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FORTY-FIVE YEARS OF THE RIGID FINITE ELEMENT METHOD

In memory of Professor Jan Kruszewski-Majewski (1929-2012)

The Rigid Finite Element Method (RFEM) is an original Polish method for modelling dynamics of complex mechanical systems with flexible elements. Its origin dates back to the sixties of the last century [8]. This was also the time when the Finite Element Method (FEM) [42] was developed. The main idea of the static and dynamic analysis of mechanical systems in this method consists in dividing the system into many deformable small elements. Then the elements are connected in such a way that the continuity of displacement filed and its derivative is preserved.

The idea of the Rigid Finite Element Method, inseparably connected with Professor Jan Kruszewski-Majewski from Gdańsk University of Technology, is different. Professor Kruszewski proposed simple (in interpretation and description) division of continuous systems into a finite number of rigid finite elements (rfe) which reflected mass (inertial) features of the discretised system connected by means of massless and non-dimensional spring-damping elements (sde) reflecting elastic and damping features. At the beginning the RFEM was used by Prof. Kruszewski and his co-workers mainly for modelling vibrations about the static equilibrium. The first monograph about the method [9] was published in 1975 and was concerned with systems with constant (not changing in time) configuration. The RFEM has been also used together with the FEM in a hybrid model of rigid and elastic finite elements, which combined advantages of both approaches [10, 24, 25].

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The RFEM has some common features with the lumped mass method, but it should be underlined that it accurately reflects inertial features of the system. In the literature, one can find formulations for multibody system dynamics similar to the RFEM, called the finite segment method.

A generalization of the RFEM for planar systems with changing configurations is presented by Wittbrodt in [26]. This monograph describes formulations for applications of the classic finite element method to modelling systems with changing configuration, as well as a hybrid method of rigid and elastic finite elements. In this formulation flexible links are divided into rfes with three degrees of freedom (planar systems – two translations and one rotation).

Monographs [11, 12, 13] contain a summary of research on the method carried out by the team of Professor Kruszewski.

Wojciech, in his monograph [33] published in 1984, presented a modification of the RFEM used for modelling of slender links of planar systems with changing configuration. The main idea of the modification is that the motion of each rfe is described in relation to its predecessor by only one generalized coordinate (rotation angle). This leads to a simpler model, in which translation and shear elasticities are not taken into account. Analysis of link mechanisms with consideration of clearance and friction in joints together with the bending vibrations is also presented.

Since the nineties of the previous century, the RFEM has been used in Bielsko and Gdańsk for modelling of working machines [28, 30], robot manipulators [1, 2, 3, 6, 17, 34, 36], textile machines [16, 35], vehicles [22, 29], band saws [37, 38] and satellite antennas [32]. These papers are connected with the generalization of the method to spatial systems with changing configuration. Joint coordinates and homogenous transformations used in robotics are proposed in order to describe the motion of rfes into which the flexible link is divided. In the analysis of spatial systems rfes have six degrees of freedom in the classical formulation, or three degrees of freedom in the modification limited to bending and torsional vibrations. Monograph [27], published in 2006 by an international publisher, is the recapitulation of this stage of the development of the RFEM.

Kaliński has used the RFEM in modelling dynamics of machine-tool systems with a coupling element for description of a machining process [7]. Modelling and dynamic analysis of machine-tools by means of the RFEM has been carried out also at other research centres in Poland, for example in Szczecin University of Technology by teams of Professors Marchelek and Berczyński. It has also been applied to dynamic analysis of systems, allowing one to take into consideration control vibrations with square quality coefficient [18], and to supervise the process of vibration at energetic quality indicator [7].

The following years are marked by the use of the RFEM for modelling offshore devices. This research area seems to be potentially one of the most challenging. Modelling of large deformations of lines, cables and risers requires consideration of not only large deflections and changing configurations caused by sea waves, but also nonlinear physical relations such as permanent deformations of pipes in the process of pipe lying by means of the reel method. The RFEM has been used for modelling dynamics of cranes mounted on vessels and drilling platforms [14, 15] as well as for modelling the process of laying pipes used for transportation of gas and petroleum from sea beds [19, 20, 21, 23]. Monograph [31] presents the most important applications of the RFEM in offshore engineering.

In 2006-2010 the RFEM was also applied to modelling vibrations of collecting plates of electrostatic precipitators, which is connected with the generalization of the method for plates and shells. Research in this area is concerned mainly with the hybrid method combining the rigid and elastic elements [4, 5]. The idea is that the elastic features of plates and shells are modelled by means of the FEM, while inertial features are reflected using RFEM. Relations enabling transformations from FEM coordinates into RFEM coordinates, which are generalized coordinates of the system, are formulated. Experimental measurements have proved the correctness and efficiency of such an approach.

During the last few years the RFEM has also been successfully used for modelling and dynamic analysis of biomechanical systems, especially for modelling the forearm-shoulder system [39, 40, 41].

This issue of The Archive of Mechanical Engineering contains articles illustrating the recent research on the development of the method carried out at Gdańsk University of Technology and Bielsko-Biała University. We hope that the Rigid Finite Element Method will continue to be developed by our younger colleagues, who have cooperated with us for many years: K. Kaliński, R. Hein, W. Wojnicz, M. Galewski and K. Lipiński from Gdańsk University of Technology, as well as I. Adamiec-Wójcik, A. Maczyński and M. Szczotka from Bielsko-Biała University. The method deserves further development. Many times the results have been verified by experimental measurements, and good compatibility with numerical results has been achieved [3, 9, 17, 33]. Good agreement has also been achieved when the results obtained by the RFEM were compared with those obtained from professional packages based on the FEM [20].

The RFEM should not be treated as a method which competes with the finite element method, but which complements it, as is best shown in the proposed hybrid model, when both approaches are simultaneously used. Formulating models, algorithms and computer programs on the basis of the RFEM is advisable when the use of commercial packages may be difficult due to the specific nature of the modelled systems and phenomena, or when programs for a specific group of devices are required. Such is the case in offshore engineering, where almost each device is designed and produced individually.

This issue also contains a biographical article commemorating Professor Jan Kruszewski-Majewski, who died last year, and who was the initiator and author of the Rigid Finite Element Method.

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