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Gender approaches to evolutionary multi-objective optimization using pre-selection of criteria

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ABSTRACT

A novel idea to perform evolutionary computations (ECs) for solving highly dimensional multi-objective optimization (MOO) problems is proposed. Following the general idea of evolution, it is proposed that information about gender is used to distinguish between various groups of objectives and identify the (aggregate) nature of optimality of individuals (solutions). This identification is drawn out of the fitness of individuals and applied during parental crossover in the processes of evolutionary multi-objective optimization (EMOO). The article introduces the principles of the genetic-gender approach (GGA) and virtual gender approach (VGA), which are not just evolutionary techniques, but constitute a completely new rule (philosophy) for use in solving MOO tasks. The proposed approaches are validated against principal representatives of the EMOO algorithms of the state of the art in solving benchmark problems in the light of recognized EC performance criteria. The research shows the superiority of the gender approach in terms of effectiveness, reliability, transparency, intelligibility and MOO problem simplification, resulting in the great usefulness and practicability of GGA and VGA. Moreover, an important feature of GGA and VGA is that they alleviate the 'curse' of dimensionality typical of many engineering designs.

KEYWORDS

Genetic algorithms; evolutionary learning; multi-objectives; Pareto optimality; engineering applications

1. Introduction

Optimal systems can be designed by following evolutionary mechanisms from nature, which allow the elimination of detrimental features and the inheritance, or development, of desirable features in the course of genetic computations. An important determinant for the use of evolutionary algorithms in learning and optimization is the experience acquired in universal realizations of iterative procedures of stochastic exploration. Elaborate evolutionary algorithms simulate the natural laws connected with inheritance, crossover and mutation. They make highly efficient tools for gaining optimal solutions of a practical nature. However, the approach proposed here is not just another evolutionary technique, but a completely new philosophy of posing and solving multi-objective optimization (MOO) tasks. Note that this article will not discuss all of the possible implemental distinctions that can be found between genetic algorithms (GAs) and evolutionary computations (ECs) (Goldberg 1989; Michalewicz 1996).

Evolutionary algorithms (Holland 1975; Goldberg 1989; Michalewicz 1996; Man *et al*. 1997) have a large number of applications (Holland 1975; Goldberg 1989; Michalewicz 1996; Man *et al*. 1997). The significance of such optimization methods emulating the evolution of biological systems is proved by their usefulness and effectiveness. Features of biological systems include their ability to regenerate, perform self-control, reproduce and adapt to the variable conditions of existence. On a similar basis, analogous features are required to characterize technical systems designed in terms of adaptation, optimality, immunity, *etc*. Thus, tasks can be formulated concerning the optimality of solutions and their robustness to small changes in environmental conditions and parameters, and to disturbances that allow effective and reliable engineering systems to be obtained.

In such decision-making and design processes it is essential to globally optimize several objectives at the same time (Goldberg 1989; Michalewicz 1996; Chen, Patton, and Liu 1996; Viennet, Fontiex, and Marc 1996; Man *et al*. 1997; Kowalczuk, Suchomski, and Białaszewski 1999). Such MOO tasks are, however, difficult to perform, as the notion of optimality is not obvious.

To join a number of objectives together, it is necessary to define relations between the partial objectives being considered, which can be done by setting suitable weights. Various methods have been proposed to solve such problems of optimality (Michalewicz 1996), including (1) weighted profits, (2) distance functions, (3) sequential inequalities (Zakian and Al-Naib 1973), (4) lexicographic ordering (Coello, Lamont, and Van Veldhuizen 2007) and (5) ranking with the use of Pareto optimality (Goldberg 1989; Srinivas and Deb 1994; Man *et al*. 1997; Kowalczuk, Suchomski, and Białaszewski 1999; Kowalczuk and Białaszewski 2006a, 2006b, 2006c; Białaszewski and Kowalczuk 2016). The substance of the first three methods lies in the direct integration of many objectives into one criterion, which is submitted to optimization using an arbitrary choice of weighting vector, demand vector or limit values for partial objective functions. Such choices are not straightforward, and they also restrict and simplify the MOO problem. The fourth method, of lexicographic ordering, relies on optimization with respect to each objective function in a sequence, starting with the most important one and proceeding according to the assigned order of importance of partial objective functions (Coello, Lamont, and Van Veldhuizen 2007).

In contrast, the fifth method, using ranking with respect to a measure of Pareto optimality, avoids the arbitrary weighting of objectives. Instead, a constructive classification of solutions is applied that takes particular goals into account more objectively. Although (on a common basis) this idea of optimality does not give any hints as to the choice of a single solution from a generated set of Pareto-optimal solutions (lying on the same Pareto-optimal front), the designer always has a chance to make an independent judgement of all the 'best' offers. The above-mentioned methods of qualifying the multi-objective solutions can be easily utilized in GAs (Goldberg 1989; Michalewicz 1996).

There are two basic reasons for and consequences of the evolution of gender in nature:

- in the long term: in the search for new mutations, beneficial improvements and adaptation
- in the short term: for genetic variation, significant in terms of resistance to parasites, bacteria and viruses.

In the above context, this article presents a new method, referred to as the genetic-gender approach (GGA), which was initiated in Kowalczuk and Białaszewski (2001). This method solves MOO problems by an evolutionary search with Pareto-optimal ranking, where information about the degree of membership to a given gender is attributed to each newly generated solution under examination. This information is used in the process of parental crossover, in which only individuals of different genders are allowed to create offspring. Another gender-related mechanism, called the virtual gender approach (VGA), concerns solely a specific method of fitness assessment of the generated individuals (Kowalczuk and Białaszewski 2006a; Białaszewski and Kowalczuk 2016).

To study the effectiveness of the gender approach, synthetic examples of the application of the proposed approaches and some known competitors are considered in computational experiments on popular multi-objective benchmark problems (Zitzler and Thiele 1999). These take into account binary and real mechanizations without elitism, as well as native implementations, and binary and real representations using elitism. Averaging and median statistics are compared, and the issues of true Pareto fronts and computation time are illustrated.

Strong points of the approach analysed and proved in this article are summarized in Section 5. The weak points of previous studies concern their inefficiency, uncertainty, ambiguity, incomprehensibility and high complexity, which generally result in practical limitations on the application and implementation of highly dimensional evolutionary multi-objective optimization (EMOO) tasks.

2. Evolutionary solutions to the multi-objective optimization problem

There are many forms of life that have resulted from natural evolution. On the basis of the existing variety of life, it can be inferred that each species is optimal with respect to a certain subset of survival criteria.

An analogy can also be found within different products of human activity and productivity. There are various goods and their variants, many kinds of constructions, bridges, buildings, automotive vehicles, aircraft, home appliances and diverse equipment. Usually, a specific set of technical criteria is defined, with respect to which a given product should be optimal, and this simplifies the choice from among all the 'equivalent' solutions. Frequently, trade-offs are made between price, reliability and safety. Thus, considering equally optimal solutions and making final decisions are part of human nature. Furthermore, based on a given (non-weighted) set of criteria, one is often stuck with a selection of solutions that are merely 'mutually non-inferior' (Kowalczuk and Białaszewski 2006a).

From a formal viewpoint, a MOO task (Goldberg 1989; Michalewicz 1996; Viennet, Fontiex, and Marc 1996) can be defined by means of the following m-dimensional vector of objective functions:

$$\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}) \quad f_2(\boldsymbol{x}) \quad \dots \quad f_m(\boldsymbol{x})]^{\mathrm{T}} \in \mathbf{R}^m \tag{1}$$

where $\boldsymbol{x} = [x_1 \quad x_2 \quad \dots \quad x_n]^{\mathrm{T}} \in \mathbf{R}^n$ means a vector of the parameters searched for, and $f_j(\boldsymbol{x})$, $j = 1, 2, \dots, m$, denotes a certain partial objective function. Assuming that all coordinates of the criterion vector (1) are profit functions, the MOO task analysed can be formulated as a multi-profit maximization task without constraints:

$$\max_{\mathbf{x}} \boldsymbol{f}(\boldsymbol{x}) \tag{2}$$

In recent decades, a great number of multi-objective genetic algorithms (MOGAs), also referred to as EMOO methods (Schaffer 1985; Goldberg 1989; Hajela and Lin 1992; Horn, Nafpliotis, and Goldberg 1994; Srinivas and Deb 1994; Viennet, Fontiex, and Marc 1996; Zitzler and Thiele 1999; Cotta and Schaefer 2004; Korbicz *et al.* 2004; Deb, Mohan, and Mishra 2005; Deb and Gupta 2006; Deb 2007; Coello, Lamont, and Van Veldhuizen 2007; Bader and Zitzler 2009; Zitzler, Thiele, and Bader 2010; Liu *et al.* 2010; Zhang and Li 2007; Kukkonen and Lampinen 2005; Emmerich, Beume, and Naujoks 2005; Yazdi 2016), has been proposed for solving multi-objective problems in multi-dimensional spaces, including the vector-evaluated genetic algorithm (VEGA), lexicographic ordering genetic algorithm (LOGA), vector-optimized evolution strategy (VOES), Hajela-Lin's genetic algorithm (HLGA), MOGA, niched Pareto genetic algorithm (NPGA and NPGA2), non-dominated sorting genetic algorithm (NSGA and NSGA2), distance-based Pareto genetic algorithm (DPGA), thermo-dynamical genetic algorithm (TDGA), multi-objective messy genetic algorithm (MOMGA), Pareto-archived evolutionary strategy (PAES), Pareto envelope-based selection algorithm (PESA and PESA2), strength Pareto evolutionary algorithm (SPEA and SPEA2), micro-genetic algorithm (μGA and μGA^2), multi-objective Bayesian optimization algorithm (MOBOA), S-metric selection evolutionary multi-objective algorithm (SMS-EMOA), harmony search–multi-objective evolutionary algorithm based on decomposition (HS-MOEA/D) and, finally, the object of this presentation, the GGA and VGA. A summary of the EMOO algorithms is shown in Table 1.

Applying a GA to MOO problems using the Pareto approach consists of selecting all the Pareto-optimal solutions to a common parental pool, which is then submitted to the selection, crossover and

Table 1. Principal characteristics of the most common and distinguished multi-objective evolutionary algorithms.

	Feature							
Algorithm	Representation	Population	Assessment	Parental selection	Crossover	Mutation	Elitism	Diversity protection
DPGA	Any	Two	Distance	Tournament	Yes	Yes	Yes	No
GGA	Any	One	Pareto ranking	Stochastic remainder	Yes (mating restriction)	Yes	No	Genetic gender
HLGA	Any	One	Weighted profits	Proportional	Yes (with mating restriction)	Yes	No	Niching
LOGA	Tree	One	Lexicographic	Tournament	Yes	Yes	Yes	No
MOBOA	Tree	Two	Non-domination	$\in$-archive selection	No	Yes	Yes	Niching
MOGA	Any	One	Pareto ranking	Stochastic sampling	Single point	Yes	No	Niching in the objective space
MOMGA	Any	Two	Fitness	Tournament threshold	Cut and splice	Cut and splice	Yes	Niching
NPGA	Any	One	Non-domination	Binary tournament	Single point	Yes	No	Decision and objective niching
NPGA2	Any	One	Pareto ranking	Binary tournament	Single point	Yes	No	Decision and objective niching
NSGA/NSGA2	Any	One	Pareto ranking	Binary tournament	Single point	Yes	Yes	Niching/crowding distance
PAES	Any	No	Non-domination	Binary tournament	No	Yes	Yes	Crowding grid
PESA	Binary	Two	Pareto ranking	Tournament	Yes (single child)	Uniform	Yes	Crowding measure
PESA2	Binary	Two	Region based	Region based	Yes (single child)	Uniform	Yes	Crowding measure
SPEA	Binary	Two	Strength value	Binary tournament	Yes	Yes	Yes	Truncated clustering
SPEA2	Any	Two	Strength value	Binary tournament	Yes	Yes	Yes	*k*th nearest neighbour
TDGA	Any	One	Energy fitness	Sorting	Yes	Yes	Yes	No
VEGA	Binary	One	Single objective	Proportional	Yes	Yes	No	No
VOES	Diploidal	Two	Fitness	Sorting	Yes	Yes	No	Niching
μGA/μGA^2	Binary/any	Two	Pareto ranking	Binary tournament	Two-point and hybrid crossover	Yes	Yes	Crowding grid
MOEA/D-DE	Real	One	Distance function	Tournament	Differential crossover	Yes	Yes	No
GDE3	Real	One	Non-domination	Binary tournament	Differential crossover	Yes	Yes	Crowding measure
SMS-EMOA	Real	One	Non-domination	S-metric sorting	Differential crossover	Yes	Yes	Hypervolume contribution
VGA	Any	One	Hierarchical Pareto	Stochastic remainder	Yes	Yes	No	No
MSGA	Binary	One	Fitness	Proportional	Yes (mating restriction)	Yes	No	Stochastic gender

Note: DPGA = distance-based Pareto genetic algorithm; GGA = genetic-gender approach; HLGA = Hajela-Lin's genetic algorithm; LOGA = lexicographic ordering genetic algorithm; MOBOA = multi-objective Bayesian optimization algorithm; MOGA = multi-objective genetic algorithm; MOMGA = multi-objective messy genetic algorithm; NPGA = niched Pareto genetic algorithm; NSGA = non-dominated sorting genetic algorithm; PAES = Pareto-archived evolutionary strategy; PESA = Pareto envelope-based selection algorithm; SPEA = strength Pareto evolutionary algorithm; TDGA = thermo-dynamical genetic algorithm; VEGA = vector-evaluated genetic algorithm; VOES = vector-optimized evolution strategy; μGA = micro-genetic algorithm; MOEA/D-DE = multi-objective evolutionary algorithm based on decomposition–differential evolution; GDE = generalized differential evolution; SMS-EMOA = S-metric selection evolutionary multi-objective algorithm; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm.

mutation operations. A typical cycle of a GA using a stochastic remainder choice (SRC), applied for a multi-objective genetic algorithm (MOGA), is briefly presented as Procedure 1.

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Procedure 1 Multi-objective Genetic Algorithm (MOGA) - a sketch
Generate randomly an initial population V containing N individuals
    {x_i}_{i=1}^N ;
while t ≤ tmax
    Compute the fitness: x_i → f(x_i)
    Assign ranks according to the fitness: x_i → r(x_i) (see Equations (3)
        and-(4))
    Select the parental pool with the use of SRC
    Create the offspring V′ by making:
        Multi-point crossover
        Binary mutation
    Replace the old population: V ← V′
    Cycle: t ← t+1.
```

Details of the applied genetic operations (selection/SRC, crossover, mutation) are given in the supplementary Appendix A. The evolutionary approaches stem directly from nature, in a similar way to how the entire domain of artificial intelligence does, although the term 'intelligence' is not appropriate in this context.

3. Genetic-gender approaches

Many and various mechanisms for generating new solutions and decision-making processes have been proposed and implemented in genetic and evolutionary algorithms. On the other hand, only a few isolated attempts at applying sexual categories in the genetic reproduction mechanisms are known from the literature, namely, the multi-sexual genetic algorithm (MSGA) (Lis and Eiben 1997), gendered genetic algorithm (G-GA) (Rejeb and AbuElhaija 2000), genetic algorithm with gendered selection (GAGS) (Sanchez-Velazco and Bullinaria 2003a, 2003b), adaptive genetic algorithm simulating human reproduction model (HRAGA) (Yan 2010), genetic algorithm with sexual selection (GASS) (Song Goh, Lim, and Rodrigues 2003), gender separation with genetic algorithm (GSGA) (Vrajitoru 2002) and multi-objective genetic algorithm with sexual selection (MOGASS) (Sodsee *et al.* 2008). Features of these algorithms are presented in Table 2.

In contrast to previous approaches (Lis and Eiben 1997; Rejeb and AbuElhaija 2000; Vrajitoru 2002; Sanchez-Velazco and Bullinaria 2003a, 2003b; Song Goh, Lim, and Rodrigues 2003; Sodsee *et al.* 2008; Yan 2010), this section presents a novel method for solving multi-criteria optimization tasks using the EMOO approach with a genetic-gender concept, which consists of assigning to each individual a specific gender relating to the degree of its membership to certain subpopulations associated with their respective subcriteria. The gendered individuals are submitted to a 'natural' crossover process of mating. To deliver a suitable view of the GGA concept and the resulting algorithm, basic issues and mechanisms of the MOO machinery (including rank estimation and global optimality concepts) are first explained, before the reasoning behind and details of the gender approach are given.

3.1. Pareto-optimal ranking

The mechanism being considered avoids arbitrary weighting of partial objectives. Instead, it uses a classification of the analysed solutions that takes into account particular objectives in a more effective way. The most important representatives of the objectives are ranks relating to Pareto optimality (Schaffer 1985; Goldberg 1989; Horn, Nafpliotis, and Goldberg 1994; Srinivas and Deb 1994; Viennet, Fontiex, and Marc 1996; Man *et al.* 1997; Kowalczuk, Suchomski, and Białaszewski 1999; Deb, Mohan,

Table 2. Characteristics of gender/sexual multi-objective evolutionary algorithms.

	Feature							
Algorithm	Determination of gender	Representation of gender	No. of genders	Assessment of quality/fitness	Selection	Mating restrictions in crossover	Mutation	Elitism
GAGS	Stochastic with parental power	Bit	Two	A sex-modifier of a single criterion	Proportional	Yes	Distinct for each gender	Yes
GASS	Stochastic	Bit	Two	Single criterion	Proportional	Yes	Yes	Yes
GGA	Dynamic, fitness-based	Label/code	Multi	Sub-Pareto-MO ranking	SRC	Yes	Yes	Possible
VGA	(Virtual)	(In hierarchical assessment)	(Multi at each level)	Sub-Pareto-MO ranking	SRC	No	Yes	Possible
G-GA	Stochastic	Bit	Two	Single criterion	Proportional	Yes	Yes	Yes
GSGA	Stochastic	Bit	Two	Single criterion	Proportional	Yes	Yes	Yes
HRAGA	Stochastic	Bit	Two	Warped single criterion ('age')	Fitness sorting	Yes and affinity	Adaptive	Yes
MOGASS	Stochastic	Bit	Two	Non-domination	Proportional	Yes	Yes	Yes
MSGA	Stochastic	Integer	No. of objectives	Fitness	Proportional	Yes/multiple	Bit negation	Possible

Note: GAGS = genetic algorithm with gendered selection; GASS = genetic algorithm with sexual selection; GGA = genetic-gender approach; VGA = virtual gender approach; G-GA = gendered genetic algorithm; GSGA = gender separation with genetic algorithm; NSGA2 = non-dominated sorting genetic algorithm 2; HRAGA = adaptive genetic algorithm simulating human reproduction model; MOGASS = multi-objective genetic algorithms with sexual selection; MSGA = multi-sexual genetic algorithm; MO = multi-objective; SRC = stochastic remainder choice.

and Mishra 2005; Kowalczuk and Białaszewski 2006a, 2006b, 2006c; Deb and Gupta 2006; Deb 2007; Coello, Lamont, and Van Veldhuizen 2007), which allow the assessment of multi-profit-maximization solutions as dominated or non-dominated (Pareto optimal).

The assessment of solutions concerning their Pareto optimality not only determines the Pareto-optimal set of solutions, but also allows some ranking of all possible solutions with respect to the degree of domination. Thus, each solution can be assigned a certain scalar quantity called a rank (Goldberg 1989; Man *et al.* 1997), which can have different definitions, interpretations and applications (Horn, Nafpliotis, and Goldberg 1994; Srinivas and Deb 1994; Coello, Lamont, and Van Veldhuizen 2007). In general, though, such a rank more or less directly relates to the number of individuals in the current population that dominate the analysed individual (or over which the analysed individual dominates) in the sense of Pareto (Kowalczuk and Białaszewski 2006a). Here, the rank $\rho(\boldsymbol{x}_i)$ of a given solution $\boldsymbol{x}_i$ among N possible solutions is calculated according to the following formulae:

$$\rho(\boldsymbol{x}_i) = \mu_{\max} - \mu(\boldsymbol{x}_i) + 1 \tag{3}$$

$$\mu_{\max} = \max_{i=1,2,\ldots,N} \mu(\boldsymbol{x}_i) \tag{4}$$

where $\mu(\boldsymbol{x}_i)$ is the degree of domination, *i.e.* the number of solutions by which $\boldsymbol{x}_i$ is dominated in the same population, while $\mu_{\max}$ is the maximum value from among all $\mu(\boldsymbol{x}_i)$. The important point is that this kind of ranking transforms the vector of profit functions into a scalar space.

3.2. Global optimality

Although, in the above ranking method, with respect to Pareto optimality, the profit vector is transformed into a scalar value, the concept of optimality does not give any directions as to the choice of a single solution from among all the Pareto-optimal solutions. Therefore, the designer has to make an independent and ultimate judgement of the offers obtained.

To utilize that freedom, a useful development of the ranking method has been proposed (Kowalczuk and Białaszewski 2001) that uses the idea of a global optimality level (GOL). In particular, the vector profit function value of each solution is transformed into a scalar (GOL) value, which allows practical ordering of the solutions. There is still a (very small) chance of obtaining equal indices of global optimality for several solutions, which constrains the opportunity of obtaining the ideal sequential ordering of all solutions without additional interference by the designer. Nevertheless, this approach significantly limits the number of 'most desired' Pareto-optimal solutions.

The method of estimating the GOL can be expressed by Procedure 2, representing a typical min–max operator (Kowalczuk and Białaszewski 2004; Białaszewski and Kowalczuk 2016):

Procedure 2 Global Optimality (GOL)

Determine a maximal acquired partial gain $f_{i_{\max}}$ as a maximum value of the partial profit function over all N solutions (or only the Pareto-optimal ones)

$$\underset{i=1,2,\ldots,m}{\forall} f_{i_{\max}} = \max_{j=1,2,\ldots,N} \{f_i(\boldsymbol{x}_j)\} \tag{5}$$

Assign each of the solutions $\boldsymbol{x}_j$, $j = 1, \ldots, N$, its global optimality level as a minimum value of its relative partial profits (i.e. over all profits):

$$\eta(\boldsymbol{x}_j) = \min_{i=1,2,\ldots,m} \frac{f_i(\boldsymbol{x}_j)}{f_{i_{\max}}} \tag{6}$$

The above procedure is not completely objective, as it includes normalization (otherwise, a typical act in most engineering considerations), which dynamically and locally refers to a current population, cycle or epoch (EN). The partial gain $f_{i_{\max}}$, instead of characterizing a current generation of solutions, may be globally set as a constant for a whole evolutionary run, on a static, *a priori* (AN) or *a posteriori* (PN) normalization basis.

Ordering of solutions with respect to the GOL index (using one of the EN, AN or PN settings) results in a considerable minimization of the problem of ambiguity of the Pareto solutions, which can be a pain for designers using the apparatus of Pareto optimality. The ordering method in terms of the GOL is very powerful both in current observation of the progress in EMOO and in the final assessment of the set of either all individuals or the Pareto-optimal ones. It also appears that the above methods of ranking and ordering are both universal and practical. They can be used in evolutionary optimization processes, as shown in the following.

3.3. High-dimensionality problem and study motivations

When considering MOO problems, one always has to be aware of the issue of dimensionality. It is a well-known fact that when the space of the optimized objectives has a higher dimension, many individuals fall within the category of being Pareto optimal, *i.e.* mutually equivalent in the Pareto sense (within a given Pareto front). As such, they are assigned the same rank, which, in turn, implies that they are indistinguishable from the Pareto-optimality viewpoint. A side-effect of such a state of the Pareto assessment is the low number of Pareto fronts, which obstructs the distinction, estimation and ordering of solutions in evolutionary cycles. This also means that during the process of selecting individuals for new generations, the Pareto-based ranking is ineffective, leading to a strongly stochastic behaviour of the GA with no 'conscious' movements.

On the other hand, when the scope of optimality is confined by a reduction (in some way) of the dimension of the analysed objectives' space, the ability of the Pareto-optimality method to differentiate between different individuals is also facilitated (Kowalczuk and Białaszewski 2006b; Białaszewski and Kowalczuk 2016).

Another motivation for the following development proposed for EC is the fact that in most cases (except for some sporadic fragmentary attempts mentioned in the introduction to this section) only integrated estimates of the fitness functions have been applied in the reproduction process of the GA/EC algorithms. This also means using the concept of one 'unisex' parental pool, unlike the usual pattern followed by species in nature.

By estimating the GOL of searched solutions, it would be easy to find a difference if one was looking only for the isolation of solutions from the same Pareto front. This, however, need not lead to whatever qualitative change in the effects of the evolutionary computations considered (Kowalczuk and Białaszewski 2006b).

3.4. Concept of genetic gender

In nature, the gender division of a species appears to differentiate individuals with reference to reproductive functions. According to this division, the concept of an artificial genetic gender consists of dividing the set of objective functions into several subsets, each of which has an attributed genetic gender Xj, $j = 1, 2, \ldots, s$ (Figure 1), and portrays an assumed partial scope (sub)optimality value of a certain utility in the designer's interpretation (Kowalczuk and Białaszewski 2001):

$$\text{GENDERS} = \{\text{X1, X2, } \ldots \text{X}j, \ \ldots., \text{X}s\} \tag{7}$$

For example,

$$\text{GENDERS} = \{\text{XX, XY}\} \tag{8}$$

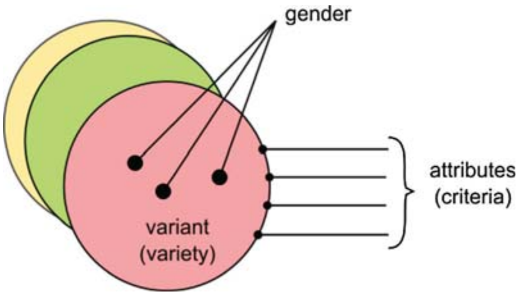


Figure 1. The overall concept of the genetic-gender approach: a gender (set) with a variant (the name of the gender) and its attributes.

symbolizes the native two-element set of the XX and XY chromosomes, or certain 'species' attributes, which are associated with some distinguishable characteristics of the individuals estimated in terms of the considered objectives.

In a particular context, a gender (a set) and a variant (a label assigned to such a gender) are associated with both a subset of criteria and a subset of individuals (best, in this context). Basically, the applied division of the set of attributes should be rational and distinctive to achieve the realization of the global optimization goal (survival) as a result of the synergy of different genders. Nevertheless, obtaining such an effect is not critical.

In this way, one gender set (X*j*) can embrace the objectives of a similar character that are only in a kind of internal (secondary) rivalry in terms of an approximately equal meaning to the user, from some pertinent point of view. Such an assortment can thus effectively discharge the designer from the cumbersome task of isolating a single solution from among all the Pareto-suboptimal solutions obtained in the course of iterative MOO.

On the other hand, different gender sets (X*j*) can express various groups of interests that the user would find difficult to judge in advance. In general, this division can be used to represent an external (primary) rivalry, which is not simple to resolve. Usually in such cases, the best method is to use the notion of Pareto optimality.

Thus, in the consequent GGA, the mechanism of gender allotment during the whole computational evolution is proposed, for the purpose of creating a few parental pools of different genders and generating new offspring by mating only apparently dissimilar individuals. Despite such a fractional perspective, the notion of the set-fitting Pareto suboptimality appears to be entirely clear and sufficient. In similar contexts, such an artificial gender is even referred to as 'sex' by several authors (Lis and Eiben 1997; Rejeb and AbuElhaija 2000; Kowalczuk and Białaszewski 2001; Vrajitoru 2002; Sanchez-Velazco and Bullinaria 2003a, 2003b; Song Goh, Lim, and Rodrigues 2003; Sodsee *et al.* 2008; Yan 2010).

The vector of the profit functions (1) can therefore be divided into s subvectors (Kowalczuk and Białaszewski 2001, 2013; Białaszewski and Kowalczuk 2016):

$$\boldsymbol{f}(\boldsymbol{x}) = [\boldsymbol{f}_1(\boldsymbol{x}) \quad \boldsymbol{f}_2(\boldsymbol{x}) \quad \ldots \quad \boldsymbol{f}_s(\boldsymbol{x})]^{\mathrm{T}} \in \mathbf{R}^m \tag{9}$$

$$\boldsymbol{f}_j(\boldsymbol{x})^{\mathrm{T}} \in \mathbf{R}^{m_i}, \quad m = \sum_{i=1}^{s} m_i \tag{10}$$

where the latter describes the jth subvector ($j = 1, 2, \ldots, s$) defining the genetic-gender perspective, which by means of some measure will be used to specify the genetic-gender set of individuals all labelled by X*j*. Within each of these sets, Pareto-suboptimality-based ranking of individuals is applied. In effect, each of the individuals is allotted a vector of ranks

$$\boldsymbol{r}(\boldsymbol{x}_i) = [r_1(\boldsymbol{x}_i) \quad r_2(\boldsymbol{x}_i) \quad \ldots \quad r_s(\boldsymbol{x}_i)]^{\mathrm{T}} \in \mathbf{R}^s \tag{11}$$

where $r_j(\boldsymbol{x}_i)$, $j = 1, 2, \ldots, s$, represents the rank of the ith individual ($\boldsymbol{x}_i$) within the jth genetic gender (Xj).

According to the proposed GGA, the assignment of the genetic gender l_i to each individual $\boldsymbol{x}_i$ in the population is performed by computing the following procedural quantities:

$$\varphi_i = \max_{j=1,2,\ldots,s} \varphi_i^j, \qquad l_i = \arg\max_{j=1,2,\ldots,s} \varphi_i^j \tag{12}$$

based on

$$\varphi_i^j = \frac{r_j(\boldsymbol{x}_i)}{r_{j\max}}, \qquad r_{j\max} = \max_{i=1,2,\ldots,N} \{r_j(\boldsymbol{x}_i)\} \tag{13}$$

where φ_i is the obtained highest degree of suboptimality, meaning a (fuzzy) measure of the memberships of the ith individual to the l_ith variant of the genetic gender, while the symbol $r_{j\max}$ denotes the maximum rank from among all individuals with respect to the jth subcriterion Xj (used for the purpose of normalizing suboptimality). As can be seen from Equation (12), the index of the maximal degree of suboptimality φ_i determines the gender of a solution under estimation in an unambiguous way.

As suggested above, it is assumed that only individuals of different genders create their offspring in the crossover process of the GGA algorithm, *i.e.* crossing individuals of the same gender is avoided. The method of selecting the parental pool is carried out according to the method known as stochastic sampling without replacement (Goldberg 1989) or stochastic remainder (Kowalczuk, Suchomski, and Białaszewski 1999), based on the highest degree of membership to the gender set (Xj) considered.

In the GGA, the number of individuals in each of the gender groups can change in the process of evolution. Only a minimum power (cardinality) of the gender sets (*e.g.* $N/(3s)$) is monitored. The missing positions (this problem occurs only when initializing the GGA procedure) can be supplemented by individuals from the lowest Pareto front of another gender set, which are left out in the course of the GGA selection process.

The sizes of the gender parental pools are similar. For instance, in the case of three genders with a population of 60 members, the cardinality of each parental genetic-gender set (pool) is equal to $60/3 = 20$, where each genetic-gender parental pool consists of individuals with one specified attribute (gender). This mechanism means that in the case of a small ($N/3s$) genetic-gender subpopulation the rivalry between these individuals is weak. Weaker individuals will then have a greater chance of survival (entering the parental pool). For large subpopulations, the rivalry is much stronger and therefore only 'truly' best individuals will appear in the parental pool. This mechanism is similar to niching, or uniform breeding (Kowalczuk and Białaszewski 2006b). The full cycle of the GGA algorithm, including preselection of the genetic-gender (GG) sets, is briefly sketched as Procedure 3:

Procedure 3 Genetic-Gender Algorithm (GGA) - a sketch
Divide the vector of profit functions into s-subvectors
Generate randomly an initial population V of N individuals $\{\boldsymbol{x}_i\}_{i=1}^N$;
while $t \leq t$max
  Compute the fitness: $\boldsymbol{x}_i \to \boldsymbol{f}(\boldsymbol{x}_i)$
  Assign the GG ranks according to each s-subcriterion: $\boldsymbol{x}_i \to \boldsymbol{r}(\boldsymbol{x}_i)$
  Compute the degree of suboptimality based on Equation (13)
  Recognize the individual's gender (GG) according to Equation (12)
  Assemble the GG sets $\{\mathrm{X}j\}_{j=1}^s$ of individuals of highest suboptimality (12)
  Select the parental GG pools $\{\mathrm{X}'j\}_{j=1}^s$ with the use of SRC

```
Create the new offspring V' by making:
    Crossover of different GG
    Mutation of the offspring
Replace the population: V ← V'
Cycle: t ← t+1.
```

As can be seen from Procedure 3, the presented GGA is unique and simple to implement. The innovative parameterization associated with GGA is confined to the 'merit' decision concerning the sense and number of genders applied. For the reader's convenience, more details are given in the supplementary Appendix A. The effectiveness of GGA was analysed and proven for various abstract challenges associated with multi-objective problems and several designs taken from engineering practice (Kowalczuk, Suchomski, and Białaszewski 1999; Kowalczuk and Białaszewski 2013). Complex standard benchmarks are analysed in Section 4.

It is thus clear that by introducing the idea of genetic gender, the issue of dimensionality by atomizing the scope of optimality and restricting the dimension of the objective spaces can largely be alleviated. This manipulation would be expected to produce a greater number of Pareto fronts (Kowalczuk and Białaszewski 2006b), leading to a measurable diversity of the generated subpopulations and to an improved effectiveness of the genetic search in the direction of both partial and total objectives (*e.g.* in the GOL sense).

Examples supporting the GGA can be found in nature; for instance, in some sexual behaviour among cuttlefish, which are particular about the transfer of two unique characteristics to their offspring (size and intelligence).

From a human point of view, and considering social characteristics and roles, 'child' can be distinguished as a 'third gender'. The three aspects of human functioning have linguistic and grammatical contexts.

3.5. Concept of hierarchical virtual gender and Pareto ranking

The idea of hierarchical Pareto ranking (HPR) or hierarchical virtual gender (HVG) of the analysed solutions (Kowalczuk and Białaszewski 2011) is based on another use of the principle of genders, and observations that human decisions performed in a process of multi-objective problem solving often have the nature of hierarchical evaluation. It is important that such an approach to MOO, referred to as the VGA, is also based on Pareto suboptimal ranking. But, this time, in contrast to GGA, during evolutionary cycles the dynamic assignment of genders to individuals is not executed, and no gender restrictions are imposed on the crossover process (thus implementing the classical 'universal-sex' scheme).

The original, individual scalar criteria are treated as primary. In the process of creating the hierarchy, first, the complete set of criteria is partitioned into disjoint subsets, referred to as secondary virtual genders. The genders (ranks), quantitatively (for a particular individual) representing the subsets of the primary criterion functions, can be marked with labels, called secondary virtual variants (labels). Next, a similar treatment is carried out with respect to the obtained set of secondary virtual genders. As a result, a master collection of some secondary virtual genders, uniquely representing a greater group of primary genders, is obtained. The resulting construction can be described as a tree structure, where lower level assessments (ranks) turn into higher level virtual individuals.

Figure 2(a) shows an example of a complete distribution of 12 primary objective functions into four disjoint secondary virtual genders. In the second step, the aggregation of two secondary virtual genders into two (disjoint) sets of third rate virtual genders (high level) is performed. Although the higher level virtual genders play a pure tool function in the Pareto ranking, suitable labels (variants) can be assigned to them. A simple case is shown in Figure 2(b), which illustrates a virtual-gender distribution of nine criterion functions using three classes (corresponding to the one-level genetic-gender resolution applied in GGA). The vector of $s = 3$ ranks of Equation (11) is applied here as

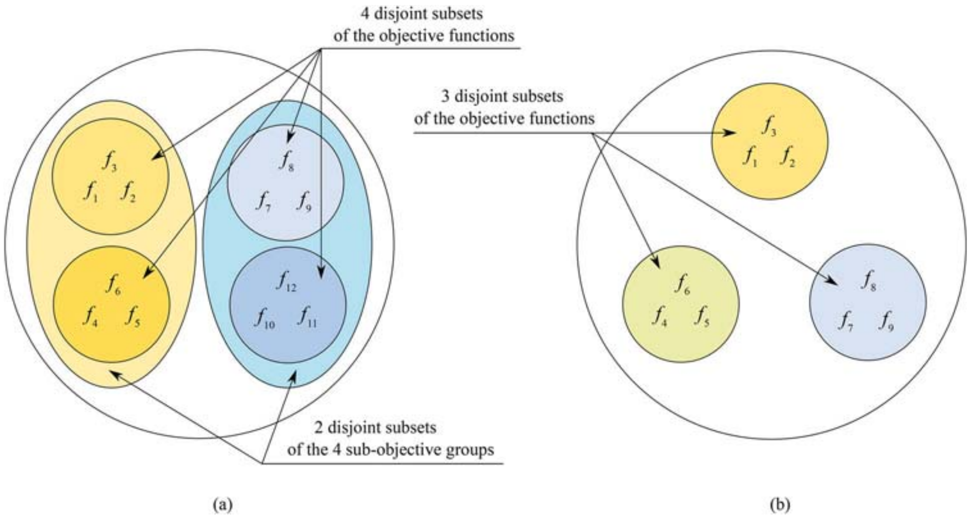


Figure 2. Exemplary distribution of objective functions into (a) four-level and (b) three-level hierarchical virtual genders.

the representation of three secondary virtual genders ('classes' or 'breeds' estimated by their own ranks), which are next subject to aggregation into a one-element set of a third rate variant. At all levels, the assessment of virtual individuals, representing fitness and ranks, is completed in the Pareto sense.

According to this scheme, at the second (II) level the assessment is performed (as in GGA) with respect to the degree φ_i^j of suboptimality (Equation (13)) within each variant. Taking into account the three resulting assessments (relative ranks) gained at this level (II) that constitute the secondary virtual genders, or virtual variants, the final distinct Pareto-optimal estimation can readily be done at the third level.

The above-described process of the sequential-derivative Pareto assessment can easily be developed for any necessary number of levels (*e.g.* up to a scalar estimation), if such an HPR with many levels will more closely match the user's needs and the nature of the analysed set of criterion functions.

Explicit hierarchies matched to the accepted way of distributing the criterion functions of Figure 2 are shown in Figure 3. Figure 3(a) describes the assessment process concerning 12 criteria, which are divided into the four groups of Figure 2(a). Inside each of them, Pareto ranking (II-level assessment) is carried out. As a result, a corresponding assessment vector of four normalized ranks is obtained. These ranks, based on the Pareto-optimal assessment of two two-fold ranks, are next used in the construction of the third rate (two-fold) level of assessment. In the VGA course of the third rate level HPR evaluation performed on the four secondary estimators (II-level virtual genders measured by four ranks, each aggregating three primary subcriteria), each individual in the analysed population will achieve a two-fold rank, which again needs normalization to be considered a III-order suboptimality degree.

Figure 3(b) presents another case of a simple two-step analysis of Figure 2(b), where nine criterion functions are divided into three virtual genders of the II level, and Pareto ranking is carried out as above. With the adopted hierarchy of multi-criteria evaluation, the III-level Pareto reranking assessment of EMOO-problem solutions concerns a corresponding (to the figure) three-dimensional criterion vector, and leads to a final scalar evaluation (already normalized).

It is clear that the use of HPR can always be extended to a desirable highest level of aggregation (IV and III, for the two examples presented in Figures 2 and 3, respectively), meaning a scalar global estimation of solutions. This ultimate application of HPR thus makes a new proposition for defining global optimality, and is a challenger to GOL.

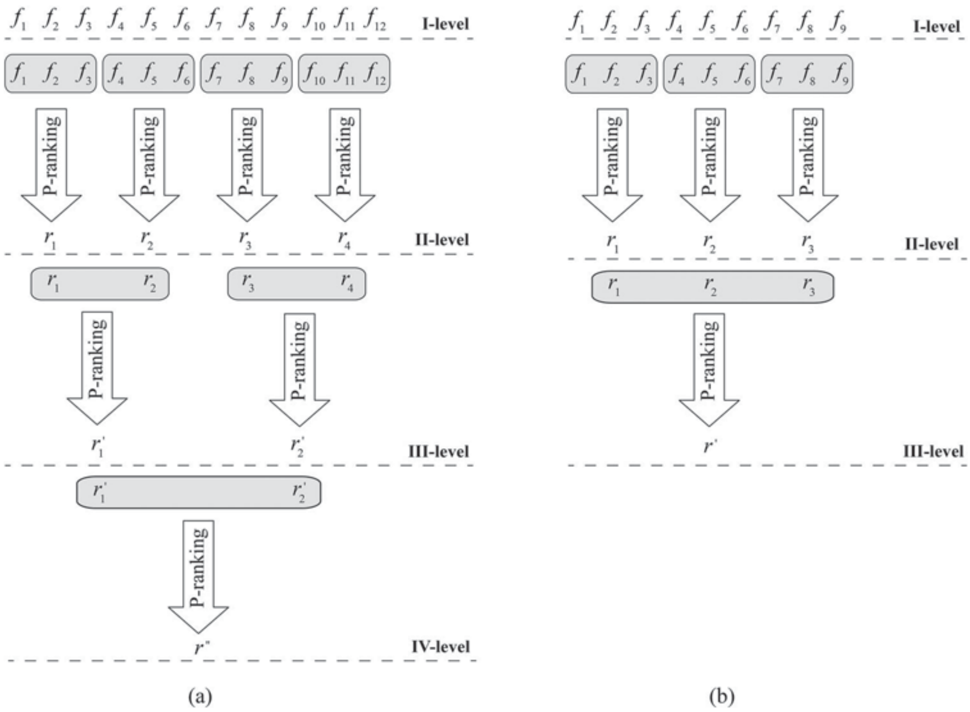


Figure 3. Hierarchical Pareto ranking according to the (a) four-level and (b) three-level hierarchical virtual genders of Figure 2.

In summary, the elaborated hierarchy represents a simple sequential procedure of assessing a scalar fitness of the considered individuals for the purpose of selecting a parental pool. This procedure is more developed than GOL, but leads to an effect similar to GOL.

3.6. Evolution quality and implemental remarks

The scalar ranks of individuals achieved in HPR can be used in a number of ways in the selection of individuals into the parental pool. This process can be supported, for example, by means of the SRC method (Kowalczuk, Suchomski, and Białaszewski 1999). In addition, the GOL can be used as a supporting measure for selecting the ultimate solution; in other words, GOL is still suitable here as a final optimality criterion for user selection. Several standard EMOO performance indices can be applied to evaluate the MOGA runs, such as the hypervolume ratio (HVR), maximum spread (MS), generational distance (GD) and spacing (SP) (Coello, Lamont, and Van Veldhuizen 2007; While *et al.* 2006; Bader and Zitzler 2009). Some of these methods will be included in the study presented here, although their use in some cases can be problematic.

Considering, for instance, the HVR measure, it must be kept in mind that the objective space needs to be convex, which is a hard condition to fulfil in practice. If it is not convex, the results may be misleading and, moreover, a true Pareto front has to be known (Cotta and Schaefer 2004; Bader and Zitzler 2009). The GD measures the mean distance between the computed Pareto front and the true one. When seeking optimal parameters in continuous search domains, the Pareto front or the Pareto-optimal solution set (Cotta and Schaefer 2004) is infinite and often difficult to find for practical optimization tasks.

The EMOO indices HVR and GD need an external (archive) population consisting of non-dominated individuals. Similarly, to determine the rate of MS the GAs must store the non-dominated solutions found during evolutionary cycles in an archive population, called a known Pareto front.

The GGA, however, does not utilize any external population, and only one set of individuals evolving through the generations is applied.

In contrast to the various other approaches and measures, the GGA algorithm returns 'good' representatives of the Pareto front of non-dominated solutions in a natural process of inheritance, and can also be assessed by other tools, such as the proposed GOL index.

Computational applications of EMOO algorithms, including those presented in this article, illustrate that determination of the true Pareto front would be computationally expensive, owing to either the form of the criteria, requiring simulation of the optimized system, or the complex numerical computations necessary for obtaining the value of the vector criterion. The final assessment of solutions could be a topic for further research studies.

Most of the known benchmarks for MOO concern only a few objectives, and they are not interesting patterns in the context of GGA and VGA, which are meant for truly multi-dimensional problems. Nevertheless, even the sample of results presented below appears to be sufficient to illustrate both the nature and power of the gender approach that is revealed in highly dimensional objective spaces.

4. Illustrative benchmark examples

This section presents the results of optimization for exemplary multi-objective problems. Two groups of evolutionary algorithms were selected for this comparative study: (1) four non-gender representatives: SMS-EMOA, multi-objective evolutionary algorithm based on decomposition–differential evolution (MOEA/D-DE), generalized differential evolution 3 (GDE3) and NSGA2; and (2) three gender-based (multi-sexual) algorithms: the gender propositions, *i.e.* GGA and VGA with HPR, and MSGA (Lis and Eiben 1997), most similar to GGA. All can be run in attended mode, using elitism. Keeping in mind that the EC methodology is to a great extent resistant in this field, the parameters of the analysed algorithms were rationally related to the considered optimization tasks and established according to experience, as suggested by Zhang and Li (2007). These implemental factors were thus set as follows: type of arithmetic, floating point; population size, 120; crossover probability, 0.8; mutation probability, 0.2; maximal number of generations, 200; and number of repeated runs, 30 (for statistical averaging). All of the algorithms start their execution from the same initial population.

4.1. Optimization results

All of the considered algorithms, both gendered and non-gendered, were compared using several optimization tasks, designated as UF1–UF10 and DTLZ1–DTLZ7 (Zitzler and Thiele 1999; Coello, Lamont, and Van Veldhuizen 2007; Qingfu *et al.* 2009). In most cases in this study, two relatively difficult sample tasks from each group, UF7, UF10, and DTLZ4, DTLZ5, are exercised.

The number of matching criteria not only represents the overall complexity of a given problem, but is also important for the structure of the gender mechanism used in the GGA/VGA. The UF7 describes a two-objective optimization test (with 30 decision variables/parameters). Such a simple set of criterion functions leads to the natural division into two one-dimensional attributes (and two genetic-gender sets), where the first gender set is characterized by the first objective $f_1(x)$ and the other is determined by the second function $f_2(x)$. This set of criterion functions can be innately divided into three one-dimensional attributes, or three genders. The UF10 is a three-objective optimization test (with 30 parameters). This set of criterion functions can be innately divided into three one-dimensional attributes, or three genders: $\{f_1(x),\ f_2(x),\ f_3(x)\}$. The DTLZ4 and DTLZ5 problems were designed for 10-dimensional criteria. In both cases, the functions of 10 objectives and 20 decision variables are divided into three default subsets (gender variants), where the first gender is set according to $[f_1(x)\quad f_2(x)\quad f_3(x)]$, the second gender is associated with $[f_4(x)\quad f_5(x)\quad f_6(x)]$ and the third gender is constructed via the criterion subset $[f_7(x)\quad f_8(x)\quad f_9(x)\quad f_{10}(x)]$. Moreover, in programming the VGA, the most simple III-level fitness hierarchy is applied (see Figures 2(b) and 3(b) for details of HVG/HPR), corresponding to the above-described genetic-gender distribution.

Table 3. Evolutionary mechanisms applied in the unattended evolutionary multi-objective optimization algorithms (without elitism) using real number representation.[a]

Mechanism	Crossover	Mutation	Selection
M.1	AX	UniM	SRC
M.2	SBX	PolyM	SRC
M.3	SBX	PolyM	TourS

Note: [a]excluding the multi-sexual genetic algorithm (MSGA), having binary representation.
AX = arithmetic crossover; SBX = simulated binary crossover; UniM = uniform mutation; PolyM = polynomial mutation; SRC = stochastic remainder choice; TourS = tournament selection.

In the experimental study concerning complex multi-dimensional and multi-parameter tasks, another choice is also exercised, of distributing the 10 objectives into the assumed three genders (variants), where the first gender is built according to $[f_1(x) \quad f_2(x) \quad f_3(x) \quad f_4(x) \quad f_5(x)]$, the second gender is determined by $[f_6(x) \quad f_7(x) \quad f_8(x)]$ and the third gender is defined as $[f_9(x) \quad f_{10}(x)]$.

All of the algorithms were compared in terms of the basic GOL (*i.e.* the maximal GOL) and the dispersion or standard spacing metric (SP), which measures the spread or dispersion of the solutions contained in the derived Pareto front (Coello, Lamont, and Van Veldhuizen 2007). The third quality index is hypervolume (HV). Owing to the computational complexity in determining the HV indicator, an offline Monte Carlo method (While *et al.* 2006) was used, which is necessary especially for the demanding multi-objective problems spanned by three to 10 objectives. When using SMS-EMOA, the HV contribution must always be determined online.

In the simplest case of UF7, it is quite easy to compute the GD index (Coello, Lamont, and Van Veldhuizen 2007) and refer the analysed solutions to a known true Pareto front. When dealing with complex problems (UF10, DTLZ4–DTLZ7), however, this index appears to be too computationally complex and therefore impractical (mainly because of the need to determine the true Pareto front). Therefore, in more complex cases, in place of the GD the median GOL is applied as the fourth indicator.

In some computations, the applied EMOO algorithms are executed without elitism, which means that the no-gender algorithms, NSGA2, MOEA/D-DE, GDE3 and SMS-EMOA, are deprived of their internal mechanism of elitism. This type of experimentation is referred to as unattended.

Apart from the easy-to-calculate index GOL, all quality indicators were estimated offline. For statistical analysis of the obtained results, the median approach was primarily used.

Consequently, the running statistical results of multi-objective unattended optimization for UF7 are shown below in terms of the following indicators: maximal GOL, SP, normalized HV and GD, whereas for the complex MOO tasks (UF10, DTLZ4–DTLZ7), maximal GOL, SP, HV and median GOL are used.

4.1.1. First experiment: three real mechanizations

In the first experiment, the floating-point representation (GGA, VGA, NSGA2, GDE3, MOEA, EMOA) is applied, except for the MSGA, which uses the binary description (by definition). In Table 3, three evolutionary mechanisms (performed without elitism) are characterized as follows:

(1) All algorithms use arithmetic crossover (AX), uniform mutation (UniM) and selection by means of SRC; the exceptions are the algorithms GDE3, MOEA and EMOA, which by definition have no selection mechanism.
(2) Simulated binary crossover (SBX) and polynomial mutation (PolyM) are exercised.
(3) Tournament selection (TourS) is implemented.

The numerical results obtained with the three mechanism cases (M.1, M.2 and M.3) applied to solve the four selected problems (UF7, UF10, DTLZ4, DTLZ5) can be seen in the supplementary material in Figures S1–S12.

Table 4. Characteristics of the operators applied in the unattended evolutionary multi-objective optimization algorithms (without elitism) using binary number representation (GGA, VGA, MSGA) and real representation (GDE3, NSGA2, MOEA, EMOA).

Mechanism	Crossover	Mutation	Selection
M.4	Native	Native	TourS

Note: GGA = genetic-gender approach; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm; GDE3 = generalized differential evolution 3; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA = multi-objective evolutionary algorithm; EMOA = evolutionary multi-objective algorithm; TourS = tournament selection.

Table 5. Evolutionary scheme applied in the evolutionary multi-objective optimization algorithms with local elitism[a] (attended) and using floating-point (real) representation (GGA, VGA, GDE3, NSGA2, MOEA, EMOA) and a binary one (MSGA).

Mechanism	Crossover	Mutation	Selection
M.5	Native	Native	TourS

Note: [a]GDE3, NSGA2, MOEA and EMOA use their native mechanisms of elitism.
GGA = genetic-gender approach; VGA = virtual gender approach; GDE3 = generalized differential evolution 3; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA = multi-objective evolutionary algorithm; EMOA = evolutionary multi-objective algorithm; MSGA = multi-sexual genetic algorithm; TourS = tournament selection.

As can be seen in Figures S3(b) and S7(b), in some generations the spacing value SP for NSGA2 (and sometimes for GGA and VGA) is not visible. This is due to the fact that in subsequent populations these algorithms produce only one Pareto-optimal solution, which makes the spacing incalculable.

The results (Figure S11) concerning the EMOO solving of problem DTLZ4 and the gendered approaches GGA and VGA were obtained using the alternative choice of distributing the 10 objectives of DTLZ4 into three (virtual) genders defined by $\{[f_1(x) \quad f_2(x) \quad f_3(x) \quad f_4(x) \quad f_5(x)]$, $[f_6(x) \quad f_7(x) \quad f_8(x)]$ and $[f_9(x) \quad f_{10}(x)]\}$. In the applied theoretical benchmark optimization problems, the choice does not appear to be critical, although it does not seem to be especially fruitful (compare the data in Figures S11 and S12).

In general, the simpler the MOO problem, the more similar the results. The best performance was observed using mechanism M.3; in this way, the combination of mechanisms SBX + PolyM + TourS can be considered a decisive factor.

4.1.2. Second experiment: binary representations

In many cases, the sheer binary algorithm MSGA appears to perform well, and sometimes even very well (see Figure S1 and the results for task UF7). Therefore, in the second experiment, attention is focused on comparison of the algorithms using binary (GGA, VGA, MSGA) and real (GDE3, NSGA2, MOEA, EMOA) representations. All of the algorithms have mechanized their native genetic operators. Binary tournament selection was applied if the algorithm needed a mechanism of selection. The implemented mechanism (still without elitism) is characterized in Table 4. 'Native' means that each algorithm uses its inborn genetic mechanisms (*i.e.* cross multi-point crossover and binary mutation). The results of mechanism M.4 used in solving problems UF10 and DTLZ5 can be seen in supplementary Figures S13 and S14.

4.1.3. Third experiment: native implementations with elitism

The third experiment represents another comparative study of the floating-point representations of all algorithms (except for MSGA), which use their native (inborn) genetic operators and elitism. In GGA, VGA and MSGA, local elitism is applied, which means that a few individuals with the best local optimality, GOL, are taken from each (virtual) gender subpopulation to the next generation. Table 5 describes the mechanisms executed using local elitism for the extra control. As above, 'native' means that each algorithm uses its inborn genetic mechanisms. The quality results of implementing mechanism M.5 for solving tasks UF10 and DTLZ5 are illustrated in supplementary Figures S15 and S16.

Table 6. Characteristics of the operators applied in the evolutionary multi-objective optimization algorithms (controlled by local elitism[a]) using binary number representation (GGA, VGA, MSGA) and real representation (GDE3, NSGA2, MOEA, EMOA).

Mechanism	Crossover	Mutation	Selection
M.6	Native	Native	TourS

Note: [a]GDE3, NSGA2, MOEA and EMOA use their native mechanisms of elitism.
GGA = genetic-gender approach; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm; GDE3 = generalized differential evolution 3; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA = multi-objective evolutionary algorithm; EMOA = evolutionary multi-objective algorithm; TourS = tournament selection.

Table 7. Characteristics of the operators applied in the evolutionary multi-objective optimization algorithms (controlled by overall elitism[a]) GGA, VGA, GDE3, NSGA2, MOEA and EMOA, using real representation, plus binary MSGA.

Case	Crossover	Mutation	Selection
M.7	Native	Native	TourS

Note: [a]GDE3, NSGA2, MOEA and EMOA use their native mechanisms of elitism.
GGA = genetic-gender approach; VGA = virtual gender approach; GDE3 = generalized differential evolution 3; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA = multi-objective evolutionary algorithm; EMOA = evolutionary multi-objective algorithm; MSGA = multi-sexual genetic algorithm; TourS = tournament selection.

4.1.4. Fourth experiment: binary representations with elitism

To verify the capabilities of the binary implementations, the fourth experiment considers the performance of binary mechanizations of the gender algorithms (GGA and VGA) compared to the other (real) algorithms using their native genetic operators, all with local/native elitism (similarly to M.5). The mechanisms are explained in Table 6. Again, 'native' concerns the inborn genetic mechanisms and elitism. The quality results of implementing mechanism M.6 for solving tasks UF10 and DTLZ5 can be seen in supplementary Figures S17 and S18.

4.1.5. Fifth experiment: real representations with (overall) elitism

At the end of this experimental review of the EMOO mechanisms and the quality of their performance, a fifth experiment on all floating-point algorithm mechanisms (and the binary MSGA) was conducted. The algorithms were implemented based on both their native genetic operators and elitism. In the GGA, VGA and MSGA, a typical mechanism of the overall (total) elitism is applied, which means that a few individuals with the best local GOLs in the whole population are carried over to the next generation. Table 7 describes the EMOO mechanisms used, along with overall elitism. The estimated results of implementing mechanism M.7 for solving tasks UF10 and DTLZ5 are illustrated in supplementary Figures S19 and S20.

The above experimental computations were carried out in view of the possibilities of implementing the algorithm SMS-EMOA, which, owing to its deposition on the mechanism of hypervolume, requires huge computational cost for the online calculation of the hypervolume contribution, which makes such calculations completely impractical for difficult multi-dimensional optimization problems. Therefore, the most complex optimization problems, represented by functions DTLZ6 and DTLZ7, were tested for the six considered algorithms, omitting algorithm SMS-EMOA. The optimization results obtained using the EMOO mechanism M.7 for problems DTLZ6 and DTLZ7 can be found in supplementary Figures S21 and S22. As the last two tasks have a highly developed objective space, as is usual in such cases, almost the entire population is not dominated in the Pareto sense. This makes the calculation of the hypervolume contribution especially difficult to perform online.

4.1.6. Sixth experiment: averaging versus median approach

The purpose of the sixth experiment is to show the effect of using simple averaging in place of median estimation. For illustration, the real number representation in algorithms deprived of elitism (as in mechanisms M.3; see also supplementary Figures S9c–S12c) is considered. The sample of results concerns problems UF7, UF10, DTLZ4 and DTLZ5. The median estimation usually yields results similar to averaging. But in some cases (see Figure S23), the averaging approach can result in a less smooth run of an index, especially concerning the hypervolume.

4.1.7. Illustrative true Pareto fronts: for the third experiment, problems UF7 and UF10, and mechanism M.5

Exemplary Pareto fronts achieved for the first experiment (mechanism M.3, without elitism) and the third experiment (mechanism M.5, with local elitism) are related to two low-order problems: two-objective UF7 and three-objective UF10, as shown in Figures 4 and 5, respectively. Solutions that make up the presented Pareto fronts belong to the last 30 generations of the 30 runs for each algorithm.

In particular, in Figure 4 (also supplementary Figure S24) the Pareto fronts can be referred to a true Pareto front, represented as a grey line segment. In contrast, in Figure 5 a tiny true Pareto-optimal

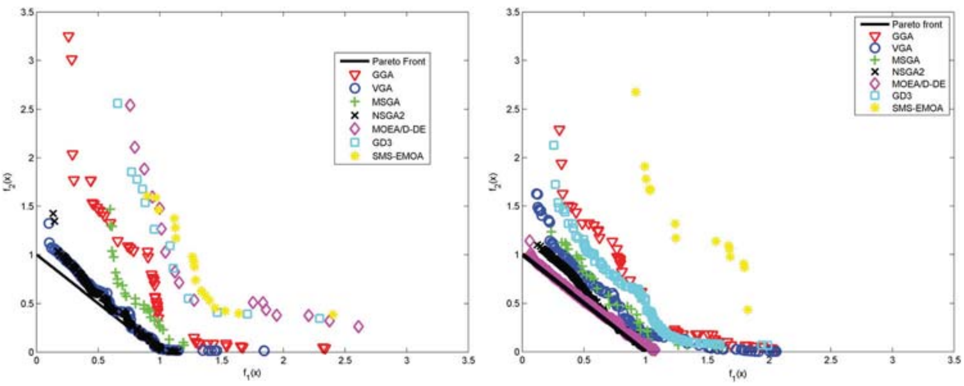


Figure 4. The true Pareto front (grey) and the Pareto fronts obtained for the simple problem UF7 using (a) mechanism M.3 (without elitism) and (b) mechanism M.5 (with local elitism). GGA = genetic-gender approach; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA/D-DE = multi-objective evolutionary algorithm based on decomposition–differential evolution; GD3 = generalized differential evolution 3; SMS-EMOA = S-metric selection evolutionary multi-objective algorithm.

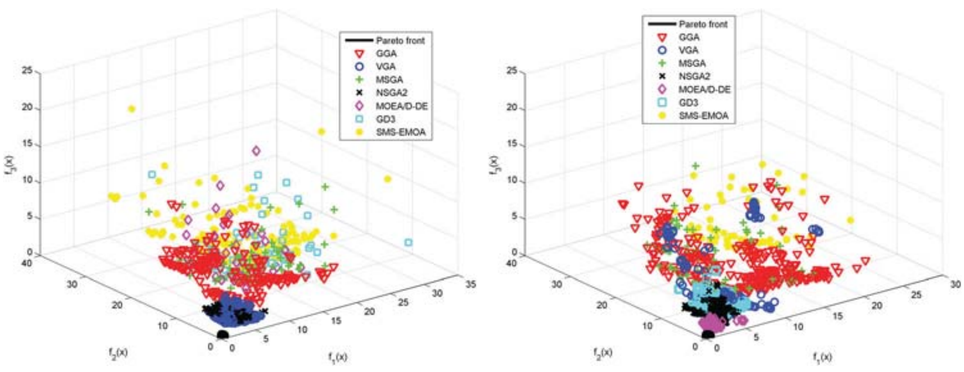


Figure 5. The true Pareto front (grey) and the Pareto fronts obtained for the simple three-dimensional problem UF10 using (a) mechanism M.3 (without elitism) and (b) mechanism M.5 (with local elitism). GGA = genetic-gender approach; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA/D-DE = multi-objective evolutionary algorithm based on decomposition–differential evolution; GD3 = generalized differential evolution 3; SMS-EMOA = S-metric selection evolutionary multi-objective algorithm.

front resembles a very small spherical grey sector centred on the origin point (0, 0, 0). As can be seen there, the gendered approaches are not most optimal. This is due to the fact that with such simple benchmark cases, the advantages of gender division are not used and integrated multi-objective optimality is not sought; instead, the low-dimensional genders (variants) care only about their own individual goals. Nevertheless, as shown for mechanism M.3 (without elitism) in Figures 4(a) and 5(a), the VGA has proven very effective and as good as NSGA2 (even for such inappropriate and unchallenging tests).

4.1.8. Computation time

Resulting from yet another study, Figure 6 presents the computation times for each of the seven analysed algorithms in the third experiment (mechanism M.5) when solving the 10-objective problem DTLZ5. Each individual bar represents the running time of a single simulation. The results (bars) for each EC algorithm are put together (30 neighbouring bars represent 30 runs). As can be seen in Figure 6, the computation times of the GGA and VGA algorithms are, on average, at least two times lower than those for the MOEA/D-DE, *i.e.* GGA and VGA take about 3 s, MOEA/D-DE about 5 s, NSGA2 about 7 s and GDE3 about 8 s. The computation times for the algorithm SMS-EMOA are much greater. The reason for this is the need to determine the HVR online. On the other hand, the computation time for the MSGA is the result of the applied binary representation, which requires the use of additional mechanisms for processing very long binary strings (20 parameters × 24 bits makes a 480 bit string). All the calculations were implemented in MATLAB® and carried out on a personal computer with a dual-core Intel Core i5 processor (2.7 GHz).

A taste of the difference resulting from the dimension and complexity of the analysed problems can also be gained by the transition from theoretical tasks to practical problems, which can increase the calculation time from a few seconds to minutes. To be more concrete, in an exemplary case of engineering design, namely the design of diagnostic observers analysed, for instance, by Kowalczuk

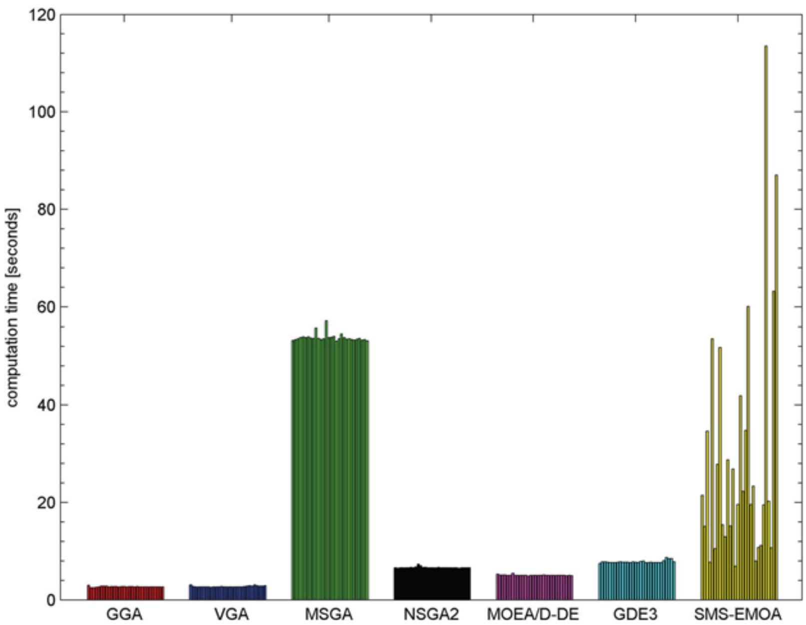


Figure 6. Experiment 3 (mechanism M.5) in problem DTLZ5: computation time for different algorithms in 30 runs. GGA = genetic-gender approach; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA/D-DE = multi-objective evolutionary algorithm based on decomposition–differential evolution; GD3 = generalized differential evolution 3; SMS-EMOA = S-metric selection evolutionary multi-objective algorithm.

and Suchomski (2004) and Białaszewski and Kowalczuk (2016), the computation times needed by the fastest algorithms (GGA and VGA) reach on average about 456 s (about 8 min), which is about 200 times longer than the running time consumed in solving the benchmark problems analysed here. This increase comes from a significant rise in dimensionality; it is also due to the fact that the applied engineering criterion functions must be determined using numerical calculations of complex analytical procedures for an appropriate infinity norm of system transmittance matrices (Kowalczuk and Białaszewski 2004, 2006b, 2013).

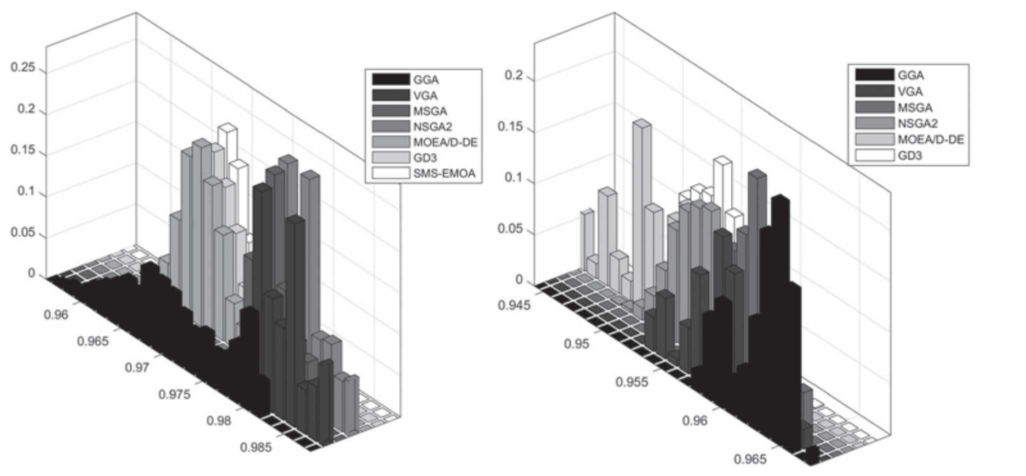
4.1.9. Statistical analysis: for selected problems UF7 and DTLZ6

A statistical analysis of the significance of differences observed among the proposed GGA/VGA and the other benchmarking procedures is also important. Therefore, the issue of the statistical significance of the results is supported here by a brief study. A limited sample of 1500 results (the last 50 generations in 30 runs), computed for the analysed benchmark MOO tasks and each of the tested six algorithms, was assessed in view of the four main performance indices: GOL, GD, HV and SP (calculated in four versions: max, mean, median and min). Corresponding approximate probability distribution functions (PDFs) were estimated (PDF is thus a probability mass function, which represents an approximation of a probability density function). The PDFs are selectively illustrated in Figure 7 (and in supplementary Figures S27–S31) in three dimensions for the two chosen problems, UF7 and DTLZ6.

In the supplementary study, the ranges of indicators GOL and HV were normalized with respect to the common maximum values obtained *a posteriori* for the given problem. These indices are normalized to the same narrow range (0.9–1). However, this is not the case for non-normalized indices GD and especially SP.

The charts have been normalized to represent a PDF. As can be seen from the results, the estimated PDFs show their different, statistically pertinent placement in the domain of possible results.

Distribution of the maximal GOL when solving tasks UF7 and DTLZ6 is given in Figure 7 for mechanisms M.3 and M.7, respectively, and shows a statistical dominance of VGA and GGA, respectively, over the others.



(a) (b)

Figure 7. Distribution of maximal global optimality level with mechanisms: (a) M.3 for problem UF7; (b) M.7 for problem DTLZ6. GGA = genetic-gender approach; VGA = virtual gender approach; MSGA = multi-sexual genetic algorithm; NSGA2 = non-dominated sorting genetic algorithm 2; MOEA/D-DE = multi-objective evolutionary algorithm based on decomposition–differential evolution; GD3 = generalized differential evolution 3; SMS-EMOA = S-metric selection evolutionary multi-objective algorithm.

Analogous results for the minimal (worst case) GOL in tasks UF7 and DTLZ6 can be found in supplementary Figure S28, where the simple (UF7) problem is solved in similar ways. In terms of the mean (as well as median) GOL, the effects of the EMOO procedures (see Figure S29) may vary over a similar narrow interval (about 0.9–1) of this performance measure.

Performance indices achieved by the EMOO algorithms, in terms of HV and SP when solving the DTLZ6 problem, and in terms of GD and SP when solving the UF7 task, are illustrated in supplementary Figures S30 and S31, respectively.

In view of the above results, it can be stated that the gender algorithms GGA and VGA, by conducting a very intensive search (making the crossing of only definitely dissimilar individuals), have more prerogative to find Pareto-optimal solutions (especially in the GOL sense). They can, however, also easily obtain weaker solutions of the EMOO tasks (although not necessarily worse than those found by other algorithms). On the other hand, looking at the subsidiary criteria (SP, HV and GD), the gender algorithms lead to more focused solutions with a lower dispersion (*i.e.* having lower values of these criteria).

4.2. Critical analysis and summary of the results

Many practical improvements in the performance of ECs are gained by the proposed gender approaches, GGA and VGA, compared to the classical, full-scope Pareto estimation used in MOO (Kowalczuk and Białaszewski 2004, 2006b):

- The classical GAs exhibit a relatively high sensitivity to the initial set of solutions.
- By the proposed GGA/VGA restatement, the issue of dimensionality can be significantly alleviated.
- The resulting gender-based Pareto fronts are more regular and appear in greater numbers.
- The applied gender suboptimality sustains diversity, representing an attractive genetic 'search power' based on both the internal and external rivalry of individuals, in a more rational way than the popular niching mechanism (Horn and Nafpliotis 1993; Michalewicz 1996; Man *et al.* 1997; Kowalczuk, Suchomski, and Białaszewski 1999; Kowalczuk and Białaszewski 2006a, 2006c; Coello, Lamont, and Van Veldhuizen 2007),
- The implemented GGA restriction in the crossover possibilities prevents premature convergence.
- The GGA/VGA Pareto-optimal solutions found are more optimal (in the sense of GOL/HPR).
- The user gains a clear outcome to support his or her decision about the selection of the ultimate solution(s).

Compared to other methods, the gender approach is fairly simple (in terms of both conception and computation), sticks to the very basics of the GA/EC methodology, more profoundly uses hints from nature, fulfils the requirements of technical design and ultimate decision making, and is completely open to amendments and developments proposed in the literature (*e.g.* Schaffer 1985; Goldberg 1989; Hajela and Lin 1992; Horn, Nafpliotis, and Goldberg 1994; Srinivas and Deb 1994; Fonseca and Fleming 1995; Zitzler and Thiele 1999; Kowalczuk, Suchomski, and Białaszewski 1999; Kowalczuk and Białaszewski 2006a; Coello, Lamont, and Van Veldhuizen 2007; Zitzler, Thiele, and Bader 2010).

The gender approach is also entirely different from other propositions. Although having several instrumental consequences, the GGA and VGA methods are conceptual in nature, consisting of the objective space decomposition of the initial problem and the effective reduction of an originally highly dimensional problem. Nevertheless, the VEGA has a few limited similarities: the VEGA partial parental pools can be assigned genders from a maximum m-element set, although it does not make use of the Pareto-optimality concept, its genders are not exclusive (one individual can have several genders) and the crossover mechanism has no gender checks.

Several examples of the application of the proposed genetic and virtual gender recognition in the evolutionary solution of different benchmark MOO tasks have been presented above; however, in solving practical problems, the gender approach generally prefers to break down the problem according to significance, *i.e.* it needs to discriminate the less relevant criteria from the more relevant ones, and to distinguish groups of similar objectives. This division does not have to be consistent with the computational complexity of the considered criteria. Moreover, as argued below, based on complex real-engineering optimization problems, when designing control or diagnostic systems, for instance, such characteristics are easily identifiable and the gender division is effortless (*e.g.* precision, insensitivity to disturbances and robustness to knowledge or modelling errors).

In the considered cases of abstract benchmark optimization problems, there is no indication of the significance of the objectives. Therefore, sometimes nothing can be gained from using the GGA and VGA, and the choice of genders does not appear to be critical or especially fruitful. Nevertheless, even in such theoretical cases and in line with the gender approach, the more complex the optimization problem, the better the results achieved by the GGA and VGA.

Other applications of the proposed GGA/VGA idea to multi-objective synthesis problems, concerning the engineering design of controllers and diagnostic observers, have been considered in many previous works, which also show the advantages and superiority of the gender approach. In particular, a typical design of a proportional-integral-derivative controller and a very complex design problem of a detection observer (Patton, Frank, and Clark 1989; Chen, Patton, and Liu 1996; Korbicz *et al.* 2004; Suchomski and Kowalczuk 2004; Kowalczuk and Białaszewski 2004, 2013), which serves as a principal element in the detection and isolation of faults, were put into practice. The latter design issue was illustrated with a benchmark problem based on a ship's propulsion system (Izadi-Zamanabadi and Blanke 1998). The obtained complex optimization design effect given in the form of a robust optimal detection observer (Kowalczuk and Białaszewski 2004) demonstrates both the usefulness and effectiveness of the proposed gender-based optimization method. Such an optimal tool of engineering system design—in accordance with suitable project prerequisites—allows systems to be designed which perform their basic task while having sufficient sensitivity (*e.g.* to errors in sensors and actuators) and simultaneously showing robustness to certain modelling uncertainties. The three desired characteristics of the system, namely the degree of functioning (performance), insensitivity and robustness, can be easily obtained using three separate genders. The optimal, robust and insensitive engineering solutions obtained in this way confirm the effectiveness of the gender approach in solving practical MOO problems.

5. Conclusions

The proposed GGA method of solving MOO tasks in an evolutionary manner is based on learning through the recognition of genetic genders. Information about the degree of membership to a given gender set is extracted in a Pareto-suboptimal process of ranking the fitness functions of analysed solutions. This information can be exploited in the crossover process of mating, in which only individuals of different genders are allowed to create offspring.

An instructive feature of the proposed EMOO approach is the utilization of the Pareto-optimization results. Thus, within each gender set of the GGA, Pareto optimization is used as an effective tool of suboptimal judgement of the 'internal' single-gender rivals for the purpose of their uniform estimation and selection (in a greater number) to the newly created parental subset (and to the next generation) in each iteration cycle of ECs. It is worth emphasizing that despite relying on this limited perspective, the notion of set-fitting Pareto suboptimality is entirely clear and practically adequate.

The GGA method can be interpreted in terms of:

- a new mechanism of preselecting both the transient and final individuals (solutions)
- mutual intergender support in the genetic search.

The standard concept of Pareto optimality can still be applied to the final set of solutions on a regular basis. Another method of processing and taking into account the full scope of optimality is based on the concept of either the GOL, calculated based on the fitness (or rank) functions, or HVG/HPR, based on a developed hierarchy of virtual genders (used in VGA).

A major success of the gender approach can be attributed to the fact that it appropriately deals with a great number of objectives by reducing the dimensionality of the Pareto-analysed spaces. In the full-scope optimization case, the number of Pareto fronts is strongly limited because of the high dimension of the objective space. This means that many individuals are estimated as being equivalent from the Pareto-optimality viewpoint (*i.e.* they have the same rank). As a result, the process of selecting individuals is not effective and the evolutionary search is overly stochastic, with no indications or progress in particular directions represented by the stated criteria.

In contrast, by introducing genetic or virtual genders, the above issue is solved by means of restricting the dimension of the objective spaces and bringing about a greater number of Pareto fronts within each population analysed in a subspace of a restricted dimension (*i.e.* solely in the space of the assigned gender objectives). This, in turn, brings about diversity among the individuals of the (GGA) subpopulations that can be easily estimated and used in effectively pushing the evolutionary exploration in the desired directions on the basis of the achievable distinctive ordering.

As explained above, in the EMOO GGA process any solution iteratively generated in particular epochs is assigned to gender, to allow for the selection of suitable members for utility parental pools. As has been shown, the genetic-gender restriction imposed upon the crossover mechanism prevents premature convergence of the optimized solutions and sustains their diversity. In this context, there is a close similarity to the results obtained by means of niching (Kowalczuk and Białaszewski 2006c). The genetic and virtual genders are easily distinguishable and they allow a global optimization via iterative suboptimization. The outcomes of the gender-based procedures are also less sensitive to the choice of the initial population than those of the other and classical genetic algorithms. In general, the GGA appears to be very robust, taking into account the various types of EMOO tasks and their complexity, as well as the values of executive parameters and initial conditions.

The GOL, according to its max–min principle, and the HPR/HVG method prefer solutions that are located in the middle of the Pareto front, and turn down the boundary solutions that are typically found in high-dimensional objective spaces under evolutionary search (owing to the known defect of Pareto evaluation). This is an important feature, as from the MOGA and engineering viewpoints the solutions that are 'excellent' with respect to a single measure are generally unacceptable. Moreover, the main idea of using the GOL for the presentation of the results during all epochs of EC lies in measuring the progress of the optimization process in terms of finding the best Pareto-optimal solutions. The waveform charts for GOL and HV are similar. GOL is not applied in the selection mechanism of GGA (and this means more room for improvement), and the concept of applying HPR/HVG in the VGA optimization procedure shows the practical advantages of this approach (Kowalczuk and Białaszewski 2004, 2006b).

In this article, significant differences were found in the main performance measures of the considered EMOO procedures in selected comparative experiments, in terms of the estimated PDFs.

Although the gender approach is more developed than other methods, as mentioned in Section 3, in similar contexts of artificial gender variation several authors (Lis and Eiben 1997; Rejeb and AbuElhaija 2000; Vrajitoru 2002; Sanchez-Velazco and Bullinaria 2003a, 2003b; Song Goh, Lim, and Rodrigues 2003; Sodsee *et al*. 2008; Yan 2010) use the term 'sex' to determine certain mechanisms anchored in observations of nature and implemented in GAs. In addition, the philosophy of GGA means an artificial and evolutionary optimization mechanism, with strong roots in nature. It can be argued that life is too short to carry out careless optimization of stochastic type, so the authors' advice is to use certified, accumulated knowledge, if possible (Kowalczuk and Białaszewski 2006b).

Disclosure statement

No potential conflict of interest was reported by the authors.

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