in hybrid propulsion depends on many factors [5], one of the most important of which is the ship's operational profile, i.e., its movement strategy. The higher the speed of the

vessel, the higher the energy demand from the propulsion system, due to the fact that the resistance generated on the hull follows a nonlinear function that increases exponentially with speed [6]. At low speeds, the energy demand is relatively small; however, slow movement is not always feasible, due to various constraints and additional requirements, such as passenger satisfaction or maintaining manoeuvrability. The optimal strategy is one that achieves an operational profile with the lowest energy consumption, while also reducing fuel consumption and emissions, or enabling the installation of fewer batteries [7].

A dynamic model is needed to determine the optimal operational profile. The knowledge of power requirement for changing dynamics is essential for short-distance cruises, such as shuttle ferries. Methods for determining the power requirement of vessels at constant speed are well-known, and have been recommended by the International Towing Tank Conference (ITTC); however, there is a need for more procedures and studies of different dynamics. The purpose of this work is to fill this gap. Such an approach, as presented in this paper, can be useful for determining energy requirements and optimisation methods based on the dynamics profile; this would be especially useful for short-range vessels, where constant speed is not the main characteristic [8]. In addition to small vessels, methods for obtaining dynamic motion profiles are also needed for underwater remotely operated vehicles (ROVs) [9,10].

Towing tank measurements made at the acceleration stage are often overlooked in conventional tests, but may serve as a valuable source of information. Recently, it was shown theoretically that information on the dependence of the hull resistance on the vessel's speed and the hydrodynamic added mass can be obtained from only one acceleration stage towing test if it is conducted up to the maximal speed [11]. In this work, we aim to demonstrate this concept experimentally.

In the context of previous studies, the main contribution of this paper is to demonstrate experimentally that towing tests at the acceleration stage can reproduce the full dynamics, including under braking and gliding conditions. It is therefore possible to model the characteristics of the total hull resistance force from acceleration stage towing tests. Moreover, we show that such tests provide valuable information about the hydrodynamic added mass needed to determine the full dynamics for a vessel.

PRINCIPLES OF TOWING TANK TESTS AT THE ACCELERATION STAGE

To explain the concept of retrieving a dynamical model from acceleration stage towing tests, we start with the equation of motion:

$$(m_v + m_{add})v' = F_p(v, v') - R_T(v) \quad (1)$$

where m_v represents the vessel's mass, and the parameter representing the hydrodynamic added mass of the water is

denoted as m_{add} , v' denotes the speed derivative over time, $F_p(v, v')$ is the propulsion force for a full-scale vessel (or the towing force in the case of a model vessel), and R_T is the total hull resistance force.

The hydrodynamic added mass represents significant inertia, and is usually equal to 10–15% of the vessel's mass. Thus, the energy required to accelerate the hydrodynamic added mass also needs to be considered when performing a seakeeping analysis for different motion profiles. In 1960, Motora was the first to conduct model testing to predict the added mass for a ship called *Mariner* [12]. Ghassemi [13] carried out a numerical calculation of the added mass for a marine propeller using the boundary element method. Bidikli [14] provided a tracking controller formulation for dynamically positioned surface vessels that took into consideration the effects of added mass. Zeraatgar [15] investigated the surge added mass of planing hulls through model experiments, and approximated it with a quasi-analytical method.

When conducting a towing test at the acceleration stage [11], the parameter representing the surge added mass can be obtained from the equations of motion in Eq. (1) by extrapolating the towing force to zero speed $F_p(0, v')$, i.e.,

$$(m_v + m_{add})v' = F_p(0, v') - R_T(0) \quad (2)$$

The total hull resistance is then zero, $R_T(0) = 0$, and it follows that:

$$m_{add} = \frac{F_p(0, v')}{v'} - m_v \quad (3)$$

In the case of constant speed, i.e., $v' = 0$, the towing force is equal in magnitude to the total hull resistance, and the second law takes the following form:

$$F_p(v, 0) = R_T(v) \quad (4)$$

Rewriting Eq. (1), we get:

$$F_p(v, v') = F_p(v, 0) + (m_v + m_{add})v' \quad (5)$$

Eq. (5) indicates that acceleration stage towing tests can provide information on both the total hull resistance at constant speed $F_p(v, 0)$ and the parameter representing the hydrodynamic added mass m_{add} .

EXPERIMENTAL SETUP

FULL-SCALE VESSEL VS. MODEL-SCALE VESSEL

The full-scale vessel is a prototype of a small inland passenger shuttle ferry on the Motława River in the city of Gdansk. It is characterised by a modern design and a hybrid propulsion system, consisting of two azimuth propellers driven by electric motors, which are powered by batteries and photovoltaic panels [8]. The main source of energy is charging from the grid. The full-scale shuttle ferry and a propulsion and power supply diagram are shown in Fig. 1. The proposed modular power supply system is installed inside the hull, allowing for optimal weight distribution within the ship.

Tests were conducted on a $\lambda = 10$ scale laminate model, as shown in Fig. 2. The main dimensions of both vessels are given in Table 1.

TOWING EXPERIMENT

Experimental tests were carried out in calm water conditions in the towing tank of the Institute of Naval Architecture at Gdańsk University of Technology. Table 2 lists the tank's water conditions and the assumptions made for the full-scale vessel. Of particular interest are the speeds of the full-scale vessel, which reach 10 km/h, and scale to the model at 0.88 m/s. The model of mass $m_v = 22.5$ kg was accelerated with $a = 0.134$ m/s².

Tab. 1. Main dimensions of the model and the full-scale ship

	Full-scale model	Model at 1:10 scale
LOA – ship's length [m]	12	1.2
B – ship's breadth [m]	5	0.5
LWL – ship's waterline length [m]	10.47	1.047
T – ship's draught [m]	0.93	0.093
V – displacement volume [m ³]	23.12	0.0231
Aw – wetted area [m ²]	50.53	0.5053



Fig. 2. Model with protruding parts (top), and during the tests (bottom)

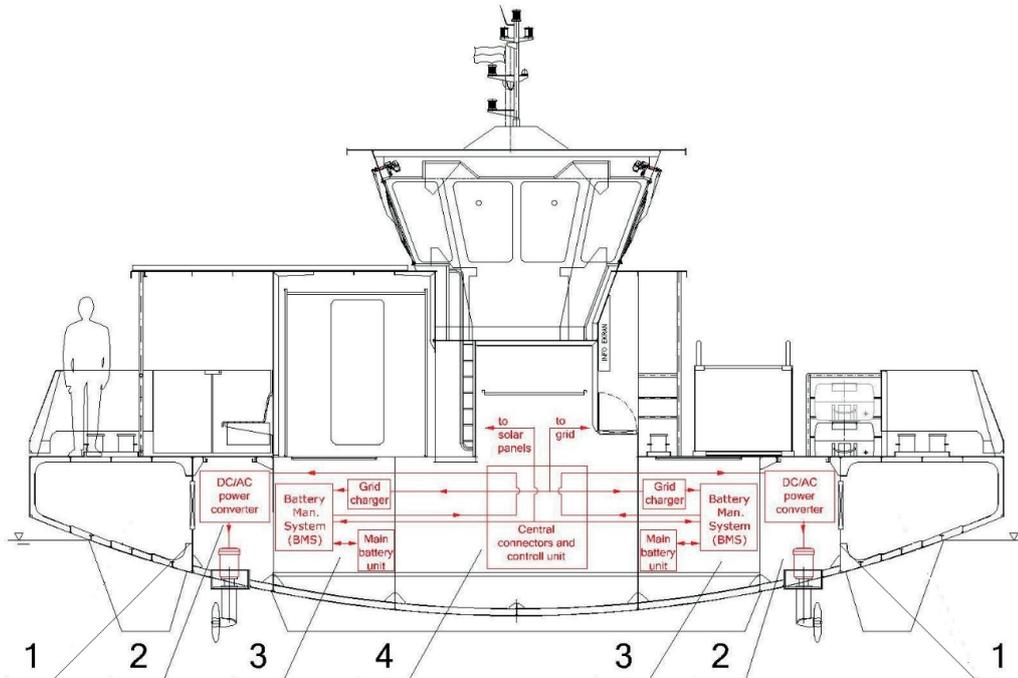


Fig. 1. Propulsion and power supply system: 1 – collision bulkhead; 2 – propulsion compartment, 3 – batteries, 4 – main switchboard

Tab. 2. Water conditions

	Full-scale model	Model at 1:10 scale
Temperature	15	19
Density	999.1	998.4
Kinematic viscosity	1.139	1.028

Conventional resistance tests involve attaching the tested model to a towing platform, where the desired speed at which the model moves is set. In general, only a constant speed is analysed in a conventional test, since the lack of acceleration allows for simple calculations of the towing force as equal to the drag force generated on the hull. However, recording of the acceleration and deceleration stages during this test is also possible. In this study, we model the towing force function in the domains of time and speed by obtaining the speed and towing force signals. The measurement results are sent to a computer from the towing platform using a data acquisition set. Measurements are controlled and processed with a special program that allows us to record the results, set the sampling frequency, apply filtering, and continuously view all measurement channels. The AMTI MC3A dynamometer enables the measurement of the towing force through a replaceable strain gauge force transducer with different ranges. For our tests, a range of 0–100 N was used.

GLIDING EXPERIMENT

As part of our research into the gliding stage, a special structure was built that utilised the force of inertia for movement. Since the ferry under test moves along an unusual route (short shuttle movement), this stage has a large impact on the total energy consumption of the propulsion system. The tests consisted of mechanically accelerating the vessel to selected speeds with the propulsion system switched off, where the speed and the distance of the freely decelerating hull were recorded. The only force acting during the test was the drag force generated on the hull. Gliding forms a kind of natural braking process; for vessels moving along short routes, utilising this stage allows a significant part of the distance to be covered without using any fuel.

Data preparation

During the calm water towing tank tests, data for the towing force and the model vessel speed were gathered. The sampling time interval for both signals was 0.002 s.

Tab. 3. Statistics on the resistance measurements for a constant speed of before and after filtering for data samples.

	Standard deviation [N]	Mean [N]	Mean deviation [N]
Raw data	4.00	2.83	3.06
After filtering	0.17	2.83	0.14
After calibration	0.17	3	0.14

The towing force signal was filtered with a 3 Hz cut-off frequency low-pass filter for data preparation, and calibrated

to 0 N at the starting time. Table 3 shows the towing force signal statistics for the constant speed stage, before and after filtering. These statistics were derived from 5000 data samples for the constant speed stage at $v = 0.88$ m/s. The table shows equal mean values before and after filtering.

The speed signal was first passed through an anomaly-type filter, and was then smoothed with a low-pass filter with a cut-off frequency of 0.1 Hz. This step was important for retrieving the acceleration of the model vessel, which was needed for the mathematical model of the towing force. An example of the results of smoothing the speed signal is shown in Figure 3. This towing test was conducted with an acceleration of $a = 0.134$ m/s² up to a maximal speed of 0.7 m/s.

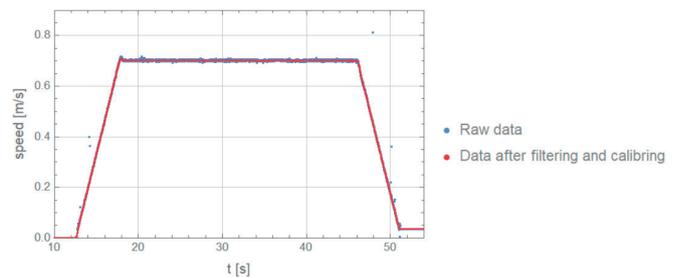


Fig. 3. Smoothing of the speed signal: raw data (blue), after filtering (red) (filtering is needed to retrieve the dynamic model)

RETRIEVING THE TOTAL HULL RESISTANCE OF THE MODEL VESSEL FROM THE ACCELERATION TOWING STAGE

In this section, we demonstrate how to retrieve the total hull resistance from acceleration stage calm water towing tank tests. In conventional towing tank experiments, the acceleration stage of the test is often neglected. In our case, data from the acceleration stage will be used to model the full dynamics. There are prerequisites that suggest that the total hull resistance may be measured more accurately from the acceleration towing stage; one of the reasons for this is that the jump in the dynamometer measurement may be underestimated for constant speed measurements. Another advantage of this method is that the hydrodynamic added mass of water can also be calculated using data from the acceleration stage.

In the mathematical model of the total hull resistance $R_T(v)$, the following function is used:

$$R_T(v) = \left(b + \frac{c}{v}\right) v^2 \quad (7)$$

where b and c are parameters to be estimated. The main motivating factors behind the choice of this function are its good alignment with experimental data and its simple form, which allows for analytical solutions to the differential

equations. A towing force $F_p(v, v')$ is needed to accelerate the vessel and to overcome the resistance $R_T(v)$. The following formula is therefore used to model the towing force from the independent speed signal:

$$F_p(v, v') = \left(b + \frac{c}{v}\right)v^2 + (m_v + m_{add})v' \quad (8)$$

To fit the model of the towing force in Eq. (8), only constant acceleration data for a value of $a = 0.134 \text{ m/s}^2$ are used, and the first high spike is excluded (see Fig. 4, which shows a jerk at the starting point). The oscillations of the towing force in the experimental data are due to the cut-off frequency filter, and are not modelled.

Fig. 4 shows experimental data from three towing tests that were carried out at different maximal speeds, and the results from our fitted model. The fitted total hull resistance model parameters are $b = 4.81$, $c = -0.28$, and the vessel's mass is $m_v = 22.5 \text{ kg}$. The added mass parameter was obtained by extrapolating the towing force to zero speed $F_p(0, v')$, where the total hull resistance is $R_T(0)=0$. Next, using Eq. (3), the added mass parameter was calculated as $m_{add} = 3.0 \text{ kg}$, representing 13% of the vessel's mass. Table 4 shows the standard errors, t-statistics, and P-values for the estimates from our mathematical model.

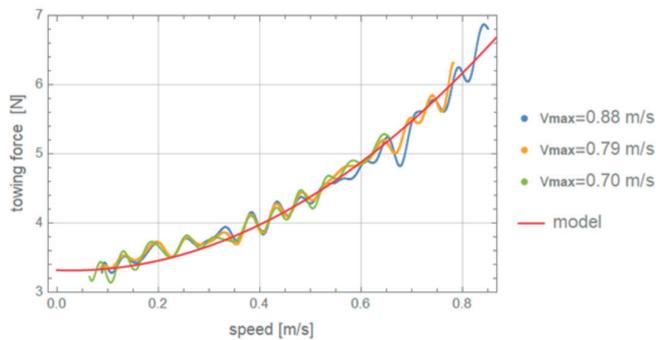


Fig. 4. Modelling dependence of the towing force on the vessel's speed in the acceleration stage, for : experimental data (blue, orange and green), and results from the proposed mathematical model (red)

Tab. 4. Estimates, standard errors, t-statistics and P-values for a towing force model function from the acceleration stage

	Estimate	Standard error	t-statistic	P-value
$(m_v + m_{add})a$	3.42	0.02	171.7	0.00
b	4.81	0.20	25.36	0.00
c	-0.28	0.19	-2.68	0.01

VALIDATION OF THE PROPOSED METHOD

ALL STAGES OF THE TOWING TANK EXPERIMENT

In this section, we validate our mathematical model for the towing force by showing how the model works in other profiles of motion. The formula in Eq. (8) with fitted parameters can be used to model all stages of the towing tests. In particular, for constant speed, the net force acting on the vessel is zero, and the towing force for the model vessel is therefore equal to the total hull resistance force, i.e.,

$$F_p(v, 0) = R_T(v) \quad (9)$$

Fig. 5 shows the experimental data for the towing force (blue) and the results from the mathematical model (red). The modelled signals for the towing force were retrieved from the independent speed signals using the estimates of b and c given in Table 4. The acceleration v' was modelled from the speed signal based on the difference quotient.

We note that the modelled towing force (red) shows lower agreement with the experimental data at the constant speed stage. This discrepancy is not a random type but a biased type, and may arise from decalibration of the dynamometer after rapid shifts between different motion stages. This discrepancy is still within 1% of the dynamometer's maximum range, but when compared with the experimental data, the mathematical model gives results that are 17% higher for a speed 0.88 m/s. Our findings from this study indicate that the towing data at the acceleration stage were better than data from the constant speed stages when used to retrieve the independent gliding experiment, as shown in the next section.

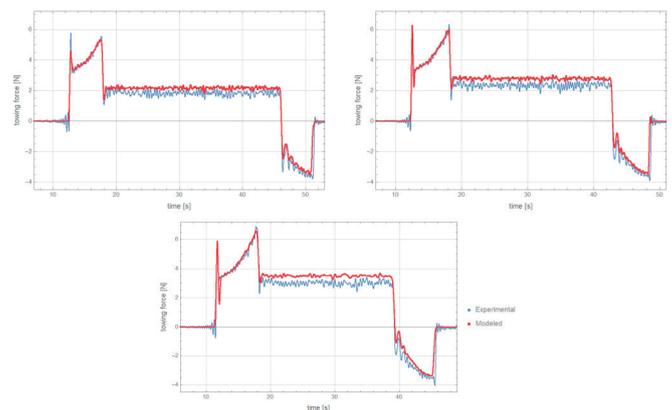


Fig. 5. Experimental data (blue), modelled data (red) for acceleration and maximal speeds m/s (left), (right), m/s (bottom)

GLIDING EXPERIMENT

For further validation, we demonstrate how the hull resistance model in Eq. (7) and the hydrodynamic added mass parameter, fitted from the towing test data at the acceleration stage, can be used to retrieve independent gliding experiments. At the gliding stage, the towing force of the model vessel is zero, giving the following equation of motion:

$$m \frac{dv(t)}{dt} = -R_T(v) \quad (10)$$

where $m = m_v + m_{add}$ denotes the total mass, i.e., the mass of the vessel and the hydrodynamic added mass. From solving the differential equation in Eq. (10) for the total hull resistance fitting function in Eq. (7), the following solution is obtained:

$$v(t) = c \left(\left(\frac{c}{v_0} + b \right) \exp \left(\frac{c}{m} (t - t_0) \right) - b \right)^{-1} \quad (11)$$

where t_0 and v_0 are the initial time and speed, respectively. The equations of motion that give us the displacement $s(t)$ can then be obtained by integrating Eq. (11), i.e.,

$$s(t) = \int_{t_0}^t v(t) dt = \frac{m \ln \left(b v_0 - (b v_0 + c) \exp \left(\frac{c(t-t_0)}{m} \right) \right) - m \ln(-c) - c(t-t_0)}{b} \quad (12)$$

Fig. 6 shows plots of the developed mathematical model (red) with the experimental data (blue) for three vessel model gliding experiments. Table 5 presents the values of the parameters b and c , together with the estimate needed to calculate the added water mass. The parameters were fitted from independent acceleration stage towing tests. The mathematical model in Eq. (11) demonstrates good agreement with the gliding experimental data for model vessel speeds down to 0.3 m/s.

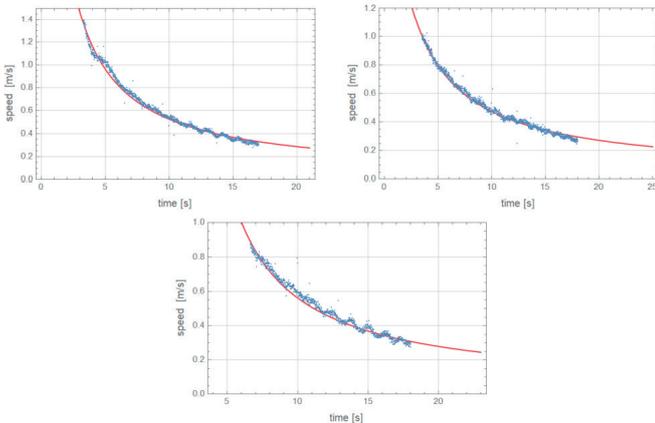


Fig. 6. Results from the proposed mathematical model (red) and experimental data (blue) for three gliding experiments with the model vessel, showing good agreement between the mathematical model retrieved from the acceleration stage towing test and the experimental data

SCALING TO FULL-SIZE VESSEL

SCALING PROCEDURE FOR THE ACCELERATION STAGE TOWING TESTS

In this section, we show how to retrieve the full-scale dynamics from acceleration stage towing tests. Most of the scaling assumptions for the accelerated motion are the same as for a conventional constant speed [16-19], which is commonly known as the form factor $(1+k)$ approach. The ITTC-1978, in its power prediction procedure for deriving the viscous coefficient needed for the scaling procedure, recommends the use of skin friction line based on flat plate results:

$$C_F(Re) = \frac{0.075}{(\log_{10} Re - 2)^2} \quad (13)$$

together with the form factor $(1+k)$. Then, the dimensionless viscous coefficient can be calculated as:

$$C_V(Re) = (1+k)C_F(Re) \quad (14)$$

The precision of 19th-century measuring apparatus was limited compared to today's devices; modern advancements now enable highly accurate measurements to be made with high sampling rates, which has improved acceleration stage towing test data. In this study, we adopt a scaling proposition based on acceleration stage data presented by Wrzask [11]. This scaling procedure relies on the second law of dynamics, which is valid for vessels with any motion profile, and on Froude's geometric similarity. It is also assumed that when the accelerating vessel moves a specific volume of the surrounding water, this volume scales based on geometric similarity.

The propulsion force for the full-scale vessel F_{pS} can be obtained from the acceleration stage data from towing experiments on the scaled model using the formula:

$$F_{pS}(v_S, v'_S) = \lambda^3 \frac{\rho_S}{\rho_M} F_{pM}(v_M, v'_M) + \lambda^3 \left(\lambda^{0.5} - \frac{\rho_S}{\rho_M} \right) m_M v'_M + \lambda^3 \frac{\rho_S}{\rho_M} (\lambda^{0.5} - 1) m_{addM} v'_M + \lambda^3 \frac{\rho_S \mu_M v_M^2}{2} \left(C_V \left(\lambda^{1.5} \frac{\mu_M \rho_S}{\mu_S \rho_M} R_{eM} \right) - C_V(R_{eM}) \right) \quad (15)$$

Here, the lower index S refers to the full-scale vessel, while the index M refers to the model. λ is the scale, μ is the dynamic viscosity, and ρ is the density of water. The ITTC recommendations used to calculate the C_V coefficient can be also used for the proposed scaling procedure from the acceleration stage towing tests.

FULL-SCALE DYNAMICS IN CALM WATER

For the scaling procedure, we took the data on the towing force $F_{pM}(v_M, v'_M)$ from the acceleration stage up to a maximal speed of 0.88 m/s. For the form factor, we used a value of $(1+k) = 1.364$, as calculated in previous studies of the model

vessel [7]. The data were scaled using Eq.(15). The results of this scaling process, i.e., the propulsion force for the full-scale vessel $F_{ps}(v_M, v'_M)$, are shown in Fig. 7 in blue.

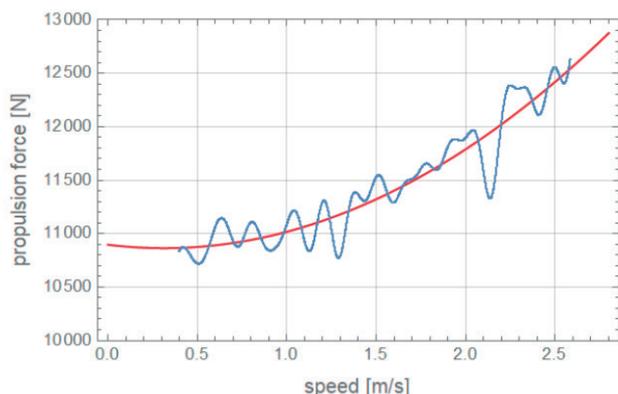


Fig. 7. Mathematical model of the propulsion force for a full-scale vessel (red) based on scaled towing force data (blue) at the acceleration stage

In the next step, the propulsion force for the full-scale vessel using the mathematical model in Eq. (8) is fitted. All the estimates are presented in Table 5. The mass of the full-scale vessel is $m_{vs} = 22500$ kg, and the hydrodynamic added mass is $m_{addS} = 3002$ kg.

Tab. 5. Estimates, standard errors, t-statistics and P-values for the acceleration stage of a full-scale vessel

	Estimate	Standard error	t-Statistic	P-value
$(m_v + m_{add})a_s$	10896.6	19.2	568	0.00
b	325.6	9.5	34.8	0.00
c	-204.5	28.5	-8.2	0.00

To illustrate the application of these results, we consider an example of a full-scale vessel in the gliding profile, starting at a speed of 10 km/h. Using Eqs. (11) and (12), we can calculate that in calm water, the vessel will cover a distance of 55 m in 26 s, decelerating to 6 km/h. This distance represents a substantial portion of the total distance travelled, which, in the case of the ferry considered here, is 100 m.

CONCLUSION

This paper has presented an evaluation of a dynamic model of a small inland ferry in calm water with a novel towing tank method, based on acceleration stage data. Experimental data obtained from the acceleration stage were found to provide valuable information about the dependence of the total hull resistance force on speed, as well as insights into the hydrodynamic added mass of the vessel. To validate our method, a simple mathematical model for the resistance force was used to obtain a model for the gliding profile of motion. The mathematical model obtained in this way exhibited very good agreement with data from independent gliding experiments, thus verifying the efficacy of the method. Our approach is

promising, especially in regard to retrieving motion models based on different operational profiles. Furthermore, this study has demonstrated that vessels operating along short routes can effectively cover a significant proportion of the distance using a gliding profile of motion, without consuming additional energy. This dynamical modelling method could therefore have a profound impact on reducing energy demand.

However, future research on other model vessels is needed to assess the performance of our method and to improve its accuracy. It would also be interesting to investigate the optimisation of the power required based on different operational profiles, and to assess the potential savings.

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